RENORMALIZATION OF A HIGHER CONSERVATION LAW
IN THE MASSIVE THIRRING MODEL

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A B S T R A C T

The renormalization of a classically conserved current in the massive Thirring model with dimension and tensor structure higher than those of the energy momentum tensor is carried out. It is shown that the possible anomalies arising from quantum corrections can be absorbed multiplicatively.

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1. - In the variety of classical field theories in two-dimensional space time, the Sine-Gordon theory and the massive Thirring model play a special rôle : both of them have an infinite number of independent non-trivial conserved currents ¹⁾.

In the case of the Sine-Gordon theory the existence of these currents is neatly connected with the integrability of the system ²⁾. For the massive Thirring model such a connection has not yet been established. However, a Bäcklund transformation associated with the set of conserved currents has been found ³⁾.

It has been observed by Polyakov ⁴⁾ that, provided the conservation laws of the massive Thirring model and the Sine-Gordon theory are respected by the quantum corrections to the classical (tree-graph) approximations of the respective theories, the corresponding S matrices will be pure phases, i.e., there will be no particle production and no non-trivial multiparticle scattering.

The existence of renormalized conservation laws in the Sine-Gordon theory has been previously established ⁵⁾.

In this note we analyze the renormalization of the first non-trivial conserved classical current in the massive Thirring model with dimension and tensor structure higher than those of the energy momentum tensor.

It is found that the possible anomalies in the integrated Ward identity can be absorbed multiplicatively.

2. - To start with we recall the derivation of the first non-trivial conservation law in the classical massive Thirring model being defined by the equation of motion

$$(i \partial_\mu \gamma^\mu - m) \psi = g \bar{\psi} \gamma^\mu \psi \gamma_\mu \psi \quad (1)$$

for an anticommuting spinor field ψ in two space time dimensions. By use of the equation of motion one easily verifies the identity

$$\begin{aligned} & \partial_\mu (i\bar{\psi} \gamma^\mu \psi_{,3u} + h.c.) = \\ & = \partial_u (\dots) - 6g \bar{\psi}_{,u} \gamma_\mu \psi_{,u} \partial_u (\bar{\psi} \gamma^\mu \psi) \end{aligned} \quad (2)$$

We use here and in the following characteristic co-ordinates $u = ((t+x)/2)$, $v = ((t-x)/2)$, $(\psi_{,nu} \equiv \partial_u^n \psi)$.

Rewriting the second term on the right-hand side of Eq. (3) in spinor components and using the equation of motion in characteristic co-ordinates

$$\begin{aligned} i\psi_{2,u} - m\psi_1 &= 2g\psi_1^+ \psi_1 \psi_2 \\ i\psi_{1,v} - m\psi_2 &= 2g\psi_2^+ \psi_2 \psi_1 \end{aligned}$$

one obtains

$$\bar{\psi}_{,u} \gamma_\mu \psi_{,u} \partial_u (\bar{\psi} \gamma^\mu \psi) = -2\psi_{1,u}^+ \psi_{1,u} \partial_v (\psi_1^+ \psi_1) \quad (3)$$

In the last equality use has been made of the Grassmann algebra property of the ψ field : terms with two or more equal spinor components vanish identically. By the same recipes (equation of motion and Fermi statistics) one realizes that the right-hand side of Eq. (3) can be rewritten as a total derivative

$$\psi_{1,u}^+ \psi_{1,u} \partial_v (\psi_1^+ \psi_1) = \partial_v (\psi_{1,u}^+ \psi_{1,u} \psi_1^+ \psi_1) \quad (4)$$

The information of Eqs. (2)-(4) can now be summarized in the conservation law

$$\partial_\mu (i\bar{\psi} \gamma^\mu \psi_{,3u} + h.c.) + \partial_u (\dots) - 12g \partial_v (\psi_{1,u}^+ \psi_{1,u} \psi_1^+ \psi_1) = 0 \quad (5)$$

For the discussion of the renormalization of the conservation law (5) we use Zimmermann's normal product formalism ⁶⁾, namely N_5 products as all composite fields appearing in the classical equation (5) carry operator dimensions not higher than five.

The potential source of quantum anomalies destroying the classical conservation law (5) arises from the well-known fact that inside higher dimension normal products one cannot apply naïvely the equation of motion ^{*}).

The extra terms appearing as deviation from the classical relations (that are the relations which one deduces from a naïve application of the equation of motion) carry at least one additional power of the Planck constant $\hbar$ proceeding in a loopwise expansion. We will exploit the appearance of additional powers of $\hbar$ in front of the possible anomalous contributions to Eq. (5).

We inspect the list of N_5 operators, which are possible candidates for anomalies to Eq. (5) taking only as restrictions for this list Lorentz covariance and charge conjugation. (The candidates have to be even under C conjugation and to transform like a third derivative ∂_u^3 under the Lorentz group.)

Leaving aside all terms with total derivatives in front, we find that there are no operators with two or six ^{**)} spinor fields left over. All possible operators with four spinor fields are of such a type that they can be brought by a repeated use of the equation of motion - the configuration of the possible anomalies is thereby iterated - into the form

$$\Delta_i = a_i N_5 [\psi_{1,u}^+ \psi_{1,u} \partial_v (\psi_1^+ \psi_1)] + \sum_j c_{ij} \hbar N_5 [\Delta_j] \quad , \quad (6)$$

$$c_{ij} \hbar N_5 [\Delta_j] = c_{ij} \hbar N_5 (\psi_{1,u}^+ \psi_{1,u} \partial_v (\psi_1^+ \psi_1)) + c_{ij} \hbar^2 \sum_k c_{jk} N_5 [\Delta_k] \quad (7)$$

^{*}) We do not make use of anisotropic normal products ⁶⁾ which formally satisfy the classical relations.

^{**)} Fermi statistics plays a crucial rôle to rule out all operators with six spinors.

Here $\{\Delta_i\}$ denotes the most general C even set of operators of dimension five with four spinor fields carrying a Lorentz tensor structure like δ_u^3 .

Equations (6) and (7) have to be understood as equalities between time-ordered functions of normal products :

$$\begin{aligned} \langle T N_5 [\Delta_i] X \rangle &= \langle T a_i N_5 [\psi_{1,u}^+ \psi_{1,u} \partial_v (\psi_1^+ \psi_1)] X \rangle \\ &+ \hbar \sum_j c_{ij} \langle T N_5 [\Delta_j] X \rangle + \delta\text{-terms} \end{aligned} \quad (6')$$

where X denotes an arbitrary bunch of elementary spinor fields. The notation "δ terms" is used for expressions where, by the use of the free field equation of motion, external propagator lines have been contracted into a vertex. These terms which do not contribute on the mass shell have been suppressed in Eqs. (6) and (7).

From Eqs. (6) and (7) one sees that the possible anomalous N_5 contributions to the conservation law have the same structure as the classical expression in Eq. (5) ; the terms with a different structure being pushed to higher and higher powers in $\hbar$ [cf. Eq. (7)].

We can therefore conclude that the classical conservation law (5) survives the renormalization procedure. The only effect of the renormalization is a redefinition of the coefficients in front of the operators appearing in Eq. (5) being defined through a formal power series expansion in $\hbar$.

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