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SUPERSYMMETRIC REGULARIZATION AND SUPERCONFORMAL ANOMALIES

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ABSTRACT

We review the status of dimensional regularization by dimensional reduction as a regularization scheme for supersymmetric field theories with particular emphasis on the calculation of the superconformal anomalies.

1. INTRODUCTION

When discussing quantum corrections to supersymmetric field theories the question arises: is supersymmetry preserved by these corrections? One way to answer this question is to find a regularization scheme that preserves supersymmetry and then prove its consistency. Alternatively one can attempt to establish by regulator independent means whether an anomaly is unavoidable, and if not to find a regularization prescription with the desired properties. The latter approach is rather less ambitious as we do not regard the regularization prescription itself as a proof that supersymmetry is preserved. In fact, Abbott, Grisaru, and Schnitzer¹ have provided general regularization independent arguments that we can have conservation of the supersymmetry current S_μ if we abandon its spin $3/2$ character, there being a clash between the two properties. That is $\partial_\mu S_\mu = 0$ implies $\gamma \cdot S \neq 0$ and vice versa. From this result we can expect that it will be possible to find a supersymmetric regularization scheme. For such a scheme to be consistent it follows that it must be able to reproduce the *known* anomalies in

*) Talk given by P.K. Townsend at the conference on Unification of the fundamental interactions, Erice, March, 1980.

the superconformal currents. These currents are the energy momentum tensor, $\theta_{\mu\nu}$, the supersymmetry current S_μ , and the axial current, j_μ^5 , which were shown to form part of a supermultiplet by Ferrara and Zumino². Related to this is the "multiplet of anomalies" containing $\gamma \cdot S$, θ_Q^α , and $\partial \cdot j^5$ which all vanish classically for a theory without dimensional parameters. As the name suggests however, they are non-zero in the quantum theory because of perturbation theory anomalies. For supersymmetry to be preserved the coefficients of these anomalies should also be related in a particular way as discussed by Curtright³ and Lukierski⁴.

Dimensional regularization was initially avoided because of a general belief in its inapplicability to supersymmetric theories. This was because verification of supersymmetry requires algebraic manipulations specific to four dimensions, such as Fierz rearrangements. Early evidence that dimensional regularization does *not* spoil supersymmetry was a calculation by Curtright and Ghandour⁵ of the two loop renormalization constants of a model with one scalar, one pseudoscalar, and one Majorana spinor field. In the supersymmetric limit (the Wess-Zumino model) only one renormalization constant remained. More recently the supersymmetry Ward identities of this model were checked at two loops⁶. The results satisfied the Ward identities not just for the infinite and finite parts but also, surprisingly, for the parts that vanish in the $n \rightarrow 4$ limit! These results relied on some calculational rules which are

$$\text{Tr} 1 = 4 \quad 1 = \text{unit Dirac matrix}$$

$$\{\gamma_5, \gamma_\mu\} = 0 \quad \forall \gamma_\mu.$$

The first of these rules is needed to ensure various boson/fermion cancellations. It expresses the equality of boson to fermion degrees of freedom. The second rule is needed to ensure various scalar/pseudoscalar cancellations typical of supersymmetric theories.

For supersymmetric gauge theories it is not so obvious how we are to maintain the equality of bosons to fermions because in ordinary dimensional regularization a field W_μ has n rather than four components. Siegel⁷ proposed that an effective Lagrangian be obtained by dimensional reduction of the $n = 4$ Lagrangian to $n < 4$. By this procedure a vector W_μ becomes an n -component vector $W_{\hat{\mu}}$ and $(4-n)$ "ε-scalars" $W_{(\mu)}$, but the total number of components of $W_\mu = [W_{\hat{\mu}}, W_{(\mu)}]$ remains the same. For example, the effective super-Yang-Mills Lagrangian in Feynman-Wess-Zumino gauge becomes

$$\begin{aligned} \mathcal{L} = & \left\{ -\frac{1}{4} \left(G_{\hat{\mu}\hat{\nu}}^a \right)^2 - \frac{1}{2} \left(\partial \cdot W^a \right)^2 + c^{*a} \partial_{\hat{\mu}} \left(D_{\hat{\mu}} C \right)^a - \frac{1}{2} \bar{\lambda}^a \gamma^{\hat{\mu}} \left(D_{\hat{\mu}} \lambda \right)^a \right\} \\ & + \left\{ -\frac{1}{2} \left[D_{\hat{\mu}} W_{(\nu)}^a \right]^2 - \frac{1}{4} W_{(\mu)}^b W_{(\nu)}^c f^{abc} - \frac{1}{2} f^{abc} \bar{\lambda}^a \gamma_{(\mu)} \lambda^c W_{(\nu)}^b \right\}. \end{aligned} \quad (1.1)$$

