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THE FERMION SPECTRUM ACCORDING TO GUTS

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A B S T R A C T

Properties of the fermion spectrum, including masses, charges, helicities, representation content, etc., are analyzed in the framework of grand unified theories. A new mechanism for "light" fermion mass generation through radiative corrections involving superheavy particles is presented. An application is made of this mechanism using an exceptional GUT, based on E_6 . Some comments concerning the neutrino mass (?) spectrum are also offered.

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PROLOGUE

Gauge theories seem to be nature's favourite candidate for describing correctly and elegantly the "observed" (fundamental?) interactions. Among other things gauge theories are supposed to shed light on and improve our understanding of many of the age-old "mysteries" of the observed fermion spectrum. Clearly such hopes have been at least partially fulfilled, especially with the advance of grand unified theories, but still a lot of "mysteries" remain unsolved. My purpose here is to report on our present status of understanding of the fermion spectrum including some old and some very recent work¹ on the subject.

1. QUESTIONS

In order to appreciate the strength of grand unified theories² (i.e., theories that unify strong and electro-weak interactions) let us see what kind of world is opened to us by the "standard" $SU(3)_C \times SU(2) \times U(1)$ ($\equiv G_1$) model. The well-known fermionic structure is as follows:

$$\text{generation} \equiv \left\{ \begin{array}{ll} \begin{pmatrix} u \\ d \end{pmatrix}_L & ; \quad u_R, d_R \\ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & ; \quad e_R \end{array} \right. \quad (1)$$

with the understanding that quarks come in three colours (leptons are colour singlets) and there are at least two more generations with identical structure ((c,s,μ,ν_μ) ; (t,b,τ,ν_τ)). Definitely such a fermionic structure is consistent with all available "low energy" phenomenology ($E < 100$ GeV (?)) but certainly it leaves unanswered too many questions. Let us enumerate some of them:

- 1) Why is $Q_{\text{proton}} + Q_{\text{electron}} = 0 (< 10^{-20})$?
- 2) Why do leptons and quarks have similar weak interactions, e.g., why is there

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and not} \quad \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}_L \quad \left(\text{or equivalently} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R \right) ?$$
- 3) Why are quarks heavier than leptons belonging to the same generation ?
- 4) How many generations are there ?
- 5) Why are the charged and neutral weak currents mixtures of V and A ? Or equivalently why do L- and R-fermionic components transform differently under SU(2) ?
- 6) Why do all the observed generations seem to have identical structure under G_1 ? (i.e., the so-called generation reproduction problem).
- 7) Can we understand the mass gaps between different generations ?
- 8) What is the origin of the Cabibbo-like angles and why are they small ?
- 9) Why is $t > b$ and $c > s$ but $u < d$? *)
- 10) Are the neutrinos really massless ?

We will show next that in the framework of GUTs most of these questions are satisfactorily answered.

2. ANSWERS ("THE CLASSICS")

The basic hypothesis of the "grand synthesis" is that there exists a simple group $G \supset G_1$, which is characterized by a single coupling constant g , and that all interactions are generated by G . Quarks and leptons are in general members of the same multiplet(s) of the

*) The names of quarks and leptons represent also their masses.

group G . Then at some superhigh energy scale ($M_1 \sim 10^{15}$ GeV) G suffers a breaking down to G_1 and then around $M_2 \sim 100$ GeV G_1 breaks down in the usual way to $G_2 \equiv SU(3)_C \times U(1)_{E.M.}$.

$$\begin{array}{ccc}
 G & \xrightarrow{M_1 (\sim 10^{15} \text{ GeV})} & G_1 \left(\equiv SU(3)_C \times SU(2) \times U(1) \right) \xrightarrow{M_2 (\sim 10^2 \text{ GeV})} \\
 & & \xrightarrow{\hspace{1.5cm}} G_2 \left(\equiv SU(3)_C \times U(1)_{E.M.} \right)
 \end{array} \quad (2)$$

If such a scenario is realized in nature then a few things follow immediately.

