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ELECTRON-POSITRON ANNIHILATION INTO TWO AND THREE PHOTONS AROUND THE Ψ F.A. Berends^{*)} and G.J. Komen^{*)}

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A B S T R A C T

We study the effect of the direct decay of a Ψ into three photons on electron-positron annihilation into two and three photons. In a charmonium - like bound state model we find a clear effect in $e^+e^- \rightarrow 3\gamma$ and a somewhat less pronounced effect in $e^+e^- \rightarrow 2\gamma$, the actual size depending strongly on the charge of the Ψ constituents. The effect is reduced when we take initial state radiative corrections into account.

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1. - INTRODUCTION

In this paper, the reaction

$$e^+e^- \rightarrow 3\gamma \quad (1)$$

is studied in presence of the ψ . The pure QED cross-section has to be modified in order to include production of a ψ which then decays directly into three photons. This modification has been studied in the literature ¹⁾, where it is found that the cross-section on top of the ψ is about five times the QED value.

We want to improve on the results of Ref. 1) in two respects. Firstly, we present analytic expressions for the differential cross-section and, secondly, we apply initial state radiative corrections to the shape of the resonance. We were motivated to make these improvements because the reaction (1) is relevant in two types of experiments. On the one hand, the reaction

$$e^+e^- \rightarrow 2\gamma \quad (2)$$

has been studied near the resonance ²⁾⁻⁴⁾. In the latest experiment the errors have decreased with respect to earlier experiments. The radiative corrections to reaction (2) contain contributions from reaction (1). Depending on the size of $\Gamma(\psi \rightarrow 3\gamma)$ these radiative corrections could induce a structure around the ψ . A comparison of the measured data of reaction (2) can in principle give a limit on this width. Assuming a non-relativistic description of the ψ as a bound state of a spin $\frac{1}{2}$ particle and its anti-particle, we can obtain information on the charge of these constituents. In practice, such a program is hampered because in actual experiments the energy spread of the beam will wash out narrow structures, unless they are very large, like, e.g., the ψ itself.

On the other hand, reaction (1) is important in a study of the decay

$$\psi \rightarrow 3\gamma. \quad (3)$$

In this decay one searched for the η_c state into which the ψ is supposed to decay under emission of a photon ^{5),6)}. In a calculation of the background of events for reaction (3) until now only the pure QED and the decay via η and η' have been taken into account. We will find that, in addition, the reaction $e^+e^- \rightarrow \psi \rightarrow 3\gamma$ is important on the top of the ψ . However, as in the 2γ case, the beam width will reduce the relevance of this effect in a realistic background calculation.

The contents of the paper are arranged as follows. In Section 2 we study the coupling of the ψ to three photons in a bound state model. In Section 3, we apply this to calculate the differential cross-section for reaction (1), considering also initial state radiation. In the last section we will give our final results on reactions (1) and (2). The analytic expressions we use to obtain these results are given in the Appendices.

2. - THE ψ - 3γ COUPLING

In this section we will discuss the decay of a ψ into three photons (Fig. 1). In order to avoid complications due to colour and strong interactions we first consider "muonium" P , whereby we mean in this case a bound state of a μ and an $\bar{\mu}$. Unlike positronium, muonium not only decays into three photons but also into one (virtual) photon, i.e., into e^+e^- . Although it is not possible to find a relativistic pointlike coupling that describes the decay $P \rightarrow 3\gamma$, it is straightforward to obtain an effective non-local relativistic Lagrangian for this coupling (Fig. 2). We therefore consider the most general interaction between a P and 3γ 's with the smallest number of derivatives possible. We then fix the strength by a comparison with the same matrix element in a bound state model. This comparison is necessarily made in the non-relativistic limit.

