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ELECTROMAGNETIC CORRECTIONS TO ASYMPTOTIC FREEDOM PREDICTIONS IN
 e^+e^- ANNIHILATION AND DEEP INELASTIC SCATTERING

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ABSTRACT

When comparing theory with experiment, both in e^+e^- annihilation into hadrons and deep inelastic μ/eN scattering, only the radiative corrections affecting the lepton vertex are taken into account. However, at high energy, other radiative corrections (propagator and hadron vertex ones) are of the same order of magnitude as asymptotic freedom corrections. In this paper we consider these radiative corrections. For e^+e^- annihilation, a complete treatment is presented (including second order gluon corrections); showing that electromagnetic and gluon corrections conspire to give a (roughly) energy independent 10% increase in the value of R predicted by the naïve parton model. For $e/\mu N$ scattering, only the (probably dominating) corrections to the propagator are calculated. Their importance is smaller than in the previous case, but they are still significant.

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1. INTRODUCTION

As is well known, the coloured quark-gluon theory¹⁾ of strong interactions (QCD) possesses the property of asymptotic freedom²⁾. In particular, this implies the prediction that the hadronic contributions to the photon propagator reproduce, at high energy, the results of the naïve parton model with known logarithmic corrections^{3),4)}, and the same holds for the structure functions in deep inelastic eN scattering⁵⁾⁻⁷⁾.

All available comparisons of the theoretical predictions with experiment, however, work to lowest order in the fine structure constant α , except for corrections to the lepton vertex. Now, other radiative corrections (corrections to the hadron vertex and hadron contributions to the photon propagator) are $O[\alpha/\pi \log(q^2/\mu^2)]$, where q^2 is the square of the c.m. energy or momentum transfer (as the case may be), and μ is a typical mass, say, $\mu \sim 0.3$ GeV. It is now clear that at already experimentally accessible values of q^2 this is of the order of a few per cent, which compares with the gluon contributions. Therefore, any precision comparison of QCD with experiment requires that account be taken of, at least, all lowest order radiative corrections. This is what will be done in this article for e^+e^- annihilation into hadrons (Sections 2 and 3) and deep inelastic eN scattering (Section 4). In the first case we will be able to present a complete discussion; in the second only the (expected to be dominant) corrections to the photon propagator shall be taken into account.

2. ELECTROMAGNETIC CORRECTIONS FOR e^+e^- ANNIHILATION: CALCULATIONS

We let, as usual,

$$R(s) = \frac{3 s \sigma(s)}{4 \pi \alpha^2}, \quad (1a)$$

where $\sigma(s)$ is the cross-section for e^+e^- annihilation into hadrons, which is the experimentally measured quantity. The relation used to compare theoretical predictions with experiment is

$$\Delta_h \Pi(s) = \alpha R(s), \quad (1b)$$

where Δ_h^Π is the hadronic contribution to (the discontinuity of) the photon propagator. Now, Eq. (1b) is only correct to lowest order in α . As explained in the Introduction, since $O(\alpha \log s/\mu^2)$ radiative corrections are comparable to gluon ones, a precision comparison^{*)} requires consideration of the first.

Among the radiative corrections, those affecting the e^+e^- initial state^{**)} have been thoroughly studied in the literature⁸⁾; they are taken into account when extracting experimental data [cf., e.g., Ref. 9)] and, therefore, shall be forgotten by us here. In view of this, we are left with radiative corrections to the final state and to the photon propagator (Fig. 1).

To first non-trivial order in α we may write

$$R(s) = R^{(0)}(s) \{ 1 + \delta_{P\ell}(s) + \delta_{Ph}(s) + \delta_V(s) + O(\alpha^2) \}, \quad (2)$$

where $\delta_{P\ell}$ is the contribution of the diagrams in Fig. 1(a), δ_{Ph} that of those in Fig. 1(b), and δ_V that of the ones in Fig. 1(c). $R^{(0)}(s)$ is then given by Eqs. (1) to zero order in α .