1) Since G is a simple group, electric charge is quantized. Actually the charge operator Q is a generator of the group G and thus traceless and so when it is acting on any representation of G containing both quarks and leptons it is bound to give some relation between quark and lepton charges. For example, in $SU(5)^3$ one finds that $Q_d = 1/3 Q_e$, where the factor of three appears because there are just three colours! So there is no mystery anymore why $Q_{\text{proton}} + Q_{\text{elect}} = 0$.

2) Since quarks and leptons are sitting in the same representation(s) of the group G , it is natural to find correlations between their helicities and similar responses to weak interactions which are contained in the G -interactions. For example, in $SU(5)$ one finds that $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ and $\bar{d}_L$ are contained in the same multiplet ($\bar{5}$) and so d_R is not available to have $SU(2)$ weak interactions. Only d_L is still available, and behold another multiplet (namely the 10_L) contains the $\begin{pmatrix} u \\ d \end{pmatrix}_L$ doublet and there is no more mystery why

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \text{ goes with } \begin{pmatrix} \nu \\ e \end{pmatrix}_L \text{ and not } \begin{pmatrix} \nu \\ e \end{pmatrix}_R .$$

3) The fact that quarks and leptons share the same representation(s) of G , has another important consequence. Namely, there are relations between quark and lepton masses. At superhigh energies where G is a good symmetry, quark and lepton masses are more or less equal (modulo some possible Clebsch-Gordan coefficients). At lower energies though, when G is badly broken, higher order corrections due to the "standard" $SU(3) \times SU(2) \times U(1)$ interactions modify these "symmetry relations" in an appropriate way: making quarks heavier than leptons, mainly because quarks have strong interactions and leptons do not. Quantitatively, one finds⁴ that if $m_b = m_\tau$ at superhigh energies as it happens in many models, then

$$\frac{m_b(Q=Y)}{m_\tau} \simeq \left[\frac{\alpha_S(Q=Y)}{\alpha_S(Q=M_1)} \right] \frac{12}{33-2f} \quad (3)$$

where α_S is the strong coupling constant evaluated at the mass of Y , $M_1 \sim 10^{15}$ GeV and f is the number of flavours. Using experimental data to determine α_S and taking $f = 6$, we get⁴ from Eq. (3) $m_b/m_\tau \simeq 2.7-3$ which is the observed value. So we are definitely on the right track.

4) A very important point to make here is that Eq. (3) is sensitive⁴ to the number of flavours (f). Two loop corrections⁵ to (3) determine uniquely⁵ that $f > 6$ would imply an unaccepted m_b/m_τ value. So GUTs not only explain why quarks are heavier than leptons but also determine that there are only three generations^{4,5}. This piece of news is most welcome to cosmologists, who have determined⁶ independently that the observed He abundance of the universe implies not more than three light neutrinos, i.e., no more than three generations. We move next to some new applications of GUTs.

3. ANSWERS ("MODERN TIMES")

Before getting to the real meat of the new applications of GUTs, let me formulate a few principles that we should find helpful in the following. Undoubtedly physics is living through its GAUGE AGE and most of us are believers in the

Central Dogma of the gauge age

- " a) All fundamental interactions are gauge interactions.
- b) Gauge symmetries may be broken only spontaneously.
- c) Exact symmetries in Nature are always local. "

I want to call attention to c) which says that that Nature does not like to have around exact global symmetries not coming from local symmetry*), which is of great importance in our case in discussing neutrino masses. Another principle of great importance is related to the symmetry breaking scheme as in Eq. (2). It is natural to assume that at each step of symmetry breaking, fermions that may receive masses that are invariant under the unbroken group indeed do so, and thus we are led to the

*) For instance non-perturbative gravitational effects break any global (but not local) symmetry⁷.

Survival hypothesis^{1,8}: "Low mass fermions are those that cannot receive G_1 -invariant masses"^{*)}.

Armed with the above principles we now try to answer the rest of the questionnaire.