In the bound state model ⁷⁾ the decay amplitude is given by

$$\mathcal{T}[P \rightarrow X] = 2\sqrt{M} \phi(0) \mathcal{M}(\mu\bar{\mu} \rightarrow X), \quad (4)$$

where $\mathcal{M}$ denotes the non-relativistic limit of the matrix element for $\mu\bar{\mu} \rightarrow X$, in which $\bar{v}_{\gamma\mu}$ is replaced by the polarization vector ϵ_{μ} of the spin 1 state of the system P . Further $\phi(0)$ denotes the muonium wave

function in the origin, M is the muonium mass, while for X the 3γ state or the virtual single γ state (i.e., e^+e^-) will be taken. In the former case six diagrams contribute to $\mathcal{M}$, one of which is depicted in Fig. 3, while the others are obtained from a permutation of the photon labels. We have

$$\mathcal{M}(\mu\bar{\mu} \rightarrow 3\gamma) = \mathcal{M}(1,2,3) + \text{all permutations of } 1,2,3,$$

$$\mathcal{M}(1,2,3) = -ie^3 \bar{v}(p_+) \frac{[-2(p_+ \epsilon_1) + \not{\epsilon}_1 \not{\epsilon}_1] \not{\epsilon}_2 [2(p_- \epsilon_3) - \not{\epsilon}_3 \not{\epsilon}_3]}{4(p_+ q_1)(p_- q_3)} u(p_-),$$

$$(p_+ \epsilon_1) = p_{+0} \epsilon_{10} - \vec{p}_+ \cdot \vec{\epsilon}_1. \quad (5)$$

This leads in the non-relativistic limit to the following decay amplitude

$$\begin{aligned} T(P \rightarrow 3\gamma) = & -\frac{4ie^3 \sqrt{M} \phi(0)}{M^2} * \\ & * \left\{ [\vec{\epsilon}_1 \cdot \vec{\epsilon}_1 (1 - \hat{q}_1 \cdot \hat{q}_2) + (\vec{\epsilon}_1 \cdot \hat{q}_2)(\vec{\epsilon}_2 \cdot \hat{q}_1)] \vec{\epsilon}_3 \cdot \vec{\epsilon} + \right. \\ & - [\vec{\epsilon}_1 \cdot (\hat{q}_2 \wedge \vec{\epsilon}_2) + \vec{\epsilon}_2 \cdot (\hat{q}_1 \wedge \vec{\epsilon}_1)] (\vec{\epsilon}_3 \wedge \hat{q}_3) \cdot \vec{\epsilon} + \\ & \left. + \text{cyclic permutations of } 1, 2 \text{ and } 3 \right\}, \\ \text{where } \hat{q}_i = & \vec{q}_i / q_{i0}. \end{aligned} \quad (6)$$

We now construct an effective coupling between the ψ and three photons. There are three couplings with four derivatives only

$$\begin{aligned} \partial_\alpha \psi_\beta F^{\alpha\beta} F_{\mu\nu} F^{\mu\nu}, & \quad F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha, \\ \partial_\alpha \psi_\beta F^{\alpha\mu} F^{\beta\nu} F_{\mu\nu}, & \\ \partial_\alpha \psi_\beta \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} F_{\gamma\delta} F^{\mu\nu} F^{\rho\sigma}. & \end{aligned} \quad (7)$$

These are not independent. It turns out that it is most simple to consider a combination of the first and third couplings. It is straightforward to verify that

$$\mathcal{L}_I = \frac{e^3 \phi(0) \sqrt{M}}{(P_{q_1})(P_{q_2})(P_{q_3})} \partial_\alpha \psi_\beta \left[F^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} F_{\gamma\delta} F^{\mu\nu} F^{\rho\sigma} \right] \quad (8)$$

leads, in the non-relativistic limit, to the matrix element of Eq. (6). From (8) we have the effective coupling for the vertex of Fig. 2

$$ie B_{\beta\mu_1\mu_2\mu_3}, \quad c = \frac{-2e^3 \phi(0) \sqrt{M}}{(P_{q_1})(P_{q_2})(P_{q_3})}, \quad (9)$$

with

$$B_{\beta\mu_1\mu_2\mu_3} = \left\{ (P_{q_3}) g_{\beta\mu_3} - q_{3\beta} P_{\mu_3} \right\} \left\{ (q_1, q_2) g_{\mu_1\mu_2} - q_{1\mu_2} q_{2\mu_1} \right\} + \epsilon(q_2, q_1, \mu_1, \mu_2) \epsilon(P, q_3, \beta, \mu_3), \quad (10)$$

where

$$\epsilon(q_2, q_1, \mu_2, \mu_1) = \epsilon_{\alpha\beta\mu_2\mu_1} q_2^\alpha q_1^\beta, \quad \text{etc.}$$