The important point is that this is equivalent to the original four-dimensional Lagrangian except that all derivatives are n -dimensional. This modified form of dimensional regularization is therefore equivalent to the prescription in which all γ -matrix algebra is done in four-dimensions while only the momenta are continued to n -dimensions. Capper, Jones, and van Nieuwenhuizen⁸, and also Majumdar, Poggio, and Schnitzer⁹ have recently used this prescription to calculate the one loop contributions to a supersymmetry Ward identity of the theory defined by the above Lagrangian. The finite parts satisfy the identity, but only in the dimensional reduction scheme, not if ordinary dimensional regularization is used.

Having established that dimensional reduction, now to be called supersymmetric dimensional regularization, preserves supersymmetry Ward identities, the next question is: can we use it to calculate the superconformal anomalies? Two problems immediately arise.

i) 't Hooft and Veltman¹⁰ used a different γ_5 prescription from that of Eq. (1.1) for their calculation of the anomaly. They had $n > 4$ and a γ_5 which anticommutes with the first 4 γ_μ but *commutes* with the last $(n-4)$ γ_μ . This γ_5 prescription would generate spurious supersymmetry anomalies.

ii) The algebraic identity $\gamma_\mu \sigma_{\alpha\beta} \gamma_\mu = 0$, valid in four dimensions ensures that $\gamma \cdot S = 0$ classically. How then can we get $\gamma \cdot S \neq 0$ while using four-dimensional Dirac algebra? We shall argue later that the index μ in the above identity should be n -dimensional as in ordinary dimensional regularization. If $\sigma_{\alpha\beta}$ is also n -dimensional we have the n -dimensional identity $\gamma_\mu \sigma_{\alpha\beta} \gamma_\mu = (n-4) \sigma_{\alpha\beta}$ leading to the usual result for $\gamma \cdot S$. However, we have a four-dimensional $\sigma_{\alpha\beta}$ and therefore ϵ -scalar contributions to $\gamma \cdot S$ proportional to $\gamma_\mu \sigma_{\alpha(\beta} \gamma_\mu = 2\sigma_{\alpha(\beta} + 0(n-4)$. This coefficient does not vanish as $n \rightarrow 4$, but the ϵ -scalar index (β) will produce a single factor of $(n-4)$. Therefore ϵ -scalars can contribute at one loop to $\gamma \cdot S$ and $\partial \cdot S$.

We have investigated both of these problems in a recent article¹¹. The first one is not new. It has been discussed by Chanowitz, Furman, and Hinchliffe¹², whose ideas we have used in our analysis, although we differ with them in detail. The second problem is new and particular to supersymmetric dimensional regularization. We will now consider these two problems in turn.

2. THE AXIAL ANOMALY IN TWO DIMENSIONS

For simplicity in this talk we will consider only the two-dimensional anomaly $\partial_\mu j_\mu^5 \propto \epsilon^{\mu\nu} \partial_\mu W_\nu$ for a fermion loop in an external vector field W_ν . For the four-dimensional case we refer to Ref. 11. The relevant diagram is:

which leads to

$$\langle j_{\mu}^{5W_{\nu}} \rangle = \int \frac{\text{Tr}[\gamma_5 \gamma_{\mu} \not{\ell} \gamma_{\nu} (\not{\ell} - \not{p})]}{\ell^2 (\ell - p)^2} d^n \ell . \quad (2.1)$$

Supersymmetric dimensional regularization tells us that only the momenta p and ℓ are n -dimensional, while all γ -matrix algebra can be done as in two dimensions. Two dimensions is sufficiently simple that we can immediately calculate (2.1), using the usual integrals of 't Hooft and Veltman¹⁰ and the usual two-dimensional γ -matrix algebra

$$\begin{aligned} \gamma_{\mu} \gamma_{\nu} &= \delta_{\mu\nu} + i \epsilon_{\mu\nu} \gamma_5 , \quad \gamma_5^2 = 1 , \quad \{\gamma_5, \gamma_{\mu}\} = 0 \\ \text{Tr}(\gamma_{\mu} \gamma_{\nu}) &= 2 g_{\mu\nu} , \quad \text{Tr}(\gamma_5 \gamma_{\mu} \gamma_{\nu}) = 2 i \epsilon_{\mu\nu} . \end{aligned} \quad (2.2)$$

