CM-P00061665

Archives

Ref.TH.2443-CERN

INELASTIC SHADOW CORRECTION
FOR HADRON DEUTERON ELASTIC SCATTERING

G. Alberi ^{*)}

CERN - Geneva
and
Istituto di Fisica Teorica
Miramare, Trieste

and

F. Baldracchini ^{*)}

Advanced School of Physics
Miramare, Trieste

A B S T R A C T

We calculate the inelastic screening effect in hadron deuteron elastic scattering, using a new technique. The main idea is to calculate explicitly the complex cut integral in M^2 plane, using an ansatz for the asymptotic behaviour of the six-point amplitude. In the hypothesis of Pomeron dominance, that contribution is real and small.

^{*)} Supported in part by INFN.

1. - INTRODUCTION

After Glauber proof ¹⁾ that there is a shadow term in hadron deuteron collisions, Abers et al. ²⁾ realized, using a diagrammatic approach, that one has to add inelastic double collision amplitudes, where one or more pions are produced virtually. It is amusing to notice that both planar and non-planar diagrams ³⁾ were included in the formal part of their work.

The ideas of Abers et al. were exploited by several authors : Pumplin and Ross ⁴⁾ calculated the inelastic shadow effect for elastic proton scattering on nuclei ; Alberi and Bertocchi ⁵⁾ evaluated the effect of virtual production of resonances on the differential cross-section for πD scattering ; Gribov ⁶⁾, using the Pomeron dominance for elastic and inelastic reactions at high energies, derived a simple formula for the total hadron deuteron cross-section

$$\sigma = \sigma_n + \sigma_p - 2 \int_0^\infty dq_\perp^2 \int_0^\infty dM^2 \frac{d\sigma}{dq_\perp^2 dM^2} S(q_\perp, \frac{M^2 - m_h^2}{2k}) \quad (1.1)$$

where $d\sigma/dq_\perp^2 dM^2$ is the inclusive cross-section for the reaction $h N \rightarrow N + X$ h being the incident hadron, and S the deuteron form factor. Since the integral on M^2 extended also to the elastic peak, this expression includes also the Glauber term.

It was soon realized that the Gribov ⁶⁾ formula had to be corrected, because his assumption of the Pomeron dominance is not satisfied at most of the presently available energies. With the inclusion of other Regge poles, however, one has to face the problem of the phase : the solution of this problem does not seem to be unique in the literature, and the reason is that there is no unique way to derive the formula for the inelastic shadow term. As stressed in Section 2, all the problems lie in the complex cut in M^2 plane generated from the deuteron form factor S . If this is neglected altogether one gets the prescriptions of Refs. 7), 8), while if it is taken into account one gets Ref. 9).

Following the line of Ref. 9), we will calculate here explicitly the contribution of the cut integral, showing that for pure Pomeron dominance it contributes only to the real part of the amplitude.

2. - INTERMEDIATE INELASTIC STATES FOR hD ELASTIC SCATTERING

According to Gribov ⁶⁾, the double scattering diagram, in general, is given in Fig. 1, where the dashed region represents all the kinematically possible states : the most important state is the hadron h itself at intermediate energies (Glauber term), but we expect other states to become more and more important at higher energies. So to the Glauber double scattering term, we have to add the contribution of the resonances and of the continuum. The contribution of the resonances is analogous to the Glauber term with the only difference that there is a longitudinal momentum transfer involved, but the continuum requires some care in the choice of phases, which is not always done in the literature. As suggested by Kwiecinski et al. ⁸⁾ and by Anisovich and Dakhno ⁹⁾, one can derive the phases thinking of the dashed region in Figs. 1 and 2 as a six-point amplitude, so that the double scattering term is

$$F_0 = \frac{i}{2\pi k} \int_{-\infty}^{+\infty} dM^2 \int d\vec{q}' F_6(s+i\varepsilon, s+i\varepsilon, M^2; (-\vec{q}'+\frac{\vec{q}}{2})^2, (\vec{q}'+\frac{\vec{q}}{2})^2, q^2) \cdot S(q', q'') \quad (2.1)$$

where k is the incident lab. momentum, and where the integration path in M^2 plane is shown in Fig. 3.

