Tidal torques, disc radius variations, and instabilities in dwarf novae and soft X-ray transients

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Abstract. The study of outer disc radius variations in close binary systems is important for understanding the structure and evolution of accretion discs. These variations are predicted by models of both quasi-steady and time-dependent discs, and these predictions can be confronted with observations. We consider theoretical and observational consequences of such variations in cataclysmic variables and low-mass X-ray binaries. We find that the action of tidal torques, that determine the outer radius at which the disc is truncated, must be important also well inside the tidal radius. We conclude that it is doubtful that the tidal-thermal instability is responsible for the superoutburst/superhump phenomena in dwarf novae, and confirm that it cannot be the reason for the outbursts of soft X-ray transients. It is likely that tidal torques play a role during superoutbursts of very-low mass-ratio systems but they cannot be the main and only cause of superhumps.

Key words. accretion, accretion discs – instabilities – Stars: dwarf novae – (Stars:) binaries (including multiple): close – X-rays: binaries

1. Introduction

Dwarf novae are a subclass of cataclysmic variables that undergo outbursts lasting at least a few days during which their brightness increases by several magnitudes (see e.g. Warner 1995 for a review). These outbursts are believed to be due to a thermal/viscous accretion disc instability which arises when the disc temperature becomes of order of \(\lesssim 10^4\) K, enough for hydrogen to become partially ionized and opacities to depend strongly on temperature (see Lasota 2001 for a review of the model). Models have become more and more sophisticated, and differ from the initial, basic version of the model proposed by Meyer & Meyer-Hofmeister (1981) and by Smak (1982), following precursor works by Smak (1971), Osaki (1974) and Hoshi (1979). They now include various effects such as illumination of the disc by the white dwarf (Hameury et al. 1999), disc truncation (Hameury et al. 1997) by evaporation or by a magnetic field, increase of the mass transfer rate from the secondary as a result of illumination (Hameury et al. 2000), and heating of the outer parts of the disc by the stream impact and tidal torques (Buat-Ménard et al. 2001). The weakest point of these models – aside from the assumption that angular momentum transport is due to a “viscosity” (i.e. is a local phenomenon accompanied by energy dissipation) described by the modified alpha prescription of Shakura & Sunyaev (1973) (bivalued \(\alpha\), low in quiescence and high in outburst), is the approximate treatment of intrinsically 2D effects at the disc edge.

There is in particular a debate about the outcome of the disc reaching the radius at which the 3:1 resonance occurs (this may happen for low secondary to primary mass ratios). SPH models (see e.g. Whitehurst 1988, Murray 1996, 1998) treat accurately the dynamics of the disc and are in principle quite appropriate to deal with these effects; they predict that when the 3:1 resonance radius is reached, the disc becomes tidally unstable, eccentric and precesses. SPH models are however limited in that it is difficult to include in them detailed microphysics; the energy equation is often replaced by isothermal or adiabatic approximations. This significantly affects the results as shown by Kornet & Rózyczka (2000) who found that the tidal instability does not occur when disc cooling is properly taken into account but is present when the disc is assumed to be isothermal. Nevertheless, the tidal instability, coupled with the standard thermal instability, is also
believed (Osaki 1989) to be the reason for the long duration of superoutbursts in SU UMa systems, a subclass of dwarf novae exhibiting large and long outbursts during which so called superhumps are seen, together with normal outbursts. These superhumps are light modulations at a period slightly longer than the orbital period, and it is often taken for granted that they result from precession of a distorted disc.