We can now calculate the width $\Gamma(P \rightarrow 3\gamma)$. In fact we have

$$d\Gamma = \frac{1}{6} \frac{1}{2M} \frac{1}{(2\pi)^5} \frac{d\vec{q}_1 d\vec{q}_2 d\vec{q}_3}{8 q_{10} q_{20} q_{30}} \delta(P - q_1 - q_2 - q_3) \frac{1}{3} \sum |m|^2, \quad (11)$$

where

$$\sum |m|^2 = \frac{4 M \phi^2(0) 32 e^6}{M^4} \left\{ (1 - \cos \Theta_{12})^2 + (1 - \cos \Theta_{23})^2 + (1 - \cos \Theta_{31})^2 \right\}, \quad (12)$$

giving

$$\Gamma(P \rightarrow 3\gamma) = \frac{64}{9} \frac{\alpha^3 \phi^2(0)}{M^2} (\pi^2 - 9). \quad (13)$$

The wave function is given in the literature by

$$\phi = \left\{ \frac{\alpha^2 m_\mu^3}{8\pi} \right\}^{1/2}, \quad (14)$$

hence

$$\Gamma(3\gamma) = \frac{2}{9} \alpha^5 (\pi^2 - 9) m_\mu. \quad (15)$$

In a completely similar way one can take, for the decay $P \rightarrow e^+ e^-$, the effective interaction

$$\mathcal{L}_I = -g \Psi_\alpha \bar{\mu} \gamma^\alpha \mu. \quad (16)$$

Comparison with the bound state description (Fig. 4) gives for the vertex of Fig. 5

$$-ig \epsilon_\mu \bar{u} \gamma^\mu v, \quad g = \frac{-2\sqrt{M} \phi(0) e^2}{M^2} \quad (17)$$

This gives a width

$$\Gamma(e^+e^-) = \frac{M g^2}{12\pi} = \frac{16\pi}{3} \frac{\alpha^2}{M^2} \phi^2(0). \quad (18)$$

We now apply these results for a description of the ψ decays. First of all, ϕ is then determined by strong interactions and is no longer given by (14). We will determine $\phi(0)$ from the leptonic width. If the ψ were just a bound state of a unit charged spin $\frac{1}{2}$ fermion and its anti-particle (Case I) we could use the above formulae with

Case I

$$\begin{aligned} \phi(0) &= \frac{1}{4} \sqrt{3} \frac{M}{\alpha} \sqrt{\frac{\Gamma}{\pi}}, \\ c &= \frac{-2e^3 \phi(0) \sqrt{M}}{(P_{q_1})(P_{q_2})(P_{q_3})}. \end{aligned} \quad (19)$$

Taking the charmonium model with three colours and charge $2/3$ for the charmed quark, we have

$$\Gamma(e^+e^-) = \frac{M g^2}{12\pi} = \frac{64}{9} \frac{\pi \alpha^2 \phi^2(0)}{M^2}, \quad (20)$$

so

Case II

$$\phi(0) = \frac{3}{8} \frac{M}{\alpha} \sqrt{\frac{\Gamma'}{\pi}},$$

$$c = \frac{-2\sqrt{3} (\frac{2}{3}e)^3 \phi(0) \sqrt{M}}{(P_{q_1})(P_{q_2})(P_{q_3})}. \quad (21)$$

Taking $\Gamma(e^+e^-) = 4.8 \text{ keV}$, we find

Case I

$$\Gamma(3\gamma) = \frac{4}{3} \alpha \frac{\pi^2 - 9}{\pi} \Gamma_{ee} = 12.9 \text{ eV},$$

Case II

$$= \frac{64}{243} \alpha \frac{\pi^2 - 9}{\pi} \Gamma_{ee} = 2.5 \text{ eV}. \quad (22)$$

