Dynamical Radion Superfield in Five-dimensional Action

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Abstract. We clarify the radion superfield dependence of 5D $\mathcal{N}=1$ superspace action. The radion is treated as a dynamical field and appears in the action with the correct mode function. Our derivation is systematic and based on the superconformal formulation of 5D supergravity. We can read off the couplings of the dynamical radion superfield to the matter superfields from our result. The correct radion mass can be obtained by calculating the radion potential from our superspace action.

1 Introduction

Five dimensional supergravity (5D SUGRA) compactified on an orbifold $S^1/Z_2$ has been thoroughly investigated. Especially, the Randall-Sundrum model [1] is attractive as an alternative solution to the hierarchy problem, and a large amount of research on this model has been done. In this model, the background geometry is a slice of the anti-de Sitter (AdS) spacetime and the metric has the form of

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{-2ky}\eta_{mn}dx^m dx^n - dy^2 = e^{-2kR_0}\eta_{mn}dx^m dx^n - R^2d\vartheta^2,$$

where $\mu,\nu,\cdots = 0, 1, 2, 3, 4$ and $m, n, \cdots = 0, 1, 2, 3$ are the 5D and 4D indices, and the coordinate of the fifth dimension is denoted as $y \equiv x^4$. The constant $k$ is the AdS curvature and $R$ is the radius of the orbifold. The physical range of the extra field $y$ is $0 \leq y \leq \pi R$. In the second line, we have changed the coordinate $y$ to the dimensionless coordinate $\vartheta \equiv y/R$.

In such a brane-world model, the radius of the compactified extra dimension is generically a dynamical degree of freedom, the radion. In the original Randall-Sundrum model [1], the radius of the orbifold is undetermined by the dynamics and thus the radion is a massless field. Hence, it remains to be a dynamical degree of freedom in low energies and should be taken into account in the 4D effective theory. A naive way of introducing the radion mode into the theory is to promote the radius $R$ to a 4D field $r(x)$. Namely, the radion $r(x)$ appears in the metric as

$$ds^2 = e^{-2kr(x)}g^{(4)}_{mn}(x)dx^m dx^n - r^2(x)d\vartheta^2,$$

where $g^{(4)}_{mn}$ is the 4D graviton. However, this is not an appropriate introduction of the radion. In fact, $r(x)$ in Eq. (2) is not a mass-eigenstate. In the usual dimensional reduction procedure, we will expand the bulk fields into the infinite Kaluza-Klein (K.K.) modes. For example, a 5D field $B$ is mode-expanded as

$$B(x,y) = \sum_n f_n(y)b_n(x).$$

In order for $b_n(x)$ to be mass-eigenstates, we have to choose the mode-functions $f_n(y)$ as solutions of the mode-equations, which are obtained from the linearized equation of motion for $B$. Otherwise, $b_n(x)$ are no longer mass-eigenstates and we cannot simply drop the higher K.K. modes after the heavy modes are integrated out. Note that the correct mode-function of the radion, which is proportional to $e^{2ky}$, is missing in Eq. (2). Thus, the above naive ansatz must be corrected. The proper treatment of the radion mode is discussed in Refs. [1, 2] in non SUSY case, and Ref. [6] discusses this issue in the context of 5D pure SUGRA.

In this talk, we will explain our work [7], which has derived the 5D superspace action including the radion superfield and clarified its couplings to the matter superfields. Our work corresponds to an extension of Ref. [6] including the matter superfields. The strategy of our work is as follows. First, we will identify how the radion mode appears in the superspace action. Then, we will promote it to a chiral superfield.

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2 Radion mode in the superspace action

In our previous work [3], we have derived the 5D superspace action directly from 5D SUGRA action by fixing all the gravitational fields to their vacuum expectation values (VEVs).\(^1\)

\[
\langle e_{\mu}^\nu \rangle = e^\sigma(y),
\langle e_y^4 \rangle = 1,
\langle \psi_{\mu} \rangle = 0,
\]

where \(e_{\mu}^\nu\) and \(\psi_{\mu}\) are the fünfbein and the gravitino, respectively. The function \(\sigma(y)\) is the warp factor, and \(\sigma(y) = -ky\) in the case that the backreaction of the radius stabilizer field on the metric is neglected.\(^2\) Since the radion mode originally belongs to the 5D gravitational multiplet, we have to modify the above treatment of the gravitational fields. To introduce the radion mode, we will replace the VEVs of fünfbein with the radion-dependent functions \(F\) and \(G\) as follows.

\[
\sigma(y) \rightarrow F(b(x), y),
\langle e_y^4 \rangle \rightarrow G(b(x), y),
\]

where \(b(x)\) is the radion field.