Another point of debate is the amplitude of the torque $T_{\text{tid}}$ due to the tidal forces, even far from the resonance. This torque transfers the outward-flowing angular momentum from the accretion disc back to the orbit, effectively truncating the disc, and therefore determining its radius. Based on their numerical simulations Ichikawa & Osaki (1994) concluded that $T_{\text{tid}}$ should be negligible everywhere in the disc, except in a small ring at the tidal truncation radius; on the other hand, it has been often assumed that $T_{\text{tid}}$ has a smoother behaviour. In this paper, we first discuss (section 2) the effect of two different prescriptions for $T_{\text{tid}}$ on the light-curve of systems for which we know that the disc does not reach the 3:1 resonance. We show that the predicted light curves are not sufficiently different to be discriminated by observations; e.g. the determination of outer disc radius in eclipsing dwarf novae is not accurate enough to clearly rule out one of the models, even though a smooth $T_{\text{tid}}$ appears to be in better agreement with observations.

We compare the predictions of the disc instability model using these prescriptions for VV SCI stars (section 3), which exhibit slow variations of their luminosity from a high to a low state and back to high luminosity. We conclude that $T_{\text{tid}}$ cannot be neglected even in the central parts of the disc. We then turn to SU UMa systems (section 4) and soft X-ray transients (SXTs) containing a black hole (section 5), and examine for these systems the consequences of assuming that superhumps are due to a black hole (section 5), and examine for these systems the stability of the disc, contrary to what happens in case (A), important only when the outer radius is close to the tidal radius, i.e. during outbursts, and therefore does not affect the stability of the disc, contrary to what happens in case (B), where an additional heating term has to be taken into account (Buat-Ménard et al. 2001). The parameters of the system are those of U Gem (primary mass $M_1 = 1 M_\odot$, orbital period $P_{\text{orb}} = 4.25$ hr, tidal radius 4.06 $10^{10}$, circularization radius 1.30 $10^{10}$ cm, average mass transfer rate from the secondary 5 $10^{16}$ g$^{-1}$).

### 2. Disc radius variations in dwarf novae

The angular momentum conservation equation for a Keplerian disc can be written as:

$$j = \frac{\partial \Sigma}{\partial t} \left( \frac{\partial}{\partial r} \left( r \Sigma v_r \right) \right) + \frac{\partial}{\partial r} \left( \frac{3}{2} \Sigma \nu \Omega_K \right) + \frac{j_2}{2 \pi r} \frac{\partial M_t}{\partial r} - \frac{1}{2 \pi} T_{\text{tid}}(r)$$

(1)

where $\Sigma$ is the surface column density, $M_t$ is the rate at which mass is incorporated into the disc at radius $r$, $v_r$ is the radial velocity in the disc, $j = (GM_1 r)^{1/2}$ is the specific angular momentum of material at radius $r$ in the disc ($M_1$ being the primary mass), $\Omega_K = (GM_1/r^3)^{1/2}$ is the Keplerian angular velocity, $\nu$ is the kinematic viscosity coefficient, and $j_2$ the specific angular momentum of the material transferred from the secondary. $T_{\text{tid}}$ is the torque due to the tidal forces. One often use the prescription of Smak (1984), derived from the linear analysis of Papaloizou & Pringle (1977) which, for radii not too close to the tidal truncation radius $r_{\text{tid}}$ at which the trajectories of test particle orbiting around the white dwarf intersect leading to high dissipation (Paczynski 1977), was confirmed by the non-linear numerical simulations of Ichikawa & Osaki (1994). This reads:

$$T_{\text{tid}} = c \omega r \nu \Sigma \left( \frac{\nu}{\alpha} \right)^n$$

(2)

where $\omega$ is the angular velocity of the binary orbital motion, $\alpha$ the binary orbital separation and $c$ a numerical constant. The index $n$ was found to be close to $n = 5$. In the particular case of U Gem, Ichikawa & Osaki (1994) obtained a value for $c$ smaller by almost two orders of magnitude than the value required for the disc to be truncated at the tidal radius in steady state. They concluded that the tidal torque is very small everywhere except very close to the tidal radius where $T_{\text{tid}}$ diverges. If true, the tidal removal of the angular momentum would occur only at the disc’s outer edge within a negligibly small radial extent (Smak 2002).