3. - THE DIFFERENTIAL CROSS-SECTION FOR $e^+e^- \rightarrow 3\gamma$

We obtain the multi-differential cross-section for the reaction

$$e^+(p^+) + e^-(p^-) \rightarrow \gamma(q_1) + \gamma(q_2) + \gamma(q_3) \quad (23)$$

from the expression

$$d^3\sigma = \frac{1}{(2\pi)^5} \delta(p_+ + p_- - q_1 - q_2 - q_3) \frac{m^2}{[(p_+ p_-) - m^2]^{\frac{1}{2}}} \frac{e^6}{32} \sum_{\text{spins}} |M|^2 \frac{d\vec{q}_1 d\vec{q}_2 d\vec{q}_3}{q_{10} q_{20} q_{30}}. \quad (24)$$

There are two types of contributions to $\mathcal{M}$

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 \quad (25)$$

coming from QED (Fig. 3) and from the resonant diagram of Fig. 6. In fact we have

$$M_1 = -\frac{1}{4}i \left\{ \bar{v}(p_+) O_{\mu_1 \mu_2 \mu_3} u(p_-) + \text{permutations of } 1, 2 \text{ and } 3 \right\} \varepsilon_{\mu_1}^1 \varepsilon_{\mu_2}^2 \varepsilon_{\mu_3}^3,$$

$$O_{\mu_1 \mu_2 \mu_3} = \frac{\gamma_{\mu_1} (-\not{p}_+ + \not{p}_1 + m) \gamma_{\mu_2} (\not{p}_- - \not{p}_3 + m) \gamma_{\mu_3}}{(p_+ q_1)(p_- q_3)}, \quad (26)$$

and

$$\begin{aligned} M_2 = & -ig \frac{c}{e^3} \bar{v}(p_+) \gamma_\alpha u(p_-) \frac{1}{s - M^2 + iM\Gamma} + \\ & + \left\{ [(Pq_3) \varepsilon_\alpha^3 - q_\alpha^3 (Pe_3)] [(q_1 q_2)(e_1 e_2) - (q_1 e_2)(q_2 e_1)] \right. \\ & \left. + \varepsilon(q_2, q_1, e_2, e_1) \varepsilon(P, q_3, \alpha, e_3) + \text{permutations of } 1, 2 \text{ and } 3 \right\}. \end{aligned} \quad (27)$$

The quantity $\Sigma |\mathcal{M}|^2$ will contain three terms : a QED, interference and resonant term :

$$X = m^4 \Sigma |\mathcal{M}|^2 = X_{\text{QED}} + X_{\text{INT}} + X_{\text{RES}}. \quad (28)$$

The QED term can be found, e.g., in Ref. 8), where it is obtained from crossing of the expression given by Mandl and Skyrme⁹⁾ for double Compton scattering. For X_{int} we find

$$\begin{aligned} X_{\text{INT}} = & -\frac{1}{8} m^2 g \frac{c}{e^3} \text{Re} \frac{1}{s - M^2 + iM\Gamma} \left[\text{Tr}(\not{p}_+ - m) O_{\mu_1 \mu_2 \mu_3} (\not{p}_- + m) \gamma_\mu \right. \\ & \left. + \text{permutations} \right] (B_{\mu\mu\mu\mu\mu_1\mu_2\mu_3} + \text{permutations})]. \end{aligned} \quad (29)$$

The expression in square brackets is evaluated with the help of Veltman's algebraic program SCHOONSCHIP¹⁰⁾. The result is given in Appendix A. Similarly, we find

$$X_{RES} = -\frac{1}{4} m^2 g^2 \frac{c^2}{e^6} \frac{1}{(s-M^2)^2 + M^2 \Gamma^2} \left[\text{Tr} (\not{p}_+ - m) \gamma_\alpha (\not{p}_- + m) \gamma_\beta \cdot \right. \\ \left. \cdot (B_{\alpha \mu_1 \mu_2 \mu_3} + \text{permutations}) (B_{\beta \mu_1 \mu_2 \mu_3} + \text{permutations}) \right] \quad (30)$$

The expression in square brackets is given in Appendix B.