There are two conditions that the functions \(F\) and \(G\) must satisfy. The first condition comes from the requirement that our superspace action reproduces the correct 5D SUGRA action. To see this, let us take a kinetic term of a chiral superfield \(\Phi\) as an example. If we fix the gravitational fields to their VEVs, such kinetic term can be written as

\[
\mathcal{L}_{\text{kin}} = e^{2\sigma} \langle e_y^4 \rangle \int d^4 \theta \bar{\Phi} \Phi
\]

\[
= e^{2\sigma} \langle e_y^4 \rangle \eta^{mn} \left\{ \frac{\bar{\varphi} \partial_m \varphi}{4} - \frac{\partial_m \bar{\varphi} \partial_n \varphi}{4} + \frac{\partial_m \bar{\varphi} \partial_n \varphi}{2} + \cdots \right\}
\]

\[
= e^{2\sigma} \langle e_y^4 \rangle \eta^{mn} \partial_m \bar{\varphi} \partial_n \varphi + \cdots,
\]

where \(\varphi\) is the scalar component of \(\Phi\). Note that we have performed the partial integral at the last step. After the replacement [4], the prefactor \(e^{2\sigma} \langle e_y^4 \rangle\) becomes \(e^{2FG}\), which generically has a nontrivial \(x\)-dependence through the radion field \(b(x)\). However, if \(e^{2FG}\) depends on \(x^m\), unwanted extra terms appears after the partial integral. To avoid the appearance of such terms, we have to impose the condition that \(e^{2FG}\) is independent of \(x^m\). Considering its background value, this condition can be written as

\[
2F + \ln G = 2\sigma.
\]

In fact, if we impose this condition, the correct radion kinetic term is also reproduced. Thus, this is the necessary and sufficient condition for the superspace action to reproduce the correct SUGRA action.

The second condition comes from the fact that the radion field should appear in the metric as if it is a modulus field. In other words, the bulk geometry should remain AdS\(_5\) with a definite curvature \(k\) when we shift VEV of the radion field by constant. This condition can be written as

\[
G = \frac{1}{k} \partial_y F.
\]

By solving the above constraints [7] and [8], explicit function forms of \(F\) and \(G\) are determined as

\[
F = \frac{1}{2} \ln \left( e^{2\sigma} + I(b) \right),
\]

\[
G = \frac{1}{1 + e^{-2\sigma} I(b)},
\]

where \(I(b)\) is some function of only \(b(x)\) and satisfies

\[
I(b) = 0.
\]

In the following, we will choose it as \(I(b) = \bar{b} \equiv b - \langle b \rangle\). In this case, the metric agrees with that of Ref. [9].

Since the most familiar definition of the radion field is a proper length \(r(x)\), we will rewrite \(b(x)\) in terms of \(r(x)\). From its definition, the proper length is written as

\[
r(x) \equiv \frac{1}{\pi} \int_0^{\pi R} dy G(b(x), y) = \frac{1}{\pi} \int_0^{\pi R} dy \frac{dy}{1 + e^{-2\sigma b}}
\]

\[
= R - \frac{1}{2k\pi} \ln \left( \frac{1 + e^{2k\pi R\bar{b}}}{1 + \bar{b}} \right),
\]

or equivalently,

\[
\bar{b} = e^{-k\pi R} \sinh k\pi (R - r(x)).
\]

By substituting this into Eq. [9], the function \(G\) can be expressed in terms of \(r(x)\).

Using this \(G\), we can express the bulk Lagrangian...
as follows.

\[
\mathcal{L} = \mathcal{L}_{\text{kin}}^{\text{rad}} + \left\{ \int d^2 \theta \left( \frac{1}{4} G_{\tau} \Phi \Phi + \text{h.c.} \right) \right. \\
+ e^{2\sigma} \int d^4 \theta \left( \partial_\tau \Phi - i \Phi \Sigma - i \Sigma \Phi \right)^2 \\
- e^{2\sigma} \int d^4 \theta \left\{ 2 M_5^3 \bar{\Sigma} \Sigma \right\}^\frac{3}{2} \\
- G^2 \left( \bar{H} e^{2\theta} H + \bar{H} C e^{-2\theta} H C \right) \\
+ e^{3\sigma} \left\{ \int d^2 \theta \left( \frac{\partial_\tau}{2} + m G + 2i \frac{G_{\tau}}{2} \right) H \right. \\
+ \left. \text{h.c.} \right\},
\]