As emphasized by Smak (2002), the tidal torque would then simply force the disc to remain within the tidal radius; contraction of the disc would result from accretion of matter with low angular momentum during quiescent phases. In order to evaluate the effect of various prescriptions for the tidal torque, we used our disc-instability model code (Hameury et al. 1998 ; see also Buat-Ménard et al., 2001) to calculate models with two different prescriptions for the tidal torque: $T_{\text{tid}} \propto \exp((r - r_{\text{tid}})/10^5)$ cm, vanishingly small for $r < r_{\text{tid}}$ and growing exponentially on a short scale ($\propto r_{\text{tid}}$) for larger $r$ (prescription (A)), and $T_{\text{tid}}$ given by Eq. (2) in which the constant $c$ has been adjusted to produce a given average radius (prescription (B)). Our code is a 1D + 1D scheme solving the radial angular-momentum and energy equations decoupled from the vertical structure equations whose solutions are represented by a grid of (pseudo)thermal equilibria (see Hameury et al. 1998 for details). The viscosity is the standard Shakura & Sunyaev (1973) one; here, we take $\alpha_{\text{cold}} = 0.04$ and $\alpha_{\text{hot}} = 0.2$, and assume a smooth (but rapid) transition between the value of $\alpha$ on the cool and hot branch (see Hameury et al. 1998 for more details). We have also assumed that in case (A), the dissipation of the tidal torque results in enhanced radiation from the disc edge, as suggested by Smak (2002), important only when the outer radius is close to the tidal radius, i.e. during outbursts, and therefore does not affect the stability of the disc, contrary to what happens in case (B), where an additional heating term has to be taken into account (Buat-Ménard et al. 2001).
Fig. 1. Outburst models in which the tidal torque is either vanishingly small for $r < r_{\text{tid}}$ and grows exponentially on a short scale for larger $r$ (solid curves), or is given by Eq. 2 (dashed curve). The top label shows the time variations of the visual magnitude (arbitrary distance), and the bottom panel gives the radius of the disc outer edge.

As can be seen, the general shape of the light curve is not strongly affected by this radical change of the prescription for the tidal torque. The recurrence time is almost the same for both prescriptions, the outburst light curve is not much modified, except that the duration is longer for prescription (B) than for prescription (A); this is a consequence of the disc heating by the dissipation of the tidal torque. Such a small difference between both sets of light curves contrasts with the large differences appearing when one assumes or not a fixed outer radius (Hameury et al. 1998). There is however a significant difference between the evolution of the outer disc radius in both cases, as expected: in case (B), the existence of a significant torque for $r < r_{\text{tid}}$ results in a significant contraction of the disc during quiescence, and, conversely, the disc can also extend to larger radii (possibly beyond $r_{\text{tid}}$!) because of the smoother variation of the tidal torque. The average disc size is identical in both cases (by construction), and its variations are smaller in case (A) than in case (B), as expected. Although this difference is significant (a factor 2.5 in the amplitude of the outer disc radius variations), it is probably not large enough to be unambiguously constrained by observations. We note for example that Smak (1996) found both in the case of U Gem and Z Cha that the outer disc radius varies by about 30%, which is quite compatible with the amplitude for prescription (B) − 37%, but we would certainly not dismiss prescription (A) on this sole basis, in view of the large observational and systematic uncertainties in the measurement of the outer disc radius.

It is also worth to be noted that, if in case (A) radiation by the disc edge were not sufficient to get rid of all of the energy dissipated by tidal heating, the overall light curve would show marked differences from Fig. 1. Because, as a consequence of the very steep variations of $T_{\text{tid}}(r)$, the energy would be released in a small annulus, the local energy dissipation rate could then be quite high in the vicinity of the tidal radius; this would result in the outer ring being quite hotter that the remaining outer disc part, with a temperature quite sensitive to radius variations. As a result, ~1 magnitude oscillations lasting for a few days would be produced shortly after the end of an outburst. Apparently such oscillations have not been observed, but their existence certainly depends on the details of the model assumptions, and in particular on the radial heat transfer at the very disc edge.