It is convenient to rewrite

$$\delta_{Ph} + \delta_{P\ell} = \hat{\delta}_{Ph} + \hat{\delta}_{P\ell} + \eta$$

where the $\hat{\delta}$ contain the contributions of leptons and hadrons excited at presently attainable energies ($\sqrt{s} \lesssim 9$ GeV): μ, τ, u, d, s, c , presumably; η contains the contribution of superheavy (mass > 4 GeV) quarks and leptons and is, of course, unknown. To get an idea of its possible importance, I have assumed η to be given by the contribution of two quarks b, t and a lepton ξ , all with masses ~ 5 GeV. This should give a reasonable bound on the realistic values of η ; it turns out to be very small at accessible energies.

*) It is sometimes also argued that, since asymptotic freedom arguments only apply for space-like s , direct comparison is impossible in the physical region. However, it is not difficult, by using either dispersion relations or operator product techniques, to get convinced that, except in the regions where R oscillates strongly, this difficulty may just be forgotten.

***) Including the electron loop contribution to vacuum polarization.

$\hat{\delta}_{Pl}$ is given by the well-known formula

$$\hat{\delta}_{Pl}(s) = - \frac{2\alpha}{3\pi} \sum_{i=\mu, \tau} I_i(s), \quad (3)$$

where

$$I_i(s) = \frac{5}{3} + \frac{4m_i^2}{s} + 2\left(1 - \frac{4m_i^2}{s}\right)\left(1 + \frac{2m_i^2}{s}\right)J_i(s), \quad (4a)$$

with

$$J_i(s) = \begin{cases} \frac{1}{2a_i} \log \frac{a_i-1}{a_i+1} & , s < 0 \\ a_i^{-1} \arctan(a_i^{-1}) & , 0 \leq s \leq 4m_i^2 \\ \frac{1}{2a_i} \log \frac{1-a_i}{1+a_i} & , s \geq 4m_i^2; \end{cases} \quad (4b)$$

$$a_i = \left| 4m_i^2/s - 1 \right|^{1/2}. \quad (4c)$$

If $|s| \gg m_i^2$,

$$I_i(s) \simeq - \log \frac{|s|}{m_i^2} + \frac{5}{3} + O\left(\frac{m_i^2}{s}\right), \quad (5)$$

while $|I_i(s)|$ is very small if $-4m_i^2 < s < 8m_i^2$. (This is the reason for the smallness of η). Indeed,

$$\begin{aligned} |I_i(s)| &\leq |I_i(s=4m_i^2)| = \frac{8}{3}, \quad -4m_i^2 \leq s \leq 8m_i^2; \\ I_i(s) &\simeq s/5m_i^2, \quad |s| \ll 4m_i^2. \end{aligned} \quad (6)$$

$\hat{\delta}_{Ph}$ is given^{10),11)} in terms of $\sigma(s)$. We shall write

$$\left. \begin{aligned} \hat{\delta}_{Ph}^{exp}(s) &= -2 \operatorname{Re} T_h(s), \\ T_h(s) &= \frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} ds' \frac{\sigma(s')}{s' - s} \end{aligned} \right\} \quad (7)$$

The "exp" in $\hat{\delta}_{Ph}^{exp}$ means that we take experimental values for $\sigma(s')$, up to $\sqrt{s_1} = \sqrt{s_1} = 9 \text{ GeV}$; above it we assume

$$\sigma(s') = R(s_1) \frac{4\pi}{3s} \alpha^2.$$

This is reasonable since we have lumped "future physics" inside η . Note also that Eq. (7) is only correct in the one photon approximation; it may be used as it stands as the error contributed to Eq. (2) is only $O(\alpha^2)$.