The cuts on the real axis in Fig. 3 are easily understood in terms of the usual representation of the six-point amplitude through Regge poles (see Fig. 4) : in Fig. 4 the dashed blob is the Reggeon-hadron amplitude ^{*}). This amplitude has the usual unitary cuts on the real axis of the M^2 plane. The cuts in the complex plane come from the deuteron form factor, which for a spectral representation ¹⁰⁾ of the wave function is

$$S(q) = \frac{i}{2q} \int_{\lambda_0}^{\infty} d\lambda \int_{\lambda_0}^{\infty} d\lambda' g(\lambda) g(\lambda') \log \frac{(\lambda+\lambda')-iq}{(\lambda+\lambda')+iq} \quad (2.2)$$

where $\lambda_0 = \sqrt{m} B$, B being the binding energy of the deuteron. The contour of integration in Fig. 3 can be deformed (Fig. 5) and we get

^{*}) One has not to confuse this diagram with the absolutely identical unitary diagram one uses to derive the triple Regge formula ⁹⁾.

$$\begin{aligned}
 F_D &= F_D(I) + F_D(II) + F_D(III) = \\
 &= \frac{i}{2\pi k} \int d^2 q'_\perp f(\vec{q}'_2 - \vec{q}'_\perp) f(\vec{q}'_2 + \vec{q}'_\perp) S(q'_\perp) + \\
 &+ \frac{i}{2\pi k} \int d^2 q'_\perp \int_{M_0^2}^{\infty} dM^2 \Delta F_6(s+i\varepsilon, s+i\varepsilon|M^2) S[q_\parallel(M^2), q'^2_\perp] + \\
 &+ \frac{i}{2\pi k} \int d^2 q'_\perp \int_{m^2-4ik\sqrt{\lambda_0^2 + \frac{q'^2_\perp}{4}}}^{m^2-i\infty} dM^2 \Delta S[q_\parallel(m^2-i\eta), q'^2_\perp] F_6(s+i\varepsilon, s+i\varepsilon|m^2-i\eta)
 \end{aligned} \tag{2.3}$$

where

$$\begin{aligned}
 \Delta F_6(M^2) &= \frac{F_6(M^2+i\varepsilon) - F_6(M^2-i\varepsilon)}{2i} \\
 \Delta S[q_\parallel(m^2-i\eta)] &= \frac{S[q_\parallel(m^2-i\eta+\varepsilon)] - S[q_\parallel(m^2-i\eta-\varepsilon)]}{2i}
 \end{aligned}$$

and

$$\eta = |\operatorname{Im} M^2|$$

Since the low mass part of the continuum contains resonances, one could represent them with poles and the result would be similar to the Glauber term

$$\frac{i}{2\pi k} \int d^2 q'_\perp \sum_n f_n(\vec{q}'_2 - \vec{q}'_\perp) f_n(\vec{q}'_2 + \vec{q}'_\perp) S(q'_\perp, q''_\parallel) \tag{2.4}$$

For the proper continuum, we can use the triple Regge formalism, provided we remember that Fig. 4 is a Feynman diagram and not a unitary diagram. The difference lies in the 2^0 Reggeon propagator (R_j) which is calculated for $s+i\epsilon$, while for the unitarity diagram it is calculated for $s-i\epsilon$. So if we want to establish a connection with the inclusive cross-sections for $q^2=0$, we consider the R_j propagator ^{*)}

$$(s)^{\beta_j} + \sigma_j (-s-i\epsilon)^{\beta_j} = \sigma_j e^{-i\pi\beta_j} ((s)^{\beta_j} + \sigma_j (-s+i\epsilon)^{\beta_j}) \quad (2.5)$$