The cross-section for $e^+e^- \rightarrow 2\gamma$ as evaluated from Eq. (28) shows a rapid variation near the ψ . It is known that for a narrow resonance like the ψ , the observed structure is strongly affected by the initial state radiation. Incorporating this effect means a modification of X as follows

$$X_{COR} = X_{QED} + F_{INT} X_{INT} + F_{RES} X_{RES}, \quad (31)$$

$$F_{INT} = A \left(1 - \frac{1}{2} \beta \frac{\Gamma}{W-M} \phi \right), \quad (32)$$

$$F_{RES} = A \left(1 + 2 \beta \frac{W-M}{\Gamma} \phi \right), \quad (33)$$

$$A = \left[\frac{(W-M)^2 + \Gamma^2}{E^2} \right]^{\beta/2}, \quad (34)$$

$$W = P_0 = \sqrt{s} = 2E, \quad \beta = -\frac{2\alpha}{\pi} \left\{ 1 - \log \left(\frac{s}{m^2} \right) \right\}, \quad (35)$$

$$\tan \phi = \frac{-\frac{1}{2} \Gamma}{W-M}. \quad (36)$$

Here we took the results of Ref. 11) to which we refer for a derivation.

4. - RESULTS

To determine the effect of reaction (1) through the radiative corrections on reaction (2), we proceed as in Ref. 8). There, two contributions to the radiative corrections are distinguished : δ_A which is analytically calculated, and δ_H which requires a numerical evaluation. The first contribution contains all virtual corrections as well as the 3γ QED contribution in the soft photon approximation integrated over part of the experimental phase space. The phase space is generally defined by the experimental apparatus and the event selection criterion. The criterion used here is :

- i) the two detected photons have tracks which make angles less than some prescribed maximum acollinearity ζ ;
- ii) the two detected photons have energies larger than a certain threshold energy, E_{th} , required to trigger the detection system.

This phase space consists of an isotropic region and an anisotropic region. The size of the isotropic region is for not too high threshold energies determined by the maximum acollinearity angle ζ only. Since δ_A is by definition calculated from the isotropic phase space, it will depend on the beam energy E , the scattering angle θ and the maximum acollinearity ζ , but not on E_{th} .

The term δ_H contains hard photon contributions from the anisotropic phase space as well as the non-soft contribution from the isotropic phase space. For details we refer to Ref. 8). Results for QED at the position of the resonance are given in Table 1. We consider $\zeta = 20^\circ$ and two different values of E_{th} . The presence of the ψ will change the value of δ_H . The size of this effect is given in Table 2. For the case where ψ is taken as a bound state of two coloured quarks of charge $2/3$, the effect is seen to be small (Case II). For the case where the ψ is a bound state of two integer charged spin $\frac{1}{2}$ fermions (Case I), there is a sizeable effect. In both cases we took the influence of the initial state radiation on $e^+e^- \rightarrow 3\gamma$ into account, i.e., we used Eq. (31). Unfortunately, in actual experiments the beam width makes it very hard to detect these effects, so that it is not possible to exclude integer charged constituents of the ψ on the basis of the recent data ⁴⁾, which show no structure around the ψ .

We now turn to the influence of the ψ on the reaction $e^+e^- \rightarrow 3\gamma$. To calculate a total cross-section for this reaction we will evaluate the following integral.

$$\sigma = \frac{1}{6} \int \frac{\partial^5 \sigma}{\partial \Omega_1 \partial \Omega_2 \partial q_{10}} d\rho. \quad (37)$$