where \( M_5 \) is the 5D Planck mass, \( g \) is a gauge coupling, and \( W \) is the superfield strength of the vector superfield \( V \). The chiral superfields \((H, H C)\) form a hypermultiplet, and \( \Sigma \) and \( \Phi \) are the 5D compensator superfield and the gauge scalar superfield, respectively. The complex quantity \( G_{\tau} \) and the radion kinetic term \( \mathcal{L}_{\text{kin}}^{\text{rad}} \) are defined as

\[
G_{\tau} \equiv G - i \kappa W, \\
\mathcal{L}_{\text{kin}}^{\text{rad}} = \frac{3 M_5^3 (k \pi)^2}{16} \left( 1 - e^{-2k \pi R} \right)^2 e^{-2\sigma} \left( e^{2\sigma} G_{\tau}^2 \right) \sinh^4 k \pi r \\
\times \eta_m \eta_n \delta_{mn} \delta_{\tau r},
\]

where \( \kappa \equiv 1/M_5 \) and \( W \) is the graviphoton field.

3 Promotion to the radion superfield

In order to obtain the desired superspace action, we will promote the radion field \( r(x) \) in Eq. (13) to a superfield. Here, we will define a complex scalar \( \tau \) as

\[
\tau \equiv r + i \kappa w,
\]

where \( w(x) \) is a gauge-invariant Wilson line of the graviphoton,\(^3\)

\[
w \equiv \frac{1}{\pi} \int_0^{\pi R} dy W_y^0.
\]

Then, we can easily check that the kinetic term for \( \tau \) becomes the Kähler form. This fact suggests that \( r(x) \) should be associated with \( w(x) \) in the form of \( \tau(x) \). For example, a complex quantity \( G_{\tau} \) appearing in Eq. (13) should be interpreted as

\[
G_{\tau} = G(\tau) = \left\{ 1 + e^{-2\sigma(y)} e^{-k \pi R} \sinh k \pi (R - \tau) \right\}^{-1}.
\]

Then, from Eq. (14), \( G \) and \( W_y^0 \) are identified as

\[
G \equiv \text{Re} \ G(\tau), \quad W_y^0 \equiv - M_5 \text{Im} \ G(\tau).
\]

Now we will promote the complex scalar \( \tau \) to a chiral superfield \( T \). Namely, \( G_{\tau} \) and \( G \) in Eq. (13) are promoted as

\[
G_{\tau} \rightarrow G(T) = \left\{ 1 + e^{2k \sigma} e^{-k \pi R} \frac{\sinh k \pi (R - \tau)}{\sinh k \pi T} \right\}^{-1},
\]

\[
G \rightarrow G_R \equiv \text{Re} \ G(T).
\]

As a result, the desired superspace Lagrangian becomes

\[
\mathcal{L} = \left\{ \int d^2 \theta \frac{1}{4} G(T) \Phi \Phi + \text{h.c.} \right. \\
+ e^{2\sigma} \int d^4 \theta G_R^{-2} \left( \partial_\tau \Phi - i \Phi \Sigma - i \Sigma \Phi \right)^2 \\
+ e^{2\sigma} \int d^4 \theta G_R^2 \left( \bar{H} e^{2\theta} H + \bar{H} C e^{-2\theta} H C \right) \\
+ e^{3\sigma} \left\{ \int d^2 \theta \left( \frac{\partial_\tau}{2} + m G(T) - 2i \frac{G_{\tau}}{2} \right) H \right. \\
+ \left. \text{h.c.} \right\} \\
- e^{2\sigma} \int d^4 \theta 3 M_5^3 \ln G_R \\
+ \sum_{\vartheta^* = 0, \pi} \mathcal{L}_{\text{brane}}^{(\vartheta)} \delta(y - R \vartheta^*),
\]

where \( \mathcal{L}_{\text{brane}}^{(\vartheta)} \) is a brane localized Lagrangian at \( y = R \vartheta^* \) given by

\[
\mathcal{L}_{\text{brane}}^{(\vartheta)} = \left\{ \int d^2 \theta \bar{f}_{AB}^{(\vartheta)} W^A W^B + \text{h.c.} \right. \\
- e^{2\sigma} \int d^4 \theta G_R^{-1} \exp \left\{ -K^{(\vartheta)}(S, \bar{S}, U) \right\} \\
+ e^{3\sigma} \left\{ \int d^2 \theta G_R^{-2} (T) P^{(\vartheta)}(S) + \text{h.c.} \right\}.
\]

Here we have assumed that the background preserves \( N = 1 \) SUSY and dropped the compensator superfield. \( \mathcal{W}^A \) is a superfield strength of \( U^A \). The chiral superfield \( S \) and the vector superfield \( U^A \) in \( \mathcal{L}_{\text{brane}}^{(\vartheta)} \) can be either brane-localized superfields or brane-induced superfields from the bulk superfields.