5) According to the survival hypothesis, fermions may not have their L- and R-components transforming identically under $SU(3) \times SU(2) \times U(1)$, because then they will all have masses $M_1 \sim 10^{15}$ GeV. Since nature chooses to have $SU(3)_C \times U(1)_{E.M.}$ "vector-like" our only hope is that $SU(2)$ is chiral.^c The simplest possibility for non-trivial weak interactions is to have

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L ; f_{1R}, f_{2R}$$

(or vice versa!) and that seems to be realized by nature. Putting things in triplets, etc., looks messy, uneconomical and ugly. So, accepting the survival hypothesis, GUTs provide a reason for chiral $SU(2)$ weak interactions (i.e., charged and neutral weak currents are a combination of V and A) namely the very existence of "light" fermions.

6) Furthermore if a second or third or ... "light" generation has to exist, then again we should make sure that it is impossible to create G_1 invariant fermion masses, not only inside the same generation but also through inter-generation couplings. For instance, a world made of

$$\begin{pmatrix} u \\ d \end{pmatrix}_L ; u_R, d_R \quad \Bigg| \quad \begin{pmatrix} c \\ s \end{pmatrix}_R ; c_L, s_L$$

is unacceptable since by coupling

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \text{to} \quad \begin{pmatrix} c \\ s \end{pmatrix}_R$$

we may create G_1 -invariant fermion mass terms, which will mean that at least some of the u,d,c,s quarks will be superheavy. Easiest way out: make every "new" generation a copy of the first one.

So we may begin to understand why weak interactions have the observed structure and why if there are two or more generations, they all need to be photocopies of the first one. Last but not least, let me briefly comment on the constraints that are

^{*)}Sometimes G_1 may be enlarged to contain some other group beyond $SU(3) \times SU(2) \times U(1)$.

imposed on the fermion representation content of the theory by demanding the absence of the Adler-Bell-Jackiw anomalies. We all agree that

- a) there is strong experimental evidence that at least "low energy" physics is well described by renormalizable field theory and
- b) the existence of anomalies spoils renormalizability,

so then we have to accept that the "effective" $SU(3) \times SU(2) \times U(1)$ theory had better be anomaly free. It is quite remarkable, as found by Bouchiat-Iliopoulos and Meyer, that each generation [Eq. (1)] is anomaly free, with a cancellation between the anomalies of the quark and lepton sector. Here lies our main point: the Survival Hypothesis forbids the existence of "low energy" vector-like weak interactions which would not only be automatically anomaly-free for each generation [Eq. (1)], but furthermore each generation's quark and lepton sector is anomaly-free separately. So again the simplest and maybe the most economic choice of an anomaly free fermion representation is that given by Eq. (1). Perhaps we would invent leptons (quarks) if we had only known about anomalies, and that quarks (leptons) exist.

Clearly the inter-relation between the existence of "low-mass" fermions, fermion representation content, and absence of anomalies is one of the most fascinating aspects of our "low-energy" theory. Before getting any further I will need some more input that I will provide next.

4. A NEW MECHANISM FOR "LIGHT" FERMION MASS GENERATION

We saw in the last section that "light" fermions are light because their L- and R-components transform differently under weak $SU(2)$. Presumably by the same token "superheavy" fermions are superheavy because their L- and R-components transform identically under weak $SU(2)$ (i.e., their weak interactions are "vector-like"). And here lies the clue of the new mechanism¹. Suppose that in some unified theory of strong-electro-weak and possibly generation changing interactions there are "light" and "superheavy" fermions lying in the same representation of the unified group $G'(\supseteq G)$. Then in general there will be gauge bosons which couple "light" to "superheavy" fermions and presumably diagrams like those in Fig. 1 are allowed:

Fig. 1

Let us look closer at Fig. 1. Clearly the gauge boson that transforms $f_L \rightarrow F_L$ cannot be the same as the one which transforms $f_R \rightarrow F_R$. For reasons that we explained before, these gauge bosons should have different weak SU(2) transformation properties and so there should be a mixing of order M_W/M_1 . But then we get a finite contribution δm of the form:

$$\delta m_f = \frac{\alpha}{\pi} \cdot M_W \cdot \frac{M_F}{M_1} \cdot O(1) \quad (4)$$

This is our main new result. Since we do not expect M_F/M_1 to be drastically different from $O(1)$, we see that amazingly enough we have been able to create "light" fermion masses of the correct order of magnitude just by radiative corrections involving "superheavy" gauge boson exchanges and "superheavy" fermions. Needless to say that such a mechanism may be used iteratively if different symmetry breaking scales are present.