3. VY Scl systems

VY Scl stars are a subgroup of cataclysmic variables which are usually in a bright high state, and have occasionally low states during which their luminosity drops by more than one magnitude, bringing them into the dwarf nova instability strip. Yet, they do not have dwarf nova outburst, even though the decline can be very gradual and prolonged, longer than the disc viscous time. It had been suggested that the apparent stability of VY Scl discs could be due to the irradiation of the inner disc by a very hot white dwarf (Leach et al. 1999), but Hameury & Lasota (2002) showed that outbursts are unavoidable unless the disc disappears completely during quiescence, most probably truncated by the white dwarf magnetic field. The disc has to remain hot at all times so that a cooling front does not start from the outer edge, until it completely disappears. This translates into a requirement on the magnetic field strength that must be sufficient for the Alfvén radius to be equal to the circularization radius when the accretion rate is just equal to the critical rate below which instabilities appear.

This model has received strong observational support (Hameury & Lasota 2005). Observations of MV Lyr in a low state show no indication for a disc; Linnell et al. (2005) have put an upper limit of 2500 K on the disc temperature. Similarly, TT Ari also shows (virtually) no sign for an accretion disc in its low state (Gänsicke et al. 1999).

These systems are therefore a key in understanding how tidal torques act on a disc, since they slowly vary from a state in which there is a fully developed disc to a discless state, and the outer disc radius has to vary con-
from the low state is slow and takes since, as noted by Stanishev et al. (2004), the recovery that radius variations are a consequence of mass redistribution (i.e. \( \dot{\alpha} \)) is not constant within the disc), so \( M_{\text{in}} = 0.7 M_{\odot} \), \( M_{\text{out}} = 0.3 M_{\odot} \), the orbital period is 3.5 hr, and the outer disc radius is \( 3 \times 10^{10} \) cm in the steady high state; the magnetic moment is \( 8 \times 10^{32} \) G cm\(^3\), and we assumed that the disc disappears. We have taken here \( M_{1} = 0.7 M_{\odot} \), \( M_{2} = 0.3 M_{\odot} \), the orbital period is 3.5 hr, and the outer disc radius is \( 3 \times 10^{10} \) cm in the steady high state; the magnetic moment is \( 8 \times 10^{32} \) G cm\(^3\), and we assumed that the disc is illuminated by the hot white dwarf (temperature of 40,000 K in quiescence). Irradiation by the white dwarf was treated as in Hameury & Lasota (2002). As above,

\[
\frac{\dot{r}_{\text{circ}}}{\dot{r}_{\text{in}}} = \frac{1}{\sqrt{G M_{1} M}} \int_{r_{\text{in}}}^{r_{\text{out}}} T_{\text{tid}}(r) dr
\]

where \( r_{\text{circ}} \) is the circularization radius (i.e. the radius at which matter originating from the secondary would circularize assuming angular momentum conservation), and where we have used the outer boundary condition \( \dot{M} \left[ 1 - \left( \frac{r_{\text{circ}}}{r_{\text{out}}} \right)^{1/2} \right] = 3 \pi \nu \Sigma \) (4)

Writing the tidal torque as \( T_{\text{tid}}(r) = \text{Const.} \nu \Sigma r^{n} \)

which is valid for both tidal torque prescriptions used here (n arbitrary large for prescription A, and \( n = 6 \) for prescription B); one finds a solution for the outer radius

\[
\int_{u_{\text{in}}}^{\dot{u}_{\text{out}}} \exp \left[ \frac{K}{(2n + 1)(u_{n}^{2n+1} - u_{\text{out}}^{2n+1})} \right] du = u_{\text{out}} - 1 \]  