where σ_j is the signature and β_j is the R_j trajectory. The discontinuity is therefore

$$\Delta F_6(s+i\epsilon, s+i\epsilon|M^2) = \sum_{ij} \sigma_j e^{-i\pi\beta_j} \Delta F_6^{ij}(s+i\epsilon, s-i\epsilon|M^2) \quad (2.6)$$

so that for $q^2=0$

$$\Delta F_6(s+i\epsilon, s-i\epsilon|M^2) = -\frac{k^2}{\pi} \frac{d\sigma^{\alpha, \beta_i, \beta_j}}{dt dM^2} (t=q^2) \quad (2.7)$$

and for $q^2 \neq 0$, neglecting the $q_{\perp}^2$ dependence in the phase factors and in the triple Reggeon vertex, for $i=j$

$$\begin{aligned} \Delta F_6^{ii} &= \sigma_i e^{-i\pi\beta_i(q^2/4)} V_{hh_d}(q^2) e^{-\alpha'_i(0) q^2 \ln M^2} \\ &\cdot \frac{k^2}{\pi} \frac{d\sigma^{\alpha, \beta_i, \beta_j}}{dt dM^2} (t=q^2/4) \cdot e^{-2\beta'_i(0) q'^2 \ln s} e^{-i\pi\beta_j(q'^2)} V_{\alpha\alpha\beta_i}^2(q'^2) \end{aligned} \quad (2.8)$$

for $i \neq j$ one can use (2.7) and the expression

^{*)} $(-s-i\epsilon)^{\beta_j} = s^{\beta_j} e^{-i\pi\beta_j}$ the angle is in the third quadrant ;
 $(-s+i\epsilon)^{\beta_j} = s^{\beta_j} e^{+i\pi\beta_j}$ the angle is in the second quadrant.

$$F_6^{ij}(s_1, s_2, M^2) = V_{hh\alpha}(t) \frac{k^2}{\pi} \frac{1}{\frac{\sigma}{\sigma_i \sigma_j} \sin \pi (\alpha - \beta_i - \beta_j)} \cdot$$

$$\cdot \left[(M^2)^{\alpha - \beta_i - \beta_j} + \frac{\sigma}{\sigma_i \sigma_j} (-M^2)^{\alpha - \beta_i - \beta_j} \right] \cdot G_{\alpha \beta_i \beta_j}(t, t_1, t_2).$$

$$\cdot \frac{(s_1)^{\beta_i} + \sigma_i (-s_1)^{\beta_i}}{\sigma_i \sin \pi \beta_i} \cdot \frac{(s_2)^{\beta_j} + \sigma_j (-s_2)^{\beta_j}}{\sigma_j \sin \pi \beta_j} \cdot V_{NN_i}(t_1) V_{NN_j}(t_2) \cdot \frac{1}{s^2}$$

(2.9)

All these vertex functions are normalized to 1, if the argument is zero

$$V_{NNR}(0) = 1$$

while all the constants are contained in the triple Reggeon vertex $G_{\alpha \beta_i \beta_j}$.

For the form factor (2.2) the discontinuity in the term F_D (III) is (see Appendix A)

$$\Delta S [q_{II}(m^2 - i\eta)] = \frac{\pi N}{2 \sqrt{\eta^2/4k^2 - q_{II}^2}} \quad (2.10)$$

where $\eta = |\text{Im } M^2|$ and $N = \left[\int_{\lambda_0}^{\infty} d\lambda \rho(\lambda) \right]^2$.

