LIGHT-GUIDE DESIGN FOR TIMING MEASUREMENTS
IN LARGE COUNTERS

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ABSTRACT

The geometrical factors influencing the time resolution in large counters are examined. Calculations of the number of photons collected in the leading edge of the light pulse are used to compare various geometries for adapting a rectangular counter to a photomultiplier. Restrictions on the sharpness of bending in twisted light-guides are examined. A simple conical adaptor between the end of a twisted guide and the photomultiplier is found to be the best solution.

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1. INTRODUCTION

Counter sizes are determined by their application and cost; photomultiplier sizes by their cost and the requirement of a good rise-time. The freedom left to the designer is mainly in the adaptor between the scintillator and the photomultiplier. It is important to stress from the start that maximum total photo-statistics does not imply optimum time resolution: one must design to obtain the maximum number of photons arriving at the photomultiplier in a time interval equal to the integration time-constant of the photomultiplier, cable, and electronics, say 2 nsec. This paper presents calculations of several factors which affect the light collection. Calculations were made using a general optical-guide simulation program GUIDE 7¹⁾.

2. TWISTED GUIDES

If the counter is wide relative to the photomultiplier diameter, it is usual in the design of high-quality counters to employ twisted-strip light-guides to transform the scintillator's exit area to a square or other shape which may more easily be transformed down to the photomultiplier size. Light losses in such guiding sections are due to the bent shape itself and to ripples which may form in the surface of the guide during the bending process. In the calculation of the light transmission in this section, the integral pulse area over the first 30 nsec of the pulse was used as a measure of the light signal.

Figure 1b shows how the light transmission varies with the radius of bending in the light-guide geometry shown in Fig. 1a which also defines the parameters used. This particular shape was chosen for convenience of computation, but the results are applicable also to smaller bending angles provided that the same bending radius is maintained over a distance which is more than several times the thickness of the strip. For shorter lengths, these results give a lower limit for the transmission. Clearly the smallest value of the bending radius determines the light loss. For example, from Fig. 1b, in order to achieve 95% light transmission, the mean radius of curvature R must be always greater than eight times the strip thickness W.

Shapes of ripples in the surface are more difficult to define so, somewhat arbitrarily, the shape shown in Fig. 2a was chosen as a model to investigate how the transmission varies with the depth of a series of cylindrical grooves across the guide. Figure 2b shows the results, which follow the line

$$T = 1 - 2d/W ,$$

where T is the transmission factor, d is the depth of the grooves, and W is the thickness of the counter.

3. PHOTON DISTRIBUTIONS FROM A RECTANGULAR GUIDE

Assume for the moment that the exit area of a group of twisted guides is equivalent to the exit area of a long square-sectioned cylinder. Consider the phase-space population of photons leaving the guide (Figs. 3a and 3b). It is the region of phase space with transverse velocities v_T below the broken lines, corresponding to light arriving in the first two nanoseconds from a 2 m counter, which is the most useful for timing information. Thus the aim should be to transform all of this section of phase space down onto the photocathode area -- a typical focusing problem. Note that, unless one is making energy or specific ionization measurements, the extra photons just confuse the measurement, and in certain situations they may even decrease the precision of the time measurement. This would be the case when there are large pulse-height fluctuations and one needs to correct for slewing effects. Common practice is to measure the pulse area and assume this to be proportional to the pulse amplitude which is required for the corrections. In this method, there are statistical fluctuations in the ratio of pulse height to pulse area. However, if only the almost prompt photons are detected by the photomultiplier then fluctuations in pulse height are strongly correlated with those in the pulse area, and so a more precise slewing correction is obtained.

4. DISTORTION OF PHOTON DISTRIBUTIONS BY TWISTED GUIDES

Prompt photons from a rectangular guide will enter a twisted guide normal to the entrance face; but in general, during transmission through the twisted guide, this correlation between delay and photon direction will be lost (see Fig. 4) and will complicate the focusing problem. Figure 5 shows a scattergram of the photon's time delay against $\cos \theta$, where θ is the angle between the normal to the exit face of the guide and the emergent photon's direction. The case illustrated here is for an (extreme) t/R ratio of 0.25. If the guide were not bent, then the photons would all be concentrated on the solid line.