where \( u_{\text{in}} = (r_{\text{in}} / r_{\text{circ}})^{1/2} \), \( u_{\text{out}} = (r_{\text{out}} / r_{\text{circ}})^{1/2} \), and K is a constant. Figure 2 shows the solution of this equation for different values of n. As expected, the disc reduces to a ring becoming smaller and smaller as \( r_{\text{in}} \) approaches \( r_{\text{circ}} \). A significant and detectable effect is found for \( n = 6 \) (prescription B), but for larger values on n, variations of \( r_{\text{out}} \) are small; for example, for \( n = 11 \), a 10% variation is found when \( r_{\text{in}} = 0.66 r_{\text{circ}} \), and a 20% variation requires \( r_{\text{in}} \) to be within 7% of \( r_{\text{circ}} \). For \( n = 21 \), \( r_{\text{in}} \) must be within 11 % and 0.8 % respectively of \( r_{\text{circ}} \) to get the same variations. The ratio of \( T_{\text{tid}} \) at \( r_{\text{circ}} \) between the three curves is 1:140:450,000; the \( n = 11 \) curve thus approximately corresponds to the numerical solution of Ichikawa & Osaki (1994) in the vicinity of \( r_{\text{circ}} \), which therefore appears totally unable to account for the observed radius variations of DW UMa.

We used our disc-instability model code (Hameury et al. 1998; see also Buat-Ménard et al., 2001) to simulate the accretion-disc properties of DW UMa. Figure 3 shows the evolution of a system with parameters similar to those of DW UMa. Figure 4 shows the solution of this equation for different values of n. As expected, the disc reduces to a ring becoming smaller and smaller as \( r_{\text{in}} \) approaches \( r_{\text{circ}} \). A significant and detectable effect is found for \( n = 6 \) (prescription B), but for larger values on n, variations of \( r_{\text{out}} \) are small; for example, for \( n = 11 \), a 10% variation is found when \( r_{\text{in}} = 0.66 r_{\text{circ}} \), and a 20% variation requires \( r_{\text{in}} \) to be within 7% of \( r_{\text{circ}} \). For \( n = 21 \), \( r_{\text{in}} \) must be within 11 % and 0.8 % respectively of \( r_{\text{circ}} \) to get the same variations. The ratio of \( T_{\text{tid}} \) at \( r_{\text{circ}} \) between the three curves is 1:140:450,000; the \( n = 11 \) curve thus approximately corresponds to the numerical solution of Ichikawa & Osaki (1994) in the vicinity of \( r_{\text{circ}} \), which therefore appears totally unable to account for the observed radius variations of DW UMa.

Recently, Stanishev et al. (2004) observed the eclipsing VY Scl star DW UMa in a state intermediate between minimum and maximum. Eclipse mapping techniques allowed to reconstruct the disc luminosity profile; they found that the luminosity difference between the high and intermediate states is almost entirely due to a change in the accretion disc radius, from \( \sim 0.5 \) to \( \sim 0.75 \times R_{L1} \), the distance between the white dwarf and the L1 point. Linnell et al. (2005 have caught MV Lyr in a similar intermediate state.) It is observed that in the intermediate state, the disc is entirely eclipsed by the secondary, while its outer parts are visible during the high state. A possible explanation of this could be that the disc is not in equilibrium (i.e. \( \dot{M} \) is not constant within the disc), so that radius variations are a consequence of mass redistribution inside the disc. This however cannot be the case, since, as noted by Stanishev et al. (2001), the recovery from the low state is slow and takes \( \sim 4 \) months; if the viscosity is not unusually small, the disc has enough time to readjust its structure to changes of the mass accretion rate. The disc should then be quasi steady, and its radius close to the tidal radius if tidal torques were exerted only in a very small annulus at \( r \sim r_{\text{tid}} \) (prescription A).