If we assume that the six-point amplitude F_6 has the same asymptotic form (2.9) in the complex M^2 plane, the cut integral becomes for $\sigma/\sigma_i \sigma_j = +1$

$$F_D^{(III)} = \frac{Nk}{4\pi} \sum_{\alpha, i, j} e^{-i\pi\beta_i(q^2/4)} \cdot \frac{1}{s^{2-2\beta_i(q^2/4)}} G_{ij}^\alpha(q_i^2) \cdot \int_{\eta_0}^\infty d\eta \eta^{\alpha(q_i^2)-2\beta_i(q_i^2/4)} \frac{e^{-b_0 q_{\perp}^2}}{\sqrt{\eta^2/4k^2 - q_{\perp}^2}} \cdot \eta^{-2\beta'_i(0)q_{\perp}^2} e^{i\pi\beta'_i(0)q_{\perp}^2} \quad (2.11)$$

where $\eta_0 = 4k\sqrt{\lambda_0^2 + (q_{\perp}^2/4)}$. Here we have assumed $i=j$, to simplify the notations : we have also included all additional constants and functions of $q_{\perp}^2$ in an effective triple Reggeon vertex G_{ij}^α . We have also neglected m^2 with respect to η .

Apart from the small phase given by the $q_{\perp}^2$ dependence of the i Reggeon propagators, the phase of $F_D^{(III)}$ is completely determined by the factor $e^{-i\pi\beta_i(q^2/4)}$. That is if $\beta_i(0)=1$ $F_D^{(III)}$ is real and does not contribute to the total cross-section. Therefore in the over-all Pomeron dominance we get the Gribov result for the total cross-section ⁶⁾. For $\beta_i(0)=\frac{1}{2}$, however, it is a shadow or antishadow contribution if $\sigma_i = +1$ or $\sigma_i = -1$.

If $\sigma_i/\sigma_j = -1$, the result (2.11) is modified only by a phase factor i . Even in this case there are negative or positive contributions to the hadron deuteron total cross-section, according to the signature σ_i .

The term (2.11) should be therefore added to the formulae of Refs. 7), 8), to calculate both total and differential cross-section for hadron deuteron scattering. The real contribution of $F_D^{(III)}$ should be visible in experiments of Coulomb interference in hadron deuteron elastic scattering ¹¹⁾ or of tensor polarization of the deuteron ¹²⁾.

We have chosen here as asymptotic behaviour of the six-point amplitude the triple Regge form (2.9) : it is clear that this is true only in a first approximation, because the discontinuity of the left-hand cut could be different from the triple Regge formula. However, we expect that even a Mandelstam ^{13),14)} type of diagram has an asymptotic behaviour of the type (2.9) : therefore, our main formal results remain valid. It is reasonable, in any case, to think that at not too high energies (e.g. $s=10^3$), the contribution of the left-hand cut is small, because the complex branch point is closer to the right-hand cut.

3. - TRIPLE-POMERON CASE

We consider in this section the case of the triple Pomeron only : this case is well defined and particularly simple to discuss, because the M^2 cut integral can be performed analytically.

For a deuteron wave function of the Hultén type ¹⁵⁾

$$\Psi(r) = \frac{1}{\sqrt{4\pi}} \frac{1}{r} \sum_i c_i e^{-\alpha_i r} \quad (3.1)$$

the spectral function is $\rho = \sum_i c_i \delta(\lambda - \alpha_i)$ and

$$S(q) = \frac{i}{2q} \sum_{i,j} c_i c_j \log \frac{(\alpha_i + \alpha_j) - iq}{(\alpha_i + \alpha_j) + iq} \quad (3.2)$$

The complex cut integral contribution to the double scattering amplitude is, using (2.11), in the hypothesis that $\alpha'(0) = 0$ and neglecting m^2 with respect to η

$$F_D^{(III)} = \frac{k}{2\pi} e^{-\frac{3}{2} b_0 q_\perp^2} c_0 \sum_{i,j} c_i c_j \int_0^\infty d^2 q'_\perp e^{-2b_0 q'^2_\perp} \int_{\eta_0(i,j)}^\infty d\eta \frac{1}{\eta} \Delta S'(\eta) \quad (3.3)$$