That the over-all pulse waveform is deteriorated is shown in Fig. 6a, where the time distribution of all photons is plotted for a 1 m long counter. A pulse rise-time of 0.6 nsec is introduced by the bending and this will be proportional to the counter's length. As the radius of bending is increased the rise-time improves, until for a t/R ratio of 0.125 the distribution already shows the sharp edge characteristic of the time distribution for a simple rectangular guide (Fig. 6b). The broken curve is the function $DI/dt = \text{constant}/t^2$ which is a universal shape for $t\text{-prompt} < t < t\text{-prompt}/\cos \theta_c$, where θ_c is the critical angle. In conclusion, if the guide is designed so as to have a high transmission ($> 95\%$) for the pulse area, then the distortion of the time spectrum will be acceptable.

5. THE COLLECTION OF EARLY PHOTONS FROM A RECTANGULAR GUIDE

Possible ways of collecting just the early photons might be

- a) a lens, with a higher refractive index than perspex, built into the light-guide;
- b) a shaped exit face on the light-guide to focus photons onto the photocathode;
- c) a parabolic-shaped adaptor with the photocathode at its focus;
- d) any simple adaptor which works.

This paper describes several parabolic solutions which are not very good, and a series of conical shapes which are surprisingly good -- an interesting result since they are simpler to make, and medium-sized production series may be manufactured by extrusion. These conical shapes are compared with the Hinterberger and Winston²⁾ type of adaptor.

6. RESULTS OF GEOMETRICAL ACCEPTANCE CALCULATIONS

Figure 7 illustrates the class of shapes which have been investigated. ABCD is a 2 m long counter with a square cross-section. FEHG is a 2.25 cm diameter, 10 cm long cylinder, and ABEF is the adaptor which is conical or is a parabola of rotation. The diagonal of the counter exit face is equal to the diameter of the upper end of the adaptor, and the adaptor is cut away so that the surface ABX is a continuation of DCBA. In the parabolic design, the length of the adaptor was varied so as to move its focus relative to the photocathode. The attenuation has been assumed zero in order to demonstrate the influence of geometry more clearly. As figure of merit was chosen the number of photons which arrive at the photocathode in the first 2 nsec after the first photon arrives, and all data are normalized to 10^4 photons produced isotropically and with a delta-function time distribution at the distant end of the counter.

Identical random number sequences were used for all calculations so that the relative accuracies of the results are better than $(\text{Number of photons accepted})^{1/2}$. In general, relative accuracies are better than 5%, but the fluctuations of points about the smooth curves should be taken as an indication of the accuracy.

Table 1 lists the geometries, conditions, and results which were obtained in a first series of calculations, and the data are presented graphically in ascending order of merit in Fig. 8. These results may be summarized as follows:

- i) a parabolic adaptor is not as good as a conical adaptor for focusing onto the photocathode the 29° cone of photons which arrive in the first 2 nsec;

- ii) an adaptor with the photomultiplier connected directly to its output instead of via a cylinder gives a higher yield;
- iii) there may be an optimum adaptor length.

Photon time distributions, for example of parabolic, conical, and rectangular geometries are compared in Fig. 9. Notice the narrow time distribution, 1.6 nsec FWHM, given by the conical adaptor.

Following these conclusions, the conical adaptor was investigated in more detail. Figure 10 shows the variation in light yield at the photocathode as a function of the length L of various conical adaptors. Curves are given for various sizes of counter and for the cases with and without a coupling cylinder to the photomultiplier. When there is no cylinder, the cone is covered with a 95% reflecting surface, but when there is a cylinder, guiding by total internal reflection is assumed since the first calculations showed no improvement in the flux of early photons when a reflecting surface is applied. The broken curve is the yield calculated for the photons collected, in the first two nanoseconds, from the end of a simple rectangular strip with length equal to [counter length (200 cm) + cylinder length (10 cm) + L]. It is thus the maximum which can be achieved. For each counter area and type of adaptor the yield at the optimum adaptor length has been read off and has been plotted in Fig. 11. The abscissa is now expressed as the width of a 2.5 cm thick counter whose area transforms, for example by twisted strips, to the square guide which is assumed in Fig. 10.

It is interesting to compare these results with the acceptance by an adaptor of the type described by Hinterberger and Winston²⁾. In this type of adaptor its shape, length, and entrance radius are all determined uniquely by the radius of the photocathode and by the maximum angle between the incident photon and the axis of the adaptor. For example, to collect all photons arriving in the first 2 nsec from a 2 m long counter, all angles up to 29° must be accepted. This angular acceptance and a 2.25 cm photomultiplier radius correspond to an adaptor of length 12.4 cm and entrance radius 4.6 cm. Hence the maximum square counter size is $6.55 \times 6.55 \text{ cm}^2$ corresponding to a 17.2 cm wide 2.5 cm thick counter.