Observations show, however, a disc radius \( \sim 50\% \) smaller than \( r_{\text{tid}} \) (Stanishev et al. 2004). Assuming a value of \( \alpha \) small enough that the viscous time is comparable to or longer than the rise-time would of course solve the problem, but there is absolutely no reason for doing so, especially that the reason for luminosity variations in VY Scl are changes in the mass-transfer rate and not modifications of the disc’s physical state. In any case playing with \( \alpha \) every time something does not fit a popular belief is not very helpful.

The relation between the inner and outer radius depends on the precise prescription for the tidal torque. Assuming steady state, one can integrate Eq. 1 over \( r \) and obtain an angular momentum conservation relation:

\[
\dot{r}_{\text{circ}} - \dot{r}_{\text{in}} = \frac{1}{\sqrt{G M_{1} M}} \int_{r_{\text{in}}}^{r_{\text{out}}} T_{\text{tid}}(r) dr
\]

Fig. 2. Variations of the outer radius as a function of the inner disc radius, for different values of \( n \) in Eq. 5. From top to bottom: \( n = 21 \), \( n = 11 \), and \( n = 6 \). The latter corresponds to prescription A. The constant \( K \) has been chosen such as to give the same \( r_{\text{out}} \) for \( r_{\text{in}} = 0 \).
\( \alpha_{\text{hot}} = 0.2 \) and \( \alpha_{\text{cold}} = 0.04 \). There are still some small oscillations left but these could be suppressed if the field were slightly stronger, and are probably not detectable. We have assumed a slow decrease of the mass transfer rate \( \dot{M}_t \) until the disc almost disappears, followed by a rapid (essentially for numerical reasons, the adaptative mesh code being unable to deal with a vanishingly small disc) increase of \( \dot{M}_t \). The second panel shows that the disc is always close to equilibrium \( (M \approx \dot{M}_t) \). It can be seen that significant variations of the outer disc radius are obtained. When the disc is fainter by 1 magnitude than the maximum, the disc is 20\% smaller than its maximal extension. This is not quite the 50\% variations that are claimed by Stanishev et al. (2004), but is acceptable in view of the fact that we define the disc outer edge as the place where the surface density vanishes, whereas Stanishev et al. (2004) use a photometric definition.

These results show that the tidal coupling cannot be negligible in the intermediate state, even at radii quite a bit smaller than the tidal truncation radius. It is worth noting in this context that the mass ratio for this system is \( q = 0.39 \pm 0.12 \), i.e. marginally compatible with the maximum value of \( q \) for the 3:1 resonance to be accessible.

4. SU UMa systems

As mentioned earlier, the popular explanation for superoutbursts in SU UMa stars combines the thermal instability with a tidal instability that is supposed to arise when the disc reaches the 3:1 resonance radius (see e.g. Osaka 1989). At this point, the disc would become eccentric and precess, allegedly causing the superhumps. The tidal torque \( T_{\text{tid}} \) is supposed to increase by at least one order of magnitude, resulting in a corresponding enhancement of dissipation and angular momentum transport. According to this scenario, the disc shrinks until the radius has decreased to an (arbitrarily chosen) critical value of order of 0.35 times the orbital separation, and the superoutburst stops. A sequence of several normal outbursts, during which the disc grows on average then follows until the next superoutburst.

This model is essentially based on SPH simulations indicating that the disc does become eccentric and precesses when the 3:1 resonance radius is reached (Murray 2000, Truss et al. 2001), and on the fact that SU UMa stars are found below the period gap, for systems in which the mass ratio \( q \) is less than 1/3, the condition for the 3:1 resonance radius to be smaller than the tidal truncation radius. According to simulations by Truss et al. (2001) the critical value of \( q \) above which no superoutburst occur is in fact rather 1/4.