The natural appearance of such radiatively induced mass terms (diagonal and off-diagonal) of the "right" magnitude fill us with hope and joy that finally we can handle the main questions of Section 1 concerning the understanding of the mass generation gap and the smallness of the Cabibbo-like angles. Indeed one may envisage a situation where we start at tree level with only the third generation massive, and the first and second ones massless, with one-loop corrections then generating masses for the second generation $[\sim (\alpha/\pi)M_W]$ and two-loop corrections generating masses to the first generation $[\sim (\alpha/\pi)^2 M_W]$. Then we would get the correct mass generation gap plus small and calculable Cabibbo-like angles, through the off-diagonal mass terms, which are induced in the diagonal-Cabibbo matrix with which we started.

We will discuss next possible scenarios where such situations may be realized.

5. SCENARIOS OF "LIGHT" FERMION MASS GENERATION

5.1 Poor man's view

The simplest case of the proposed "radiative" mechanism is to assume that no "light" fermion gets a tree level mass (let us call it a "direct" mass to distinguish it from the radiatively induced mass: a "radiative" mass). All "light" masses are "radiative" and the observed fermion mass spectrum is nothing else but a "reflection" of the superheavy fermion mass spectrum; i.e., play with the M_F/M_1 ratio of Eq. (4). In other words Eq. (4) acts like a "transport" equation and the real game is played at superhigh energies. This approach has the virtue that we may start with only one mass parameter say M_{Planck} and get all other masses for free (radiatively).

Actually this has been already proposed⁹ for the gauge bosons and Higgs particles ("light" and superheavy) in connection with the solution of the famous gauge-hierarchy problem. So there is a good chance that given M_{planck} all other masses of all particles are created radiatively in a most agreeable and natural way. It should be noticed that it is much easier to restrict the Yukawa couplings of "superheavy" fermions using discrete or continuous symmetries, involving superheavy multi-Higgs systems because these do not suffer from the well-known naturality constraints on "light" multi-Higgs systems¹⁰.

5.2 Rich man's view

One may try to be a bit more imaginative. Thus suppose that we consider a unified theory of strong-electro-weak and generation changing interactions described by a gauge group $G' \supset G$ ($\equiv$ the strong and electro-weak interactions grand unified group). Let us assume now that fermions are thrown in different inequivalent representations ψ_i ($i = 1, 2, \dots$) of G' , with only the constraint

$$\left. \begin{array}{c} \sum_i \psi_i \\ \text{anomalies} \end{array} \right|_G = 0 \quad *)$$

and then presumably (if we are lucky) we will be left finally with only three-"light" generations of the "standard" model and everything else will be superheavy. Then it is not inconceivable to imagine that G' -invariance forbids at the tree level all but the third generation to get a "direct mass". This is possible because the different generations belonging in general to inequivalent reps of G' . Then one may imagine that a one-loop "radiative" mass is allowed for the second generation, but not for the first one, since again they belong to inequivalent reps of G' and the gauge transitions desired to complete the one-loop diagram for the first generation may not exist. Finally one may hope that in a natural way the first generation gets a two-loop-"radiative" mass. So we may get a satisfactory qualitative explanation of questions 7) and 8) but we need explicit and detailed calculations to answer 9). One may ask why nature has done such random thing? i.e., why has she thrown different generations in inequivalent G' -reps? Well, the obvious answer that comes to one's mind is that after all perhaps quarks and leptons (together with gauge bosons and Higgs particles) are not fundamental particles but composites of more basic object, preons (?). The energy scale where this compositeness is realized may be superhigh ($M_1 \sim 10^{15}$ GeV ; $M_{\text{pl}} \sim 10^{19}$ GeV ?) so we need not worry much about it yet. But if this is the case then we should also not care about nature's randomness because composite things may indeed fall into surprising representations. Actually, recent efforts¹¹ to unify grand unified theories with gravity through

*) In this case G' is very strongly broken and may only be useful as global classification symmetry.

supergravity, provide just such a picture. Then I strongly believe that the scenario proposed in this subsection when worked out in detail may solve the last "mysteries" of the fermion spectrum.