On checking QCD against experiment it is perhaps more consistent to replace $\hat{\delta}_{Ph}^{exp}$ by a theoretical expression for the same quantity. To do this, we use the QCD prediction that, in the one photon approximation,

$$\hat{\delta}_{Ph}^{th} = \delta_{PL}^{th} + \delta_{Pc}^{th} \quad (8a)$$

δ_{PL}^{th} being the contribution of the light quarks (u, d, s) and δ_{Pc}^{th} that of the charmed quark. We have⁴⁾

$$\begin{aligned} \delta_{PL}^{th}(s) &= \frac{4\alpha}{3\pi} \left(\log \frac{s}{\mu^2} - \frac{5}{3} \right) \left(1 + \frac{\alpha_G(s) \log \log(s/\mu^2)}{\pi} \right), \\ \delta_{Pc}^{th}(s) &= \frac{-8\alpha}{9\pi} \left(1 + \frac{\alpha_G(s) \log \log(s/\mu^2)}{\pi} \right) I_c(s); \end{aligned} \quad (8b)$$

here μ is a reference mass, to be obtained by fitting^{*)} $\hat{\delta}_{Ph}^{th}$ to $\hat{\delta}_{Ph}^{exp}$, and $\alpha_G(s)$ is the running gluon coupling constant^{1),2)}. Since we want to be accurate

*) The (expected) result is that μ is fixed between 0.15 and 0.3 GeV.

to the per cent level, we have to take an expression for it, correct not only to $O(1/\log s)$, but to $O((\log \log s)/\log^2 s)$: for the four-quark model¹²⁾,

$$\frac{\alpha_G(s)}{\pi} = \frac{300}{625 \log \frac{s}{\Lambda^2} + 462 \log \left[\frac{625}{537} \log \frac{s}{\Lambda^2} + 1 \right] + 75}; \quad (9a)$$

Λ^2 is a parameter defined by the condition

$$\alpha_G(\Lambda^2) = 4\pi. \quad (9b)$$

Experimentally^{6),7)}, Λ may vary between 0.15 and 0.45 GeV, approximately. The errors committed in Eqs. (8) and (9) are^{*)}

$$O\left(\frac{\alpha}{\log s}\right), \quad O(\alpha^2 \log^2 s).$$

It only remains δ_V . Using the QCD model we see that it is given by the discontinuity of the sum of the diagrams of Fig. 2, if we neglect terms of order $\alpha_G(s)\alpha$. Except at $c\bar{c}$ threshold where the calculations are unreliable anyway, we may forget the effect of m_c ; we may then use the old Jost-Luttinger calculation¹³⁾ to get

$$\delta_V(s) = \frac{3\alpha}{4\pi} \cdot \frac{\sum_a Q_a^4 \theta(s - 4m_a^2)}{\sum_a Q_a^2 \theta(s - 4m_a^2)}, \quad (10)$$

$$a = u, d, s, c.$$

*) An extra error $O(\alpha_G)$ for $s \sim 4m_c^2$, $O(\alpha_G s/5m_c^2)$ for $s \ll 4m_c^2$, and $O(\alpha_G 4m_c^2/s)$ for $s \gg 4m_c^2$ is added to the gluonic correction due to our having neglected the effect of the mass of the c quark, in this gluonic correction. This could be avoided [Ref. 6)] but there is little point in doing so: the error is appreciable at all only near $c\bar{c}$ threshold where Eqs. (8) fail anyway due to the presence of resonances.

This is very small. Not only it has not got a logarithmic behaviour but, when compared with, say, $\hat{\delta}_{Ph}$, the enhancement factor $\sum Q_a^2 \sim 3.3$ is replaced (because the charges of the quarks are <1) by the "depression" factor $(\sum Q_a^4)/(\sum Q_a^2) \sim 0.3$.

3. e^+e^- ANNIHILATION INTO HADRONS: RESULTS AND DISCUSSION

The results of the previous section are, if using $\hat{\delta}_{Ph}^{exp}$, and except for the very small η, δ_V , fairly model independent. However, we want to compare the quark-gluon QCD model with experiment. Therefore, we should use $\hat{\delta}_{Ph}^{th}$, and separate off gluon corrections to $R^{(0)}$, Eq. (2). We then write $R^{(0)} = (1 + \delta_G)R^P$ and hence

$$R^{th}(s) = [1 + \delta_{all}(s)] R^P(s); \quad (11)$$

$R^P(s)$ is obtained with purely point-like quarks (naïve parton model result): for $s \gg \mu^2$.