However, two elements seem to have seriously put in doubt the viability of the thermal-tidal model of superhumps and outbursts. First, Kornet & Różycka (2000) found that the outcome of numerical simulations depends on the equation of state that has been used; they found that eulerian models gave the same results as SPH codes in the isothermal approximation, but obtained very different results (no superhumps) when using the full thermal equation in their eulerian code. More recently, Smak & Waagen (2004) found superhumps in the famous U Gem 1985-superoutburst. The component masses of this prototypical binary are rather well constrained giving a rather large value of \( q = 0.364 \pm 0.017 \) (see Naylor et al. 2005). Clearly the tidal instability cannot apply to this system. In addition, a permanent superhump was detected in TV Col (Retter et al. 2003) a binary with an estimated mass-ratio between \( q = 0.62 \) and 0.93 (Hellier 1993), consistent with its 5.39 hour orbital period. One could try to save the tidal model by arguing that these superhumps are not the usual ones, and that, therefore U Gem and TV Col superhumps are phenomena different from those observed in SU UMa stars. Smak & Waagen (2004) examined this possibility in detail, and we repeat here their argument that this is not the case. The superhumps in U Gem have an amplitude typical of normal superhumps; even more, when plotted on superhump excess-period vs orbital period (logarithmic) plane, U Gem and TV Col...
fall on the linear extension of the relations defined by shorter period dwarf novae and permanent superhumpers (see Patterson et al. 2003, Fig. 20). Absolutely nothing distinguishes them from superhumps in low-q systems. One should note in addition that SPH models fail to reproduce the exact period excess: this is always found to be larger by a factor 1.5 – 2 than the measured values, making the position of U Gem on the superhump excess-period vs orbital period plane even more significant.

For the moment there are no alternative models for SU UMa’s (but see however Bath 2004). It is likely that irradiation and enhanced mass transfer play a significant role (Small 1993; Hameury et al. 2000), but at present there is no universally accepted explanation for the superhump/superoutburst phenomenon. It nevertheless remains true that the tidal torque has to be modified when the 3:1 resonance is reached; the disc response and the amplitude of the change are still a matter of debate, but they are certainly very different that what has been assumed in the TTI model.

5. Soft X-ray transients

In soft X-ray transients, the mass ratio is usually very small, and one would expect to find superhumps in these systems, if indeed the explanation of this effect is related to the tidal instability. Indeed there is observational evidence of modulations at periods slightly longer than the orbital period (Zurita et al. 2002). There are however several important differences between SXTs and SU UMa systems: first, because the mass ratios are extremely small, the disc should remain permanently eccentric, and second, the outer parts of the disc can remain unaffected by the thermal instability in systems where the disc is large. Finally, in SXTs superhumps would arise from a modulation of the reprocessed flux by the changing disc area and not from an increase of viscous dissipation as in SU UMa stars (Haswell et al. 2001). We do not discuss here problems that might arise from the amount of reprocessing required to account for the observed modulations, and that have been addressed by Haswell et al. (2001).

As an example, for \( q = 0.05 \), the circularization radius is 0.42\( a \), where \( a \) is the orbital separation, larger than the critical radius below which the tidal instability cannot be maintained (typically 0.33\( a \), see e.g. Osaki 1984). The disc can therefore never shrink enough for the tidal instability to stop, and it should remain in a permanently eccentric state. This will occur for values of \( q \) below 0.10, i.e. for most SXTs. Therefore the tidal instability cannot be the cause of SXT outbursts – and, as far as we know, this has never been suggested. It is then interesting to note that SXTs have sometimes been compared to WZ Sge systems (e.g. Kuulkers 2000), a particular class of SU UMa stars that have no normal outbursts between large and unfrequent superoutbursts. If the observational similarities between both classes reflect similar physics, this is then a additional problem for the tidal-thermal instability model in WZ Sge systems, and, by extension, in all SU UMa stars.