Taking for the region ρ $45^\circ < \theta_i < 135^\circ$ and $q_{i0} > E_{th}$ ($i=1,2,3$), for which we take 20 MeV, we can rewrite this as

$$\sigma = \frac{\alpha^3}{8\pi^3 m^2 s} \frac{2\pi}{6} \cdot \int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} d\cos\theta_1 \int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} d\cos\theta_2 \int_0^{2\pi} d\phi_2 \int_R dq_{20} \frac{q_{10} q_{20}}{2E - q_{20}(1 - \cos\delta)} X, \quad (38)$$

with X given by Eq. (31), unless $|\cos\theta_3| > \frac{1}{2}\sqrt{2}$ where we set $X=0$. The region R is determined from the condition that $q_i > E_{th}$. We denoted the angle between $\vec{q}_1$ and $\vec{q}_2$ by δ . For the photon energies we find

$$q_{10} = E \frac{E - q_{20}}{E - \frac{1}{2} q_{20} (1 - \cos\delta)}, \quad (39)$$

$$q_{30} = 2E - q_{10} - q_{20} \quad (2E = P_0 = \sqrt{s}). \quad (40)$$

From this R follows :

$$R \left\{ \begin{array}{l} a) \quad E_{th} < q_{20} < E \frac{1-E}{1-\frac{1}{2}E\Delta}, \quad \text{if } \Delta < \Delta_c \\ b) \quad E_{th} < q_{20} < E(1-\frac{1}{2}E) \left\{ 1 - \left[1 - \frac{\Delta_c}{\Delta} \right]^{1/2} \right\}, \\ \quad E(1-\frac{1}{2}E) \left\{ 1 + \left[1 - \frac{\Delta_c}{\Delta} \right]^{1/2} \right\} < q_{20} < E \frac{1-E}{1-\frac{1}{2}E\Delta}, \quad \text{if } \Delta > \Delta_c, \end{array} \right. \quad (41)$$

with

$$\Delta = 1 - \cos\delta, \quad \Delta_c = 2 \frac{1-E}{(1-\frac{1}{2}E)^2}, \quad \varepsilon = \frac{E_{th}}{E}. \quad (42)$$

With this phase space we obtain a QED cross-section of 0.43 ± 0.01 nb at the ψ . Using the full X of Eq. (28) we obtain without radiative corrections $\sigma(e^+e^- \rightarrow 3\gamma) = 1.93 \pm 0.05$ nb. This, however, is certainly an overestimate of the effect, since it is known that for narrow resonances bremsstrahlung from the initial state has a dramatic effect. Using the correct expression [Eq. (31)] we obtain the result of Table 3. Although the effect of the ψ is clearly there, experimental detection is as in the 2γ situation, hindered because of the beam spread in the present experiments.

APPENDIX A

We define

$$S_{\mu}(1,2,3) = \text{Tr}(\not{p}_+ - m) \frac{\not{\epsilon}_{\mu_1}(\not{p}_+ + \not{q}_1 + m) \not{\epsilon}_{\mu_2}(\not{p}_- - \not{q}_3 + m) \not{\epsilon}_{\mu_3}(\not{p}_- + m) \not{\epsilon}_{\mu}}{(p_+ q_1)(p_- q_3)} \quad (\text{A.1})$$

and

$$t_{\mu}(1,2,3) = [(q_1 q_3) g_{\mu_1 \mu_2} - q_{1\mu_1} q_{2\mu_2}] [(P q_1) g_{\mu_2 \mu_3} - P_{\mu_2} q_{3\mu_3}] + \epsilon(q_2 q_1, \mu_2, \mu_1) \epsilon(P, q_3, \mu_3, \mu_2) \cdot \quad (\text{A.2})$$

Defining

$$\begin{aligned} F(p_+, p_-, 1, 2, 3) &= S(p_+, p_-, 1, 2, 3) t(p_+, p_-, 3, 2, 1) \\ G(p_+, p_-, 1, 2, 3) &= S(p_+, p_-, 2, 1, 3) t(p_+, p_-, 3, 2, 1) \end{aligned} \quad (\text{A.3})$$

the expression in the square brackets of Eq. (29) becomes

$$[] = 2 \sum_{\substack{\text{permutations} \\ 1, 2, 3}} \left\{ F(p_+, p_-, 1, 2, 3) + F(p_-, p_+, 1, 2, 3) + G(p_+, p_-, 1, 2, 3) \right\} \quad (\text{A.4})$$