$$R^P(s) = 2 + \sum_H Q_H^2 \left(1 + \frac{2m_H^2}{s}\right) \left(1 - \frac{4m_H^2}{s}\right)^{1/2} \theta(s - 4m_H^2); \quad (12)$$

the sum runs over heavy quarks, $H = c, b, t, \dots$. The correction $\delta_{all}(s)$ contains two pieces:

$$\delta_{all}(s) = \delta_G(s) + \delta_{e.m.}(s) \quad (13)$$

where $\delta_G(s)$ is the gluon correction,

$$\delta_G(s) = \alpha_G(s)/\pi \quad (14)$$

and $\delta_{e.m.}$ the electromagnetic one:

$$\delta_{e.m.}(s) = (1 + \delta_G(s)) \left(\frac{\delta_{Ph}^{th}(s)}{\delta_{Ph}^{exp}(s)} + \delta_{\ell}(s) + \delta_V(s) + \eta(s) \right). \quad (15)$$

These equations, (11) to (15), take into account terms of

$$O(1), O(\alpha \log s), O(\alpha), O\left(\frac{1}{\log s}\right), O\left(\frac{\log \log s}{\log^2 s}\right);$$

The errors committed are

$$O(\alpha^2), O(\alpha^2 \log^2 s), O\left(\frac{1}{\log^2 s}\right).$$

To be able to give a quantitative error estimate, I have composed, as if they were statistical and independent, the following errors: i) Errors in $\hat{\delta}_{\text{ph}}$, taken to be just equal to the errors induced in $\hat{\delta}_{\text{ph}}^{\text{exp}}$ by the experimental errors in σ [cf. Eq. (7)]. ii) Errors in δ_G due to variations of Λ between 0.15 and 0.45 GeV. iii) As it is unknown, η is treated as if it were an error. iv) "Electromagnetic" errors, taken to be just $\delta_{\text{e.m.}}^2$. v) "Gluonic" errors, taken to be just δ_G^2 . All this gives an over-all expected error, $\varepsilon(s)$.

A last comment. The values of $\hat{\delta}_{\text{ph}}^{\text{exp}}$ were calculated using Eq. (7) and SPEAR data¹⁴⁾. The difference with the corresponding values in Ref. 11) is due to the fact that I have separated out the contribution of the τ lepton.

The results may be summarized as follows: first, numerically (Table 1). Second, graphically, the comparison with experimental R ^{14),15)} (Fig. 3)^{*)}. It is remarkable that diminishing gluon corrections, and raising electromagnetic ones, conspire to give a roughly constant $\sim 10\%$ upward shift to R^{P} . Thus, in particular, the fact that R stays flat not only is not proof against QCD but quite the converse: if δ_G were neglected then R should increase with energy, contrary to experiment.

4. RADIATIVE CORRECTIONS FOR DEEP INELASTIC eN SCATTERING

In the one-photon approximation [Fig. 4(a)] we have the well-known expression

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{[Q^2]^2} \left\{ 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} + W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} \right\}, \quad (16)$$

*) R in Fig. 3 contains the contribution for τ , counted as 1, according to Perl¹⁴⁾.

where θ is the scattering angle and E' the energy of the final electron in the lab. system and

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}, \quad \nu = E - E'$$

with E the initial electron energy.

The lowest order radiative corrections to Eq. (16) may be separated into three categories. Firstly, we have bremsstrahlung, corrections to the electron vertex and electron loop contribution to the photon propagator. These are taken into account when extracting experimental data^{16),17)}, and will therefore be forgotten in the present paper. Secondly (Fig. 4c), μ, τ and hadron corrections to the photon propagator. It will be shown that this introduces corrections of the approximate form

$$\Delta_{e.m.} \simeq \frac{2\alpha}{3\pi} C \log \frac{Q^2}{Q_0^2}, \quad (17a)$$

where Q_0 is a reference energy, that we will take to be $Q_0^2 = 1.8 \text{ GeV}^2$. C is given by