Let us first do the integrals. This gives contractions of the γ -matrices with n -dimensional $g_{\mu\nu}$'s leading to a factor of $(n-2)$ which cancels the pole in $(n-2)$ of the integral. The final finite result is

$$\langle j_{\mu}^{5, W_{\nu}} \rangle = 4\pi \left[\epsilon_{\mu\lambda} \frac{p_{\lambda} p_{\nu}}{p^2} - \epsilon_{\mu\nu} \right] + O(n-2) . \quad (2.3)$$

There is nothing dubious about this calculation, nor anything surprising about the result which satisfies for $n \rightarrow 2$,

$$p_{\nu} \langle j_{\mu}^{5W_{\nu}} \rangle = 0 ; \quad p_{\mu} \langle j_{\mu}^{5W_{\nu}} \rangle = -4\pi \epsilon_{\mu\nu} p_{\mu} . \quad (2.4)$$

This says that the vector current coupled to W_{μ} is conserved while the axial vector current has an anomalous divergence. What is surprising is that had we done the calculation in a slightly different way we would have found a different result! For example, let us first do all the γ -matrix algebra. This produces factors of $g_{\mu\nu}$ which contract the factors of ℓ^{μ} , p^{μ} appearing in the integrals.

The $g_{\mu\nu}$ is four-dimensional but only its n -dimensional part is retained when dotted into n -component momenta. Performing the resulting integrals gives

$$\langle j_{\mu\nu}^5 \rangle = 2\pi \left[\epsilon_{\nu\lambda} \frac{p_\lambda p_\mu}{p^2} + \epsilon_{\mu\lambda} \frac{p_\lambda p_\nu}{p^2} \right] + 0(n-2) . \quad (2.5)$$

This satisfies

$$p_\nu \langle j_{\mu\nu}^5 \rangle = -2\pi \epsilon_{\lambda\mu} p_\lambda ; \quad p_\mu \langle j_{\mu\nu}^5 \rangle = -2\pi \epsilon_{\lambda\nu} p_\lambda , \quad (2.6)$$

which distributes the anomaly equally between the vector and axial vector currents. The result of this calculation is inherently ambiguous because the n -dimensional contraction ($n < 2$) of a two-dimensional expression depends on the precise way in which the two-dimensional expression is written. This ambiguity can be seen to be a consequence of the use of two-dimensional γ -matrix algebra, in particular an anticommuting γ_5 , because we could have initially moved the γ_5 in (2.1) from the μ index to the ν index *without* change and then the first of the above calculational routes would have established conservation of the axial current and an anomalous divergence of the vector current. In all cases we get an anomaly, but where it lies is undetermined.

To further examine the consequences of a totally anticommuting γ_5 we shall redo the calculation of $\partial \cdot j^5$ as it is generally done in four dimensions. Accordingly, we multiply (2.1) by $p_\mu = (p-\ell)_\mu + \ell_\mu$ to get

$$p_\mu \langle j_{\mu\nu}^5 \rangle = - \int \frac{\text{Tr}[\gamma_5 (\ell-p) \ell \gamma_\nu (\ell-p)]}{\ell^2 (\ell-p)^2} + \int \frac{\text{Tr}[\gamma_5 \gamma_\nu (\ell-p)]}{(\ell-p)^2} d^n \ell . \quad (2.7)$$

The last integral in (2.7) vanishes on shifting the integration variable, $\ell \rightarrow \ell+p$, a manipulation that is allowed in dimensional regularization. Now if we assume in addition that

(1) γ_5 is totally anticommuting;

(2) $\text{Tr}(AB) = \text{Tr}(BA)$; A, B strings of Dirac matrices;

then the first integral in (2.7) can be written in the same form as the second to show that it vanishes also. This would give $\partial \cdot j^5 = 0$. A similar argument shows that also the vector current is conserved, a clear contradiction with the *known* existence of the anomaly. To avoid this we must give up one of the above assumptions. We require assumption (1) for supersymmetry, so we give up assumption (2), the cyclic property of the trace. Before commenting on this unorthodox proposal, let us continue the calculation without

using $\text{Tr}(AB) = \text{Tr}(BA)$. We pull the factor of $(\ell - p)$ in the first integral of (2.7) to the end of the trace using the usual two-dimensional algebra. This gives

$$p_\mu \langle j_\mu^5 W_\nu \rangle = -4i \int \frac{(\ell - p)^\mu \ell^\lambda (\ell - p)^\rho d^n \ell}{\ell^2 (\ell - p)^2} A_{\nu\rho\lambda, \mu} \quad (2.8)$$

$$A_{\nu\rho\lambda, \mu} = \left(\epsilon_{\nu\rho} g_{\lambda\mu} + \epsilon_{\rho\lambda} g_{\nu\mu} + \epsilon_{\lambda\nu} g_{\rho\mu} \right). \quad (2.9)$$