5.1. Short period systems

For systems with periods less than about one day, such as A 0620-00, the whole disc is affected by the outburst. One therefore expect that the superhump modulation, if related to the 3:1 tidal instability, should be visible in these systems. Systems for which a superorbital modulation has been claimed (XTE J1118+480, Nova Muscae 1991, GRO J0422+32 and GS 2000+25; Zurita et al. 2002; O’Donoghue & Charles 1990), all have periods shorter than one day. However, the disc is always larger than the 3:1 resonance radius as confirmed by the presence of the superhump in quiescence (Zurita et al. 2002), hence outbursts in SXTs cannot be related to tidal interactions. Interestingly superhumps in quiescence have been also observed in SU UMa stars (Patterson et al. 1995).

5.2. Long period systems

In systems with long (>1 day) orbital periods, the outer disc will not be affected by the outburst, except possibly by illumination effects. Hence if the tidal instability sets in, it will always remain present, and superhumps are expected to be present both in quiescence or in outburst. It turns out however that in quiescence, the outer disc is too cold to radiate efficiently even in the infrared: most of the luminosity originates from the central regions, despite the reduced emitting area. Figure 4 shows the structure of a disc in a system with the orbital parameters of “typical” long period binary (primary mass: 12 M\(_{\odot}\); secondary mass : 0.7 M\(_{\odot}\); orbital period: 78 hr): the mass transfer rate was taken to be \( 10^{16} \text{ g s}^{-1} \). We assume here \( \alpha_{\text{cold}} = 0.02 \) and \( \alpha_{\text{hot}} = 0.2 \). The irradiation of the disc is treated as in Dubus et al. (2001): we assume that the disc is illuminated by an X-ray flux that is proportional to the X-ray luminosity; we take here \( \sigma T_{\text{irr}}^4 = 5 \times 10^{-3} L_x/4\pi r^2 \), and solve for the vertical disc structure assuming that the disc effective temperature is such that it allows for reradiating the X-ray flux and the viscous flux. We also assume that the inner disc is truncated by some mechanism, presumably evaporation and/or formation of an ADAF, and that the inner disc radius has the same functional form as the Alfvén radius with a magnetic moment of \( 5 \times 10^{31} \text{ G cm}^3 \).

In quiescence, the effective temperature is of order of 1000 K, and less than 0.1 % of the total luminosity is emitted by the outer parts of the disc; any modulation of the light emitted by these cool regions would therefore be undetectable.

In outburst, the disc heats up as a result of irradiation from the primary (we assumed here the same prescription for irradiation as in Dubus et al. 1999 and in Dubus et al. 2001). A significant fraction of the disc luminosity in the infrared band should therefore be emitted by regions of the disc possibly affected by the tidal insta-
bility. However, the opacity is minimum for temperatures of the order of 3,000 K, typical in these regions that are affected by illumination, and it turns out that the optical depth is very small. As a consequence, the assumption of blackbody emission does no longer hold, and the colour temperature will be significantly higher than the effective temperature; light will be shifted in the optical or blue band, and will again be diluted in the total light emitted by the inner disc.

We therefore do not expect to detect any superorbital modulation in these systems, in the hypothesis that it would originate from the outer parts of the accretion disc (whether it is caused by a tidal instability or not).

6. Conclusion

The effects of the companion on the accretion disc are still not well explained. Even in the simple case where no strong resonance is present, observations of VY Scl stars indicate that the tidal torques must be strong enough to truncate the disc at radii much smaller than the so called tidal truncation radius. This calls for a theoretical reexamination of the tidal torques in binaries, an enterprise beyond the scope of this paper. Such reexamination should also take into account on results of SPH codes. The presence of superhumps during the superoutburst of U Gem casts doubt on the validity of the tidal-thermal model of these phenomena. This conclusion is strengthened by theory and observations of superhumps in TV Col. If we make the assumption that superhumps are in some way related to the 3:1 resonance, we predict that superhumps should be observed in short period SXTs in quiescence and in outburst, but not in long period systems. The outburst mechanism should then be unrelated to this resonance, and the tidal-thermal instability model does not apply to these systems, casting even more doubts on the applicability of the tidal-thermal instability to SU UMa systems.

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