The acceptance calculated with this geometry is shown by the square on Fig. 11. It demonstrates that for timing applications in solid (or liquid) counters, a silvered conical adaptor is as efficient as the more complicated Hinterberger and Winston adaptor. Not only is the conical guide simpler to construct, but it fits more easily into the magnetic shielding of the photomultiplier.

7. A NOTE ON ATTENUATION LENGTH MEASUREMENTS

It has been pointed out above that the time distribution of photons detected via a conical light-guide adaptor is narrow. Corresponding to this, there is a narrow peak in the path-length distribution as shown in Fig. 12, and the mean path length relevant in a pulse area measurement is only 10% more than the nominal path length. This should be compared to the mean path length extension for a rectangular counter and guide which is 30% or more, depending on the length of the time gate over which the pulse area is integrated.

8. CONCLUSIONS

It has been shown that conical adaptors on light guides are a convenient and good solution of light guide design for timing measurements (though they are not claimed to be the best design for pulse amplitude measurements). They have the advantage that they select predominantly the early fraction of the photons, they give a sharp time signal for timing measurement, and they introduce a strong correlation between statistical fluctuations in pulse area and pulse height thus aiding correction for slewing effects. The optimum solution is a silvered conical guide connected directly to a photomultiplier, but in many applications, for reasons of mechanical support and magnetic shielding design, the second best solution of a conical adaptor followed by a cylinder may have to be selected. The fractional difference between the photon efficiencies for these two solutions decreases as the counter size is decreased (Fig. 11).

In the cone plus cylinder solution, the time distribution of photons is narrow, and the collection efficiency for photons within the first two nanoseconds varies slowly with distance. (Inspect Fig. 9 and remember that the abscissa will scale linearly with the distance from the light source to the photocathode.) Figure 13 shows the factor F by which the time resolution varies over a 2 m long counter. It is normalized to unity at 2 m distance:

$$F = \left[\frac{\text{Number of photoelectrons at distance 2 m}}{\text{Number of photoelectrons at distance D}} \right]^{\frac{1}{2}}.$$

In practice, this factor will multiply a similar factor for loss in photostatistics by bulk attenuation in the scintillator. For counters less than 1 m long, the effective attenuation length to be used is 0.9 times the true attenuation length, and for very long counters the true and effective attenuation lengths become equal.

To ensure that the quality of the time distribution is preserved, any bends in the light-guide should have a bending radius greater than eight times the counter thickness. This criterion also ensures less than 5% loss of light in the bent section, which may be compared with the 5% loss in light which would result from 0.3 mm amplitude waves in the surface of a 1 cm thick guide.

REFERENCES

- 1) T. Massam, GUIDE 7: A general program for evaluating the properties of scintillation and Čerenkov counter optical systems, CERN Yellow Report, in preparation.
- 2) H. Hinterberger and R. Winston, Rev. Sci. Instrum. 37, 1094 (1966).

Table 1

Light collected from a 2 m long counter in the first 2 nsec.
Counter cross-section = 10 cm × 10 cm

Adaptor shape	Length (cm)	10 cm long extension cylinder	Surface treatment	$\left(\begin{array}{c} \text{Photons} \\ \text{collected} \end{array} \right)$ $\left(10^4 \text{ photons produced} \right)$
Conical	30	Yes	Polished	245
"	15	"	"	233
"	30	"	95% reflecting	222
"	15	"	"	228
"	30	No	"	324
"	15	"	"	337
"	11.3	"	"	359
"	7.5	"	"	274
"	3.75	"	"	137
Parabolic	22.5	Yes	Polished	80
"	18.0	"	"	80
"	13.5	"	"	99
"	10.0	"	"	100
"	4.5	"	"	120
"	10.0	"	95% reflecting	108
"	22.5	No	"	99
"	18.0	"	"	105
"	13.5	"	"	108
"	10.0	"	"	108
"	4.5	"	"	127
Total from a rectangular counter without adaptor				575
$575 \times \text{Area factor} = \frac{575 \times \pi \times 2.25^2}{100}$				= 99