The tensor $A_{\nu\rho\lambda, \mu}$ is antisymmetric in $\nu\rho\lambda$ and so must vanish in two-dimensions. But, as it appears in (2.8), all indices are n -dimensional and $A_{\nu\rho\lambda, \mu}$ need not vanish in n -dimensions. In fact, the n -dimensional trace is

$$A_{\nu\rho\lambda, \mu} g^{\mu\nu} = (n-2) \epsilon_{\rho\lambda}. \quad (2.10)$$

Doing the integral in (2.8) produces such a trace so that the final result in the limit $n \rightarrow 2$ is

$$p_\mu \langle j_\mu^5 W_\nu \rangle = -4\pi \epsilon_{\mu\nu} p_\mu \quad (2.11)$$

as obtained previously. It is to be expected that this calculation is ambiguous and indeed it is. Different calculational routes lead to additional contributions proportional to tensors antisymmetric in three indices which simply serve to move the anomaly around between the vector and axial vector current. An equivalent ambiguity is already apparent from the fact that we could have started the trace in (2.1) at a different point. This leads to differing results because $\text{Tr}(AB) \neq \text{Tr}(BA)$.

Unlike the previous direct calculation this one involves some less obviously allowable manipulations. The result (2.10), for instance, depends on a completeness relation in n -dimensions, i.e. using the notation $\hat{\mu}$ for an index running from 1 to n , that

$$\epsilon_{\hat{\lambda}\hat{\mu}} g_{\hat{\nu}\hat{\rho}} g^{\hat{\mu}\hat{\nu}} = \epsilon_{\hat{\lambda}\hat{\rho}}. \quad (2.12)$$

Our justification for using this relation is really *a posteriori* in that the existence of the anomaly requires (2.10) from which it follows that $\text{Tr}(AB) \neq \text{Tr}(BA)$, our initial assumption. One might also wonder whether the unstated assumption that shifting integration variables is allowed may be relaxed. The point here is that without using properties of integrals in n -dimensions one can already derive that the relation $\text{Tr}(\gamma_5 \gamma_\mu \gamma_\nu) = 2i \epsilon_{\mu\nu}$ is inconsistent if the cyclic property of the trace is assumed, so that abandoning this assumption is the minimal relaxation of the usual assumptions

that gives the anomaly. We should say here that giving up the *assumption* that $\text{Tr}(AB) = \text{Tr}(BA)$ does not mean that $\text{Tr}(AB) \neq \text{Tr}(BA)$ *always*. This is to be established by calculation. The result is that all traces with an even number of γ_5 's *are* cyclic; while those with an odd number of γ_5 's are not, except for the simplest case $\text{Tr}(\gamma_5 \gamma_\mu \gamma_\nu)$. This means that no spurious anomalies will arise, a point particularly emphasized in Ref. 12. Mathematically, the failure of the cyclicity of the trace means that the γ -matrices should not be represented by finite dimensional matrices, but a specific infinite dimensional representation satisfying the desired properties is unknown. This is one fact that makes a genuine proof of the consistency of this regularization scheme difficult.

The fact that supersymmetric dimensional regularization leads to an ambiguous result for the axial current is not necessarily to be regarded as a defect. It is equally a property of every other regularization scheme, and is clearly an unavoidable concomitant of the anomaly. It is plausible that Siegel's recent argument¹³ that this regularization scheme is inconsistent is simply a restatement of this ambiguity. It is true that any ambiguity could be regarded as an inconsistency, in which case the only resort is to a regulator independent method of renormalization such as BPHZ. This method has been used successfully in supersymmetric theories by Clark, Piguet and Sibold¹⁴. However, it is reasonable that we should not have to regard the ambiguity in where to put the anomaly as a real inconsistency. Imposing gauge invariance will allow us to fix anomalous diagrams uniquely.