where

$$\Delta S'(\eta) = \frac{\pi}{2 \sqrt{\eta^2/4k^2 - q'^2_\perp}}$$

and

$$\eta_0(i,j) = 2k \sqrt{q'^2_\perp + (\alpha_i + \alpha_j)^2}$$

The integral in η can be performed analytically and we get

$$F_D^{(III)} = \frac{k}{4\pi} e^{-\frac{3}{2} b_0 q_\perp^2} c_0 \sum_{i,j} c_i c_j \int_0^\infty d^2 q'_\perp e^{-2b_0 q'^2_\perp} \arcsin \frac{q'_\perp}{\sqrt{q'^2_\perp + (\alpha_i + \alpha_j)^2}} \quad (3.4)$$

It is instructive to compare this expression with the corresponding contribution of the cut on the real axis

$$F_0^{(II)} = \frac{ik}{2\pi} e^{-\frac{3}{2}b_0 q_\perp^2} c_0 \int_0^\infty dq'_\perp e^{-2b_0 q'^2_\perp} \int_{\eta_0^2}^\infty dM^2 \frac{1}{M^2} S[q'_\perp, q_\parallel] \quad (3.5)$$

While the above expression (3.4) is not energy-dependent if we exclude the trivial kinematical factor k , (3.5) is increasing with energy because S depends on k through $q_\parallel = (M^2 - m^2)/2k$. In the expression (3.4) there is actually a slight energy dependence, if we relax the approximation $m^2 \ll \eta$. The calculation can be still done analytically and the results are presented in Appendix B.

It is clear that since $F_D^{(III)}$ is real, it cannot interfere with the imaginary part as suggested recently by Dakhno¹⁶⁾.

To have a quantitative feeling of the complex cut contribution, we have calculated the ratio between (3.4) and (3.5), which turns out to be independent of c_0 . Using a standard value¹⁷⁾ for $b_0 = 6.5 \text{ (GeV/c)}^2$ and the McGee¹⁵⁾ parametrization of the deuteron wave function we get for $q^2 = 0$ and $s = 1500 \text{ GeV}^2$

$$R = \frac{F_0^{(II)}}{\sum F_0^{(I)}} = .08$$

This means that the calculation of the inelastic scattering using only $F_D^{(II)}$, equivalent to Refs. 7), 8), is reliable, for calculation of hadron deuteron differential cross-section. Only experimental quantities which are particularly sensitive to the real part should take into account also $F_D^{(III)}$.

4. - CONCLUSION

It seems from the recent Dakhno ¹⁶⁾ analysis that the calculation of the inelastic part of the total cross-section defect using the prescriptions of Ref. 8) or Ref. 9) over-estimates the experimental values. It seems, from our estimate of the complex cut integral, that this discrepancy cannot be filled by taking in account it in some way ¹⁶⁾, but only modifying the dynamics of the six-point amplitude, with more complicated diagrams than the triple Regge.

ACKNOWLEDGEMENTS

G.A. is grateful to L. Bertocchi and A. Schwimmer for many illuminating discussions. F.B. is grateful to C.I.S.R.I.F., Trieste for financial support. We thank G. Goggi for raising our interest in the problem.

APPENDIX A

In order to calculate the discontinuity of

$$S(q) = \frac{i}{2q} \int_{\lambda_0}^{\infty} d\lambda \int_{\lambda_0}^{\infty} d\lambda' g(\lambda) g(\lambda') \log \frac{(\lambda + \lambda') - iq}{(\lambda + \lambda') + iq} \quad (\text{A.1})$$

we study the behaviour of the argument of the logarithm for $M^2 = m^2 - i\eta + \epsilon$ ($\eta = |\text{Im } M^2|$)