We have the following results for F and G :

$$\begin{aligned} (p_+ q_1)(p_- q_3) F(p_+, p_-, 1, 2, 3) &= \\ &= 16 \left\{ 2(q_1 q_3)(p_+ q_1)(p_+ q_3)(p_- q_1) - 2(q_1 q_3)(p_+ q_1)^2(p_- q_3) + \right. \\ &\quad + (p_+ q_1)(p_+ q_2)(p_- q_1)(p_- q_3) - 2(p_+ q_1)(p_+ q_3)(p_- q_1)(p_- q_2) + \\ &\quad \left. + (p_+ q_1)^2(p_- q_2)(p_- q_3) \right\} + \\ &\quad + 8S \left\{ -(q_1 q_2)(p_+ q_1)(p_- q_3) - 2(q_1 q_2)(q_2 q_3)(p_+ q_1) + \right. \end{aligned}$$

$$\begin{aligned}
 & + 2(q_1 q_3)(p+q_1)(p-q_2) - (p+q_1)(p-q_2)(p-q_3) - (p+q_2)(p-q_1)(p-q_3) \} + \\
 & + 4s^2 (q_1 q_2)(p-q_3) + \\
 & + 16m^2 \{ (q_1 q_2)(p+q_1)(p-q_3) - 2(q_1 q_3)(q_2 q_3)(p-q_1) - (q_1 q_3)(p+q_1)(p+q_2) \\
 & + (q_1 q_3)(p-q_1)(p-q_2) + 2(q_2 q_3)(p+q_1)(p+q_3) - 2(q_2 q_3)(p+q_1)(p-q_1) \\
 & - (q_2 q_3)(p+q_1)^2 + 2(q_2 q_3)(p+q_3)(p-q_1) - (q_2 q_3)(p-q_1)^2 \\
 & - (p+q_1)(p+q_3)(p-q_2) + (p+q_1)(p-q_2)(p-q_3) - (p+q_3)(p-q_1)(p-q_2) \\
 & + (p-q_1)(p-q_2)(p-q_3) \} + \\
 & + 8m^2 s \{ 2(q_2 q_3)(p+q_1) - 2(q_1 q_3)(q_2 q_3) + (q_1 q_3)(p-q_2) \\
 & - (q_1 q_2)(p-q_3) \} \\
 & - 32m^4 (q_2 q_3)(p-q_1) ,
 \end{aligned}$$

(A.5)

$$\begin{aligned}
 & (p_+ q_2)(p_- q_3) G(p_+, p_-, 1, 2, 3) = \\
 & = 32 \left\{ - (p_+ q_1)(p_+ q_2)(p_+ q_3)(p_- q_3) + (p_+ q_1)(p_+ q_3)^2(p_- q_2) \right. \\
 & \quad \left. - (p_+ q_2)(p_- q_1)(p_- q_2)(p_- q_3) + (p_+ q_3)(p_- q_1)(p_- q_2)^2 \right\} + \\
 & \quad - 16s \left\{ (q_2 q_3)(p_+ q_1)(p_+ q_3) + (q_2 q_3)(p_- q_1)(p_- q_2) + \right. \\
 & \quad \left. + (p_+ q_1)(p_+ q_3)(p_- q_2) + (p_+ q_3)(p_- q_1)(p_- q_2) \right\} + \\
 & \quad + 8s^2 (q_2 q_3) (P q_1) + \\
 & \quad + 16m^2 \left\{ - (q_1 q_2)(p_- q_2) (P q_3) + (q_1 q_2)(p_+ q_3) (P q_3) \right. \\
 & \quad \left. - (q_1 q_3)(p_+ q_3) (P q_2) + (q_1 q_3)(p_- q_2) (P q_2) + \right. \\
 & \quad \left. + (q_2 q_3)(p_+ q_1) (P q_3) + (q_2 q_3)(p_- q_1) (P q_2) - 2(p_+ q_3)(p_- q_2) (P q_1) \right\} \\
 & \quad + 8m^2 s (q_2 q_3) \left\{ (q_1 q_2) + (q_1 q_3) \right\} + \\
 & \quad - 32m^4 (q_2 q_3) (P q_1) .
 \end{aligned}$$