After all these speculations let me come to a more concrete proposal.

5.3 A Greek-Latin view¹: AN EXCEPTIONAL GUT

A modest realization of the new ("radiative") mechanism¹ for "light" fermion mass generation is offered by the group E_6 . Of course, this application is less ambitious than the scenario just described above, since E_6 claims to unify only strong and electro-weak interactions, and does not contain any generation changing interactions. Still, E_6 has enough structure to serve as a useful and simple example. E_6 has very nice and interesting properties and it is a serious candidate for being the correct group that unifies strong and electro-weak interactions. Here I am going to use those E_6 properties that serve my purposes, but the interested reader should consult Ref. 1) for all the details of the specific E_6 model that I am using here. Fermions of each family lie in the fundamental $\underline{27}$ representation. The nice property of the $\underline{27}$ is that under $SU(3) \times SU(2) \times U(1)$ it splits into the observed 15-plet (counting helicity states) of "light" fermions of Eq. (1) plus a self-conjugate 12-plet of fermions which will become superheavy in accord with our survival hypothesis^{1,8}. The 12-plet of superheavy fermions transforms under the subgroup $SU(5)$ as $\underline{5} + \bar{\underline{5}} + \underline{1} + \bar{\underline{1}}$; we call them (D_i, E^C, N^C) , $(\bar{D}_i, \bar{E}, \bar{N})$, ν^C and L . Of the last two neutral superheavy leptons, L is also an $SO(10)$ singlet, whereas ν^C is the member of the 16-plet under $SO(10)$ which is apparently missing from the "observed" spectrum. As to the Higgs representations we use the $\underline{351}_S$ and the $\underline{27}$, the first is capable of breaking $E_6 \rightarrow SU(3) \times SU(2) \times SU(1)$, and, giving superheavy masses to the self-conjugate 12-plet of fermions whereas the $\underline{27}$ breaks $SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{E,M}$, and it should at the same time give masses to the top generation. We assume at this level, that it will be possible for the charm and up-generation to remain massless in a natural way. What we have in mind is that eventually E_6 will be contained, together with some group of generation changing interactions in some bigger group G' so that the scenario presented in the preceding subsection will be fully realized. A relatively simple pattern of "radiative" masses emerges for the charm generation if one assumes a hierarchy in the "superheavy" fermion masses according to the suggestive cascade $E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ (i.e., $m_L \gg m_{\nu^C} \gg m_{D,E}$). In this case a careful inspection shows that the dominant contribution to the μ - and the s -quark masses comes from L -exchange (Fig. 2).

Fig. 2

whereas the charm quark receives contributions from L exchange and from ν^c exchange (Fig. 3)

Fig. 3

The contributions are in general comparable, because the higher mass of L relative to ν^c is compensated by the higher masses of the gauge bosons exchanged in Fig. 2 relative to those of Fig. 3. This comes from the hierarchical pattern being postulated, which in turn suppresses all the diagrams with D_i or E exchanges. A similar suppression actually takes place for the contributions to the $Q = -1/3$ and $Q = -1$ masses coming from ν^c exchange, basically because they have superheavy partners unlike the $Q = 2/3$ quark. From the diagram of Fig. 2 $s \approx \mu$ arises at superhigh energies whereas the additional contribution to the c quark mass from Fig. 3, although not possible to estimate numerically because of the extra parameters involved, gives a rather straightforward qualitative explanation^{*)} why $c > s \approx \mu$. All the charm generation masses produced in this way are $O((\alpha/\pi)M_W)$, which is what we find experimentally. Concerning the up-generation masses, one finds that if we stay within E_6 interactions (as we did for the charm generation masses), then we find zero masses to all orders in perturbation theory. Maybe this is an encouraging result, since we know that $u < d$ and $d \neq e$ in contrast with $c > s$, $s \approx \mu$ and $t > b$, $b = \tau$. How then does the up-generation get mass? Well, we would employ post E_6 interactions, which are there anyway (generation changing interactions, gravity, or $?^{10}$) and they may well generate masses of a few MeV (or $(\alpha/\pi)^2 M_W$ i.e., probably mass generation through two loops). So that is no big sweat. I hope to have shown with this simple example how we expect to get reasonable (and hopefully correct answers) to questions 7), 8) and 9) within the general framework described in this section.