$$C = \sum_{f=\mu, \tau, u, d, s, c} Q_f^2 = 5 \frac{1}{3}. \quad (17b)$$

Lastly, we have the two photon exchange (Fig. 4c) corrections. This also contributes $\log Q^2$ (and $\log \nu$) terms, but the coefficient C is now replaced by $\langle |Q_q| \rangle < 1$: we expect these contributions to be $\lesssim 20\%$ of the previous ones. In view of this, we will consider only the μ, τ and hadron contributions to the corrections to the photon propagator (Fig. 4b). This affects the $1/Q^4$ term in Eq. (16), effectively replacing

$$W_i(\nu, Q^2) \rightarrow [1 + \delta_P(Q^2)] W_i(\nu, Q^2). \quad (18)$$

Here

$$\delta_P(Q^2) = \hat{\delta}_{Pl}(-Q^2) + \hat{\delta}_{Ph}(-Q^2) + \eta(-Q^2) \quad (19)$$

with $\hat{\delta}_P$, $\hat{\delta}_{Ph}$, η being as before [Eqs. (3), (4), (7) and (8)]. As the δ contain $\log Q^2$ terms it is clear that they will contribute to violations of Bjorken scaling that compete with the ones due to gluon corrections. To see this, let us consider for example, the case of W_2 . If we define

$$M_n(Q^2) = \int_0^1 dx \, x W_2(x, Q^2) x^{n-2}, \quad n \geq 2, \quad x = \frac{Q^2}{2m_P v}, \quad (20a)$$

then to lowest order in α , QCD gives [Refs. 5), 6) and especially Ref. 7) whose notation I follow] for, e.g., the valence contribution,

$$M_n(Q^2) = \left\{ 1 + \frac{\alpha_G(Q^2)}{\pi} [H_n + P_n + L_n(Q^2)] \right\} e^{-\gamma_{n0} \lambda / 2 \beta_0} A_n, \quad (20b)$$

where

$$\lambda = \log \left[\log \frac{Q^2}{\Lambda^2} / \log \frac{Q_0^2}{\Lambda^2} \right],$$

$$L_n(Q^2) = -\frac{3\beta_1}{2\beta_0} \gamma_{n0} \log \log \frac{Q^2}{\Lambda^2};$$

the constants H_n , P_n , γ_{n0} , β_0 , β_1 are defined in Ref. 7). By taking experimentally measured^{17), 18)} $vW^2(v, Q_0^2)$ one can calculate $M_n(Q_0^2)$ and hence eliminate the A_n , allowing thus a reconstruction of $vW^2(v, Q^2)$ for all values of $Q^2 > Q_0^2$. Since, then, $vW^2(v, Q_0^2)$ is fixed by experiment, it follows from (18) that, to take into account the electromagnetic corrections it is sufficient to replace

$$v W(v, Q^2) \rightarrow [1 + \Delta_{e.m.}(Q^2)] v W^{(0)}(v, Q^2) \quad (21a)$$

where $W^{(0)}$ is calculated via Eq. (20), i.e., neglecting radiative corrections, and where

$$1 + \Delta_{e.m.}(Q^2) = \frac{1 + \delta_P(Q^2)}{1 + \delta_P(Q_0^2)}. \quad (22)$$

It is useful to get an approximate expression for $\Delta_{e.m.}$. The δ_P are of the form

$$\delta_P(Q^2) \simeq \sum_f \frac{2\alpha}{3\pi} Q_f^2 \left\{ \log \frac{Q^2}{m_f^2} - \frac{5}{3} + O\left(\frac{m_f^2}{Q^2}\right) \right\}$$

where the sum runs over $f = \mu, \tau, u, d, s, c \dots$. Now, as δ_P is small, we can write, from (22),

$$\Delta_{e.m.}(Q^2) \simeq \delta_P(Q^2) - \delta_P(Q_0^2).$$

The dependence on the m_f^2 drops, and we get expression (17). It is this fact that makes radiative corrections less important than for e^+e^- annihilation, although, as shown in Fig. 5a, $\Delta_{e.m.}$ is still as large as each of the $H_n, P_n, L_n(Q^2)$ terms. By itself, however, $\Delta_{e.m.}$ is unable to make a significant change in the agreement of theory with experiment. The most favourable case is shown in Fig. 5b, with theoretical data from Ref. 7) and experimental points from Ref. 18).