(A.6)

APPENDIX B

We can safely neglect the lepton mass in the calculation of the expression inside the square bracket of Eq. (30). We obtain

$$\begin{aligned}
 [] = & -8 \left\{ 6(q_1 q_2) (P_{q_1}) (P_{q_2}) (Q_{q_3})^2 - 12(q_1 q_2) (P_{q_1}) (Q_{q_2}) (P_{q_3}) \right. \\
 & \cdot (Q_{q_3}) + 6(q_1 q_2) (Q_{q_1}) (Q_{q_2}) (P_{q_3})^2 + \\
 & + s \left[- (q_1 q_2)^2 (Q_{q_3})^2 + 5(q_1 q_2)^2 (P_{q_3})^2 + \right. \\
 & \left. - 6(q_1 q_2) (q_1 q_3) (P_{q_2}) (P_{q_3}) + 6(q_1 q_2) (q_1 q_3) (Q_{q_2}) (Q_{q_3}) \right] \\
 & \left. + s^2 \left[2(q_1 q_2) (q_1 q_3) (q_2 q_3) \right] + \text{permutations of } \right. \\
 & \left. 1, 2 \text{ and } 3 \right\},
 \end{aligned}
 \tag{B.1}$$

where we defined

$$Q = P_+ - P_- \quad . \tag{B.2}$$

	$E_{th} = 100 \text{ MeV}$		$E_{th} = 600 \text{ MeV}$	
	45°	90°	45°	90°
$\sigma_o(\text{nb})$	6.49	2.16	6.49	2.16
$\delta_A(\%)$	-2.9	-2.9	-2.9	-2.9
$\delta_H(\%)$	3.3	0.9	2.9	0.8
$\delta_{tot}(\%)$	0.3	-2.0	0.0	-2.1

TABLE 1 QED results for the reaction $e^+e^- \rightarrow 2\gamma$ at the $\psi(\sqrt{s} = 3095 \text{ MeV})$. We took $\zeta = 20^\circ$, $\Gamma_{ee} = 4.8 \text{ keV}$, $\Gamma_{tot} = 69 \text{ keV}$.

$\sqrt{s} - M_\psi$	Case II				Case I
	$E_{th} = 100 \text{ MeV}$		$E_{th} = 600 \text{ MeV}$		$E_{th} = 600 \text{ MeV}$
	$\theta = 45^\circ$	90°	45°	90°	90°
0.1	3.3	1.3	3.0	1.1	4.2
0.05	3.8	2.4	3.4	2.1	10
0.0	5.8	6.7	5.2	6.1	27
-0.05	4.6	3.7	4.2	3.3	11
-0.1	3.9	2.2	3.6	2.0	5.1
QED	3.3	0.9	2.9	0.8	0.8

TABLE 2 Part of the radiative corrections (δ_H) to the reaction $e^+e^- \rightarrow 2\gamma$ as a function of $\sqrt{s} - M_\psi$, for various angles and threshold energies. Case II corresponds to the standard coloured charmonium picture, Case I to the situation where the ψ is a bound state of two integer charged fermions.

	$\sqrt{s} - M_\psi$			
	QED	-0.1	0	0.1
$\sigma \text{ (nb)}$	0.43	0.57	1.13	0.48

TABLE 3 $\sigma(e^+e^- \rightarrow 3\gamma)$ as a function of
 $\sqrt{s} - M_\psi$ (Case II).

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FIGURE CAPTIONS

- Figure 1 $\psi \rightarrow 3\gamma$ in the charmonium model.
- Figure 2 $\psi \rightarrow 3\gamma$ effective coupling.
- Figure 3 $e^+e^- \rightarrow 3\gamma$.
- Figure 4 $\psi \rightarrow e^+e^-$ in the charmonium model.
- Figure 5 $\psi \rightarrow e^+e^-$ effective coupling
- Figure 6 $e^+e^- \rightarrow 3\gamma$ through a resonating ψ state.

FIG.1

FIG.2

FIG.3

FIG.4

FIG. 5

FIG.6