4 Supersymmetric radius stabilization

Finally, we will demonstrate the radius stabilization in a model proposed in Ref. [10], which corresponds

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\(^3\)Here we will assume that \( W_y^0 \) = 0.
to a supersymmetric extension of the Goldberger-Wise mechanism \[11\].

The stabilization sector consists of a hypermultiplet \((H, H^c)\) with a bulk mass \(m\). The following tadpole superpotentials are introduced at both boundaries.

\[
P^{(0)} = J_0 H, \quad P^{(\tau)} = -J_\tau H,
\]

where \(J_0\) and \(J_\tau\) are real constants.

Then, we can calculate the radion potential from our superspace action obtained in the previous section.

\[
V_{\text{rad}}(\tau) = \frac{|G^{-\tilde{\eta}}(\tau, y = 0)|^2 \cdot |J_0 - J_\tau e^{-\gamma \pi \tau}|^2}{4 \int_0^{\pi R} dy \, e^{(k-2m)\eta} G_R^2 [G^{-\tilde{\eta}}(\tau)]^2 + \mathcal{O}(l^4)},
\]

where \(\gamma = \frac{3}{2} k + m\) and \(l = \kappa^{3/2} |J_\tau|\). Here we have restricted the section of \(h = 0\), where \(h\) is the scalar component of \(H\). From this potential, we can easily see that the radius \(\langle \tau \rangle = \text{Re}(\tau)\) is certainly stabilized to a finite value

\[
\langle \tau \rangle = \frac{\ln(J_\tau/J_0)}{\gamma \pi}.
\]

By differentiating this potential with respect to \(\tau\), we can also calculate the radion mass as

\[
m_{\text{rad}}^2 = \frac{l^2 k^2}{6} \left(1 - \frac{2m}{k}\right) \left(\frac{3}{2} + \frac{m}{k}\right)^2 \times \frac{e^{-2k\pi R} (1 - e^{-2k\pi R})}{1 - e^{-(k-2m)\pi R}} + \mathcal{O}(l^4).
\]

This completely agrees with the result obtained by directly solving the equation of motion. This agreement supports the validity of the \(T\)-dependence of our superspace action.

\section{Summary}

We have derived 5D superspace action including the dynamical radion superfield, and clarified its couplings to the bulk and the boundary matter superfields. Our result is obtained in a systematic way based on the superconformal formulation of 5D SUGRA in Ref. \[12\].

The \(T\)-dependence of our action is different from that of Ref. \[13\], which is based on the naive ansatz. \[4\]. Especially, the marked difference appears in the couplings between \(T\) and the K.K. modes of the matter superfields.

Note that we cannot redefine \(G(T)\) as a single chiral superfield by holomorphic redefinition of the superfield because \(G(T)\) has an explicit \(y\)-dependence. (See Eq. \[20\].) Thus, the 4D effective Kähler potential has a quite complicated form in the case of the warped geometry. In contrast, the \(T\)-dependence becomes greatly simplified in the flat spacetime (i.e., \(k = 0\)). The \(y\)-dependence of \(G(T)\) disappears and

\[
G(T) = \frac{T}{R},
\]

The 4D effective Kähler potential in this case becomes the following no-scale form up to a constant.

\[
K_{\text{rad}}^{(4)}(T, \tilde{T}) = -3M_P^2 \ln (T + \tilde{T}),
\]

where \(M_P = (\pi R M_3^2)^{1/2}\) is the 4D Planck mass.

Here, we have assumed that the background preserves \(N = 1\) SUSY, and dropped the dependence of the compensator superfield. However, it plays an important role when we consider the SUSY breaking effects. Thus, our next task is to extend our result including the compensator superfield. Note that what appears in the 4D effective action is the 4D compensator superfield. Although the superconformal formulation of 5D SUGRA has a compensator multiplet, they are 5D fields. Since the compensator multiplet is not dynamical, we cannot mode-expand its component fields into the K.K. modes in the ordinary manner. In fact, the \(F\)-terms of the compensator and the radion superfields are closely related to each other. Therefore, the result we have derived here provides an important hint for identifying the dependence of the 5D superspace action on the 4D compensator superfield. The research along this line is now in progress.

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