Finally, values of $\delta_P, \Delta_{e.m.}$ are collected in Table II.

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Table I

Electromagnetic and gluon corrections for R.

a) Extrapolated fit errors, obtained varying μ between 0.15 and 0.30 GeV.

b) In brackets: error committed if taking the lowest order expression for α_G when calculating δ_G .

$\sqrt{s}$ (GeV)	2	4	6	10	30	50
$100 \times \hat{\delta}_p(s)$	0.64	0.50	0.97	1.38	2.09	2.42
$100 \times \hat{\delta}_{ph}^{th}(s)$	1.55 ± 0.21	1.83 ± 0.20	2.44 ± 0.28	3.07 ± 0.37	4.29 ± 0.27 ^{a)}	4.84 ± 0.28 ^{a)}
$100 \times \hat{\delta}_{ph}^{exp}(s)$	1.13 ± 0.21	1.60 ± 0.23	2.56 ± 0.28	3.20 ± 0.37	4.57 ± 0.53	5.31 ± 0.77
$100 \times \hat{\delta}_V(s)$	0.05	0.02	0.02	0.02	0.02	0.02
$100 \times \eta(s)$	-0.01	-0.04	-0.10	-0.18	0.57	1.23
$100 \times \delta_{em}(s)$	2.44	2.53	3.64	4.71	6.67	7.56
$100 \times \delta_G(s)$ (b)	9.30 ± 2.14 (3.0)	7.12 ± 1.22 (1.9)	6.28 ± 0.94 (1.6)	5.47 ± 0.71 (1.3)	4.31 ± 0.44 (0.8)	3.92 ± 0.36 (0.7)
$100 \times \delta_{all}(s)$ $100 \times \epsilon(s)$	11.7 ± 2.3	9.7 ± 1.3	9.9 ± 1.1	10.2 ± 0.9	11.0 ± 0.7	11.5 ± 0.7

Table II

Correction terms for the structure functions. a) Errors in $\hat{\delta}_{ph}$ as before. As these errors are strongly correlated, it is hard to compute an error for $\Delta_{e.m.}$, although it is certainly smaller than 5%, from errors in δ_p , and perhaps $\sim 20\%$ from the two-photon exchange contribution.

Q^2 (GeV ²)	1.8	4	6	10	16	25	100
$100 \times (\delta_{p\mu} + \delta_{p\tau} = \hat{\delta}_{p\ell})$	0.56	0.70	0.77	0.87	0.97	1.08	1.45
$100 \times \hat{\delta}_{ph}^a$	1.35 ± 0.27	1.64 ± 0.26	1.81 ± 0.26	2.02 ± 0.26	2.23 ± 0.26	2.29 ± 0.24	3.12 ± 0.25
$100 \times \delta_p(Q^2)$	1.91	2.34	2.58	2.89	3.20	3.5	4.57
$100 \times \Delta_{e.m.}(Q^2)^a$	0	0.4	0.7	1.0	1.3	1.6	2.6

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FIGURE CAPTIONS

Fig. 1 : Radiative corrections to the final state, and contributions other than the electron loop to vacuum polarization in e^+e^- hadrons.

Fig. 2 : Relevant diagrams for final state vertex corrections.

Fig. 3 : Comparison with experiment for R . The contribution of the heavy lepton τ has been included in the theoretical values as it is in the experimental data. Experimental points from Ref. 14) (solid dots) and 15) (small circles). The hatched band gives the theoretical prediction. The continuous line is the prediction with purely point-like quarks; the broken line separates gluon and e.m. contributions.

Fig. 4 : Relevant diagrams for deep inelastic eN scattering.

Fig. 5 : a) Comparison of electromagnetic and gluon corrections to νW_2 . Gluon corrections from Ref. 7). P_n : adding P_n . H_n : adding also H_n . L_n : adding also L_n . $\Delta_{e.m.}$: adding also $\Delta_{e.m.}$.
b) Comparison with experiment for νW_2 . Theoretical data from Ref. 7), experimental points from Ref. 17). For the definition of ASF I, MASF II, cf. Ref. 7).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5a

Fig. 5b