^{*)} I believe that this explanation survives in more complicated schemes than the one considered here.

Before embarking on the discussion of neutrino masses let me make a few more remarks on the structure of the charged fermion mass spectrum. The old dream that some masses in the fermion spectrum are "radiatively" generated seems to be realized. Namely, quark-lepton mass differences are generated^{4,5} through radiative corrections involving the "standard" strong and electroweak interactions - essentially, quarks are heavier than leptons because quarks participate in strong interactions and leptons do not. On the other hand inter-generation mass differences - or quark doublet mass differences - may be explained¹ through radiative corrections involving "post" - $SU(3) \times SU(2) \times U(1)$ - interactions, plus the fact that under the unification group G' of everything, different generations transform inequivalently and so gauge invariance of G' naturally prevents certain types of "direct" Yukawa couplings from appearing. A final comment concerning the mass of the as yet unseen top quark. In some grand unified theories like $O(10)$, E_6 , etc. suitably (and reasonably) constrained the following relation holds¹:

$$\frac{t}{\tau} = \frac{c}{\mu} \quad . \quad (6)$$

At the toponium mass (T), i.e., taking into account finite renormalization effects when extrapolating from the charmonium region (ψ), this relation becomes

$$t(T) \approx \frac{\tau}{\mu} \cdot c(\psi) \cdot \left[\frac{\alpha_S(T)}{\alpha_S(\psi)} \right]^{\frac{4}{7}} \quad (7)$$

Using appropriate values for α_S and $c(\psi)$ we get¹

$$t \approx 18 - 22 \text{ GeV} \quad , \quad (8)$$

almost inside the energy range of PETRA or PEP. Thus, we should not be surprised by the discovery of the top quark very soon.

6. NEUTRINO MASSES (?)

Neutrinos, these mysterious and fascinating chargeless, massless (or nearly so), feebly interacting objects are now with us for exactly fifty years, after their "theoretical" discovery by Pauli, in 1930. Though until recently, we have not really had a satisfying answer to the question of their masses (question 10). In the "standard" $SU(3) \times SU(2) \times U(1)$ model a direct Dirac mass term of the form $\bar{\nu}_R \nu_L$ is prohibited by making the neutrino a two component spinor. A Majorana mass term of the form $\bar{\nu}_L \nu_L$ is prohibited since it violates lepton number, which is supposed to be an exact global symmetry.

Concerning the absence of a Dirac mass term, the argument is clearly circular, and is certainly not a theoretical explanation. With the advance of GUTs the mechanism of L conservation that forbids a Majorana mass term cannot hold water. Lepton number is violated, as it should, because it is only a global symmetry and thus should be broken, according to the "central dogma of the gauge age". So we are left in the woods. This is not exactly the case, because in grand unified theories we have mechanisms¹² that may give an appropriate explanation of the neutrino masses. In general, there are particles around (N) that may serve as the right-handed missing partners of ν_L , but since they are $SU(3) \times SU(2) \times U(1)$ singlets, they acquire according to the survival hypothesis superheavy Majorana masses (M). So a general neutrino mass matrix will eventually look^{*)} like:

$$\begin{array}{cc} & \begin{array}{c} \nu \\ N \end{array} \\ \begin{array}{c} \nu \\ N \end{array} & \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \end{array} \quad (9)$$

where the off-diagonal terms (m) correspond to "normal" Dirac mass terms and thus they are naturally related to the energy scale that $SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$ E.M.. Clearly (9) implies a neutrino mass:

$$m_{\nu_L} \approx \frac{m^2}{M} \quad (10)$$

which may be anything between (10^{-5} eV $\rightarrow$ few eV), due to the uncertainties concerning the values of m and M used in (10), which in turn depend crucially on the specific model in use. This is the GUT answer for question 10). Certainly such values of neutrino masses are acceptable phenomenologically and they are very interesting from an experimental point of view, since the upper part of these predictions lies close to the presently available experimental upper bounds. Looking for effects of such masses, neutrino oscillations, distortion of the end-points of the energy spectrum in β decay, etc. may be a fruitful and worth while enterprise. I want to stress the point that Eq. (10) emerges naturally and so we understand⁴ that corrections to the relation $M_Z \cos \theta_W = M_W$ are negligible, because the $SU(2)$ symmetry breaking is predominantly through a Higgs doublet.

The point is that a Majorana mass term $\nu_L \nu_L$ may effectively break $SU(2) \times U(1)$ similarly to a Higgs triplet. Then the relation $M_Z \cos \theta_W = M_W$ gets spoiled if the breaking remains unchecked, but Eq. (10) assures us that GUTs take care of this problem in a very satisfying way⁴. Among grand unified theories, minimal $SU(5)$ ³

^{*)} In fact the top left-hand entry should not be exactly zero: for reasons of naturalness¹² it will in general be $O(m^2/M)$.

uniquely predicts massless neutrinos in a straightforward way. The neutrinos do not have right-handed components because there is no space for them in the $(\bar{5} + 10)$. The neutrinos do not have Majorana mass terms because they violate B-L, which is a global (accidental) symmetry of minimal SU(5)³. Recently efforts have been made to undo both the above-mentioned arguments. It is possible that there are other representations of SU(5) around¹¹ (presumably self-conjugate under "SU(3) $\times$ SU(2) $\times$ U(1)" and thus possessing superheavy masses) which may contain particles which act as right-handed components of the "observed" neutrinos, or they may be "post" SU(5) interactions that break B-L and thus allow Majorana mass terms¹³. In both cases, though, it should be mentioned that the predictions for the neutrino masses should lie very close to the lower part of the range mentioned above (say, closer to 10^{-5} eV than to 1 eV). Other GUTs, like O(10), E_6 , etc., since they are L-R symmetric contain naturally*) in the same representation both ν_L and ν_R and thus Eqs (9) and (10) apply directly to them, with the neutrino masses able to cover the whole range of 10^{-5} eV $\rightarrow$ few eV. So there seems to be a crucial distinction between the "extended" SU(5) model and other GUTs concerning the probable range of neutrino masses. It remains to be seen if nature uses her last chance to re-establish L-R symmetry at superhigh energies (which could perhaps be needed if we want to unify with gravity, which respects L-R symmetry) or if she prefers to remain always left-handed and so we have to swallow the fact that we are living in a lopsided world! A perhaps not unrelated matter concerns quark-lepton symmetry. In contrast with SU(5) where quarks and leptons are thrown randomly in $(\bar{5} + 10)$, in all other GUTs at superhigh energies SU(4)_{colour}¹⁴ becomes a good symmetry, i.e., lepton-number serves as a fourth colour and so the observed quark-lepton symmetry looks less mysterious and finds a rather aesthetically and conceptually appealing explanation. On the other hand proponents of the compositeness of quarks and leptons may argue that the randomness of SU(5) should not worry us because since quarks and leptons are not fundamental they may find themselves sitting in rather random-looking representations. Clearly a lot of theoretical and experimental effort is needed before we decide which picture makes more sense, and clearly the neutrino mass spectrum may play a key role.

*) We recall that O(10) and E_6 are the only Lie-groups with the following desirable properties:

- i) they are automatically anomaly free.
- ii) they have useful complex representations and
- iii) the fermions of each generation belong to an irreducible representation 16 [O(10)], 27 [E_6].

EPILOGUE

The most tantalizing predictions of GUTs concerning the fermion spectrum refers to the number of quarks (not more than six)^{4,5} and the neutrino mass spectrum^{12,13} (10^{-5} eV $\rightarrow$ few eV). In the next few years, experimentalists no doubt will put these predictions under scrutiny, but I hope that their results will not jeopardize my firm belief and conviction that grand unified theories are right.

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