3. THE SUPERCONFORMAL ANOMALY

The supersymmetry current for super-Yang-Mills theory, now in four-dimensions again, is

$$S_\mu = \sigma_{\alpha\beta} G_{\alpha\beta}^a \gamma_\mu \lambda^a.$$

The first question that must be settled is whether the indices are n or four-dimensional. The natural definition of S_μ within the dimensional reduction scheme is such that the Noether index μ is n -component, while all others are four-dimensional. This follows directly by applying Noether's theorem to the dimensionally reduced Lagrangian. This allows us to get a non-zero result for $\gamma \cdot S$. At first sight it seems obvious that this non-zero result will be the same as obtained using ordinary dimensional regularization, at least at one loop, because if $\gamma_\mu \sigma_{\alpha\beta} \gamma_\mu = (n-4) \sigma_{\alpha\beta}$ produces one factor of $(n-4)$ and ϵ -scalars another, then it seems that ϵ -scalars could only contribute at two loops. In fact, this argument is not correct, as pointed out earlier, and the ϵ -scalars do contribute at one loop. The matrix element of interest is $\langle \bar{S}_\mu(p+q) \lambda_\alpha^b(q) W_\rho^c(p) \rangle$ and the ϵ -scalar contribution comes from the following graphs

in which the dotted lines represent ϵ -scalars. As we have shown in Ref. 11, these contributions give a non-vanishing contribution to the matrix element, but their contributions to $\gamma \cdot S$, $\partial \cdot S$ vanish. Neither do they destroy gauge invariance for the vector W_μ^0 . This reduces the calculation to that done by Hagiwara, Pi, and Tsao¹⁵ using ordinary dimensional regularization. These authors find $\gamma \cdot S = 0$ but $\gamma \cdot S = 3N/8\pi^2 \sigma \cdot G^a \lambda^a$ for $SU(N)$ (in our notation). They presumably calculated with $n > 4$ rather than $n < 4$, but this should not make a difference. We have redone this calculation and confirm their result.

Had we used a completely four-dimensional S_μ then necessarily $\gamma \cdot S = 0$ so that we should expect to get $\partial \cdot S \neq 0$. This is in fact the result obtained recently by Majumdar, Poggio, and Schnitzer¹⁶. In rigid supersymmetry this difference is unimportant as no gauge field is coupled to this current. However in local supersymmetry, i.e. supergravity, it is important that the anomaly be in $\gamma \cdot S$ *not* $\partial \cdot S$. Our results are consistent with a recent modification of the dimensional reduction scheme for local supersymmetry proposed by Siegel¹⁷. The axial superfield $H_\mu(x, \theta)$ that contains the graviton and gravitino fields $h_{\mu\nu}$, ψ_μ , is split under dimensional reduction as $H_\mu \rightarrow [H_\mu^\wedge, H_{(\mu)}]$. Now $H_{(\mu)}$ must be dropped from the Lagrangian altogether in order not to violate Ward identities of local supersymmetry¹⁷. This is clearly equivalent to a coupling of supergravity fields to currents whose Noether indices are n -component. Were this not the case ψ_μ would not couple to a conserved current.

We should also mention a recent paper¹⁸ in which the transversality of the gravitino of supergravity was checked at one loop using dimensional regularization. This transversality condition was satisfied only for the supersymmetric form of dimensional regularization.

CONCLUSIONS

We have not yet mentioned the trace anomaly. This is because $\theta_\alpha^\alpha \propto (n-4)\mathcal{L}$ and ε -scalars provide an extra factor of $(n-4)$, so it is manifest that the one loop result¹⁹ for θ_α^α will not be affected. Certainly for the axial current and possibly for the others, there is an ambiguity in where to place the anomaly. Imposing physical requirements, such as gauge invariance and $\partial_\mu S_\mu = 0$ eliminates this ambiguity. We then find the usual results for $\partial \cdot j^5$, θ_α^α and $\gamma \cdot S$, such that these anomalies form a supersymmetry multiplet at one loop. This result agrees with the superfield results reported at this conference by Grisaru²⁰. At two loops the situation is not quite so clear. It seems likely, from our results and from the superfield approach, that the anomalies will continue to form a supersymmetry multiplet. If so, this would be an explicit demonstration that the Adler-Bardeen non-renormalization theorem for the axial anomaly does not apply in this model.

In conclusion we can say that supersymmetric dimensional regularization has been shown to deserve its name.

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