Figure captions

- Fig. 1 : a) Showing the geometry used in the calculation of transmission by a twisted guide. The rectangular counter and the twisted guide are both 20 cm wide. The thickness W was 1 cm. The radius of curvature R of the median surface in the guide was varied.
- b) Showing how the transmission of the light guide in Fig. 1a varies with the bending radius. The transmission is normalized to the light intensity calculated for direct coupling of the counter to the final cylinder.
- Fig. 2 : a) The geometry used to calculate the effect of surface ripples. Five equally spaced cylindrical grooves were assumed. Their widths and spacing were as indicated and were kept fixed; only the depth d was varied.
- b) Showing how the transmission of the guide shown in Fig. 2a varies with d , the depth of the grooves. The transmission is normalized to the light intensity calculated with no grooves.
- Fig. 3 : The phase-space population of photons leaving a rectangular guide; v_T is the component of photon velocity perpendicular to the axis of the counter, and R is the displacement relative to this axis; nc is the photon's velocity.
- a) The phase-space diagram with some indication of the density distribution.
- b) The population density of the phase space. For a long counter, the density dI/dv_T is independent of R . Below $v_T/nc = 0.7$, the density is $v_T/(n^2c^2 - v_T^2)^{1/2}$. Above this value of v_T , the density falls to zero in a geometry-dependent way which has not been calculated explicitly.
- Fig. 4 : The loss in angle-time correlation in a twisted guide.
- Fig. 5 : Scattergram showing the correlation between the photon direction θ (relative to the counter axis) and the time delay. The solid line corresponds to the strict correlation which exists when photons are transmitted in a rectangular counter. The points are simulated photons which have propagated along a 1 m long counter and then through a twisted guide with a bending radius equal to four times its thickness.

- Fig. 6 : a) The over-all pulse waveform corresponding to a 1 m long counter followed by a twisted guide with t/R ratio 0.25 (as in Fig. 5). The curve is drawn by hand.
- b) The over-all pulse waveform for the same counter with a t/R ratio of 0.125. The broken curve is the shape expected for a straight guide.

Fig. 7 : The class of shapes which have been investigated (described in the text).

Fig. 8 : Comparison of various trial geometries. F is the number of photons collected in the first 2 nsec divided by the number collected in the first 2 nsec from a simple rectangular guide.

Fig. 9 : Examples of time distributions for various geometries. The time delay is the time relative to the earliest possible photon. All curves are normalized to the same initial light intensity. The counter is 2 m long $\times$ 7.07 cm $\times$ 7.07 cm.

- a) The output of the 2 m long section.
- b) The output after coupling to a 4.5 cm diameter phototube via a 15 cm long silvered cone.
- c) The output after coupling via a 15 cm long cone and a 10 cm long cylinder, reflection being by total internal reflection.
- d) The output after coupling via a 4.5 cm long silvered parabolic adaptor.

Fig. 10 : The variation in light yield at the photocathode as a function of adaptor length. Curves are given for square counters of various widths W , for various surface properties, and with the final connecting cylinder present or absent.

Curve	Counter width	Surface finish	Final cylinder
(a)	5 cm	95% reflecting	No
(b)	5 cm	Polished	Yes
(c)	7.07 cm	95% reflecting	No
(d)	8.66 cm	95% reflecting	No
(e)	7.07 cm	Polished	Yes
(f)	10 cm	95% reflecting	No
(g)	8.66 cm	Polished	Yes
(h)	7.07 cm	Polished	Yes

The broken curve is the transmission of a rectangular adaptor with identical cross-section to that of the counter.

Fig. 11 : Photon yields at optimum adaptor length as a function of counter width for two types of adaptor.

- a) A silvered conical adaptor (95% reflection coefficient).
- b) A polished conical adaptor together with a 10 cm long polished cylinder to connect it to the photocathode.

Fig. 12 : The photon path-length distribution for a 1 m long counter, with 100 cm^2 cross-section area, coupled to a 4.5 cm diameter photomultiplier by a 15 cm long polished cone and a 10 cm long polished cylinder.

Fig. 13 : The factor F for the variation in resolution due to the fact that the number of photoelectrons collected in the first 2 nsec varies with the distance between the source and the photomultiplier.

Fig. 1

a)

Distance along the counter (cm)

Light produced
at 0 cm

Light collected
at 90 cm

Transmission Factor for
a 1 cm thick counter

b)

Fig. 2

(a)

(b)

Fig. 3

Prompt photons
entering a
twisted guide

Prompt photons
leaving the guide

Fig. 4

Fig. 5

Photons / 0.2 nsec / 3000 generated

a)

Photons / 0.2 nsec / 1800 generated

b)

Fig. 6

Fig. 7

Fig. 8

Fig. 9

No of photons in first 2 nsec
for 10^4 generated.

Fig. 10

Fig. 11

Fig. 12

F Resolution factor

Fig. 13