$$\begin{aligned} \lambda + \lambda' - iq &= \lambda + \lambda' - i \sqrt{q_{\perp}^{\prime 2} + \frac{(-i\eta + \epsilon)^2}{4k^2}} = \\ &= \lambda + \lambda' - i \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2} - \frac{i\epsilon\eta}{2k^2}} = \\ &= \lambda + \lambda' - i \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} \left(1 - \frac{1}{2} \cdot \frac{i\epsilon\eta}{2k^2} \cdot \frac{1}{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} \right) = \\ &= \lambda + \lambda' - i \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} - \frac{\epsilon\eta}{4k^2} \frac{1}{\sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}}} \\ &= \lambda + \lambda' - \left| \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} \right| - \frac{i\epsilon\eta}{4k^2} \frac{1}{\left| \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} \right|} \quad (\text{A.2}) \\ &\quad \left(\sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} = -i \left| \sqrt{q_{\perp}^{\prime 2} - \frac{\eta^2}{4k^2}} \right| \right) \end{aligned}$$

In this way, we explore the negative real axis ; choosing the cut of the logarithm on this side, we obtain the discontinuity of $\ln(\lambda + \lambda' - iq)$:

$$\Delta \ln(\lambda + \lambda' - iq) = -\frac{2i\pi}{2i} = -\pi \quad (\text{A.3})$$

The discontinuity of $\ln(\lambda + \lambda' + iq)$ is not interesting here, because it is relative to the other cut in the M^2 plane (from $m^2 + 4ik\sqrt{\lambda_0^2 + (q_\perp'^2/4)}$ to $m^2 + i\infty$), so the discontinuity of S is :

$$\Delta S = \frac{i}{2q} \int_{\lambda_0}^{\infty} d\lambda \int_{\lambda_0}^{\infty} d\lambda' g(\lambda) g(\lambda') \left[\Delta \log \frac{\lambda + \lambda' - iq}{\lambda + \lambda' + iq} \right] =$$

$$= \frac{-i\pi}{2q} \left[\int_{\lambda_0}^{\infty} d\lambda g(\lambda) \right]^2 = \frac{-i\pi \mathcal{N}}{2\sqrt{q_\perp'^2 - \pi^2/4k^2}} =$$

(A.4)

$$= \frac{\pi \mathcal{N}}{2\sqrt{\pi^2/4k^2 - q_\perp'^2}}$$

APPENDIX B

In order to solve the integral

$$I = \int_{m^2 - 4ik\sqrt{\lambda_0^2 + q_{\perp}^2/4}}^{m^2 - i\infty} \frac{dM^2}{M^2 \sqrt{q_{\perp}^2 + q_{\parallel}^2}}$$

we use the relation $q_{\parallel} = (M^2 - m^2)/2k$ and then the transformations $q_{\parallel} = i\xi$ and $t = 1/(\xi - im^2/2k)$:

$$\begin{aligned} I &= \int_{-2i\sqrt{\lambda_0^2 + q_{\perp}^2/4}}^{-i\infty} \frac{dq_{\parallel}}{(q_{\parallel} + \frac{m^2}{2k}) \sqrt{q_{\perp}^2 + q_{\parallel}^2}} = (-i) \int_{-2\sqrt{\lambda_0^2 + q_{\perp}^2/4}}^{-\infty} \frac{d\xi}{(\xi - i\frac{m^2}{2k}) \sqrt{\xi^2 - q_{\perp}^2}} = \\ &= (+i) \int_{-2\sqrt{\lambda_0^2 + \frac{q_{\perp}^2}{4}} - i\frac{m^2}{2k}}^{\frac{1}{-\infty - i\frac{m^2}{2k}}} \frac{dt}{\sqrt{1 + \frac{im^2}{k}t - (q_{\perp}^2 + \frac{m^4}{4k^2})t^2}} \end{aligned}$$

(B.1)

This integral can be solved analytically and the result is :

$$\begin{aligned} I &= \frac{+i(-1)}{\sqrt{q_{\perp}^2 + \frac{m^4}{4k^2}}} \arcsin \left\{ \frac{[-2(q_{\perp}^2 + \frac{m^4}{4k^2})t + i\frac{m^2}{k}]}{2q_{\perp}} \right\} \Bigg|_{\frac{1}{-2\sqrt{\lambda_0^2 + \frac{q_{\perp}^2}{4}} - i\frac{m^2}{2k}}}^{\frac{1}{-\infty - i\frac{m^2}{2k}}} \\ &= \frac{(-i)}{\sqrt{q_{\perp}^2 + \frac{m^4}{4k^2}}} \left\{ \arcsin \left[\frac{im^2}{2q_{\perp}k} \right] - \arcsin \left[\frac{im^2}{2q_{\perp}k} + \frac{q_{\perp}^2 + \frac{m^4}{4k^2}}{q_{\perp}(2\sqrt{\lambda_0^2 + \frac{q_{\perp}^2}{4}} + \frac{im^2}{2k})} \right] \right\} \end{aligned}$$

(B.2)

REFERENCES

- 1) R.J. Glauber - Lectures in Theoretical Physics, Vol. I, Ed. W.E. Brittin (New York, 1959).
- 2) E.S. Abers, H. Burkhardt, V.L. Teplitz and C. Wilkin - Nuovo Cimento 42A, 363 (1966).
- 3) L. Bertocchi - "High Energy Physics and Nuclear Structure - 1975" (Santa Fe and Los Alamos), AIP No 26 ;
J. Weis - Acta Physica Polonica B7, 851 (1976).
- 4) J. Pumplin and M. Ross - Phys.Rev.Letters 21, 1178 (1968).
- 5) G. Alberi and L. Bertocchi - Nuovo Cimento 61A, 203 (1969).
- 6) V.N. Gribov - Zh.Eksp.Teor.Fiz. 56, 892 (1969).
- 7) A.B. Kaidalov and L.A. Kondratyuk - Nuclear Phys. B56, 90 (1975).
- 8) J. Kwiecinski, L. Lesniak and K. Zalewski - Nuclear Phys. B78, 251 (1974).
- 9) V.V. Anisovich, P.E. Volkovitskii and L.G. Dakhno - Zh.Eksp.Teor.Fiz. 63, 1576 (1972) [English Translation JETP 36, 835 (1973)] ;
V.V. Anisovich and L. G. Dakhno - Nuclear Phys. B85, 208 (1975).
- 10) L. Bertocchi, C. Ceolin and M. Tonin - Nuovo Cimento 18, 770 (1960).
- 11) V. Franco - Phys.Rev.Letters 27, 1541 (1976) ;
V. Franco and G.K. Varma - Phys.Rev. C12, 225 (1975).
- 12) D.R. Harrington - Phys.Letters 29B, 188 (1969) ;
J.F. Germond and C. Wilkin - Phys.Letters 59B, 317 (1976).
- 13) S. Mandelstam - Nuovo Cimento 30, 1127 (1963).
- 14) L. Caneschi, J.G. Halliday and A. Schwimmer - CERN Preprint TH. 2384 (1977), and private communication by A. Schwimmer.
- 15) L. Hultén and M. Sugawara - in Handbuch der Physik, Ed. by S. Flügge (Springer Verlag, Vol. 39, p. 1), (1957).
M.J. Moravcsik - Nuclear Phys. 7, 133 (1958) ;
I. McGee - Phys.Rev. 151, 772 (1960).
- 16) L.G. Dakhno - Leningrad Preprint IFVE 117 (1976).
- 17) Y. Akimov et al. - Phys.Rev.Letters 35, 763 (1975).

FIGURE CAPTIONS

Figure 1 Intermediate inelastic states in double scattering diagram.

Figure 2 six-point amplitude.

Figure 3 Branch points and cuts in the M^2 plane for the function $S \cdot F_6$.
Path of integration in the double scattering integral.

Figure 4 Regge pole representation of the six-point amplitude F_6 .

Figure 5 Deformation of the integration path.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5