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ATTEMPTS AT SUPERUNIFICATION^{*)}

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A B S T R A C T

We describe attempts to construct a phenomenologically viable model which embeds a unified theory of strong, weak and electromagnetic interactions in an extended supergravity theory. Following Cremmer and Julia, we assume that the local unitary symmetry which they found for $SO(N)$ supergravity theories is realized dynamically, and that the usual quarks, leptons, gauge bosons and Higgs scalars of the renormalizable gauge theory are composite states which appear point-like on a mass scale below the Planck mass. We argue that this picture leads uniquely to $SO(8)$ (or $SO(7)$) supergravity with $SU(5)$ as the grand unification gauge group. We find that conventional symmetry breaking schemes according to which particles acquire all allowed invariant masses at each stage of a step by step symmetry breakdown do not yield an acceptable particle spectrum. We argue that the original symmetries of the composite supermultiplet may break dynamically to an effective anomaly free renormalizable sub-theory, with the underlying supersymmetry serving at most to restrict the particle spectrum and to specify the couplings of the effective renormalizable Lagrangian at the Planck mass. Preliminary investigations suggest that this approach may be more promising.

^{*)} Presented at the Europhysics Study Conference - "Unification of the Fundamental Interactions", Erice, 17-24 March 1980.

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1. INTRODUCTION

At present most particle physicists believe that the known interactions are described by a renormalizable gauge theory, namely

$$SU(3)_{\text{colour}} \otimes SU(2)_{\text{left}} \otimes U(1) \quad (1)$$

and many theorists believe in addition that these interactions become unified¹⁾ at a mass scale of about 10^{15} GeV. The minimal grand unified theory (GUT) which can incorporate the product group (1) is the $SU(5)$ of Georgi and Glashow²⁾. This picture has met with some phenomenological success. The assumptions of a common unification mass scale for the three low energy gauge groups and of essentially no new physics below this scale determine³⁾ the weak angle which parameterizes neutral current couplings at the value^{4),5)}

$$\sin^2 \theta_w = 0.206 \pm 0.006, \quad (2)$$

compatible with values obtained from analyses^{6),7)} of the present experimental data

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$$\left. \begin{aligned} \sin^2 \theta_w &= 0.229 \pm 0.008 \text{ (exp.)} \pm 0.006 \text{ (theor.)} \quad (\text{Ref. 6}) \\ \sin^2 \theta_w &= 0.238 \pm 0.011 \quad (\text{Ref. 7}) \end{aligned} \right\} \quad (3)$$

In addition, the minimal GUT relates quark and lepton masses²⁾ at 10^{15} GeV. Evaluation of symmetry breaking effects⁸⁾ gives, for example the ratio^{4),9)}

$$m_b/m_\tau \approx 3, \quad (4)$$

again compatible with experiment. A further crucial test of at least the minimal GUT will soon be forthcoming with searches for nucleon decay with the predicted lifetime¹⁰⁾

$$\tau_{\text{proton}} \approx 10^{30 \pm 2} \text{ years.} \quad (5)$$

However, the existing models for grand unification also have conspicuous failures which strongly suggest that they are incomplete. They contain far too many free parameters: most of the fermion masses and weak mixing angles are undetermined, as well as the couplings and even the field content of the scalar sector. There is a notorious "gauge hierarchy" problem which is the difference of about 12 orders of magnitude in the mass scales at which the initial breaking

$$\text{GUT} \rightarrow \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1) \quad (6)$$

and the final breaking

$$\text{SU}(2)_L \otimes \text{U}(1) \rightarrow \text{U}(1)_{\text{e.m.}} \quad (7)$$

are found to occur. This is in fact the same problem of parameter adjustment, but it appears particularly unpalatable as it requires a readjustment of parameters in the scalar potential to many orders in perturbation theory. Finally, what is perhaps the most glaring defect of grand unification is that it excludes gravity which we know must become important at a mass scale $m_p \approx 10^{19}$ GeV, and another hierarchy problem¹¹⁾ is to understand why the GUT unification scale is about four orders of magnitude lower.

On the other hand, while there is no sign of the phenomenological relevance of supersymmetry, let alone supergravity, the only known technology which might allow a marriage of renormalizable gauge theories with gravitational interactions is extended supergravity¹²⁾. The largest such theory known is $N = 8$ supergravity in which the elementary fields include 28 vector bosons which form the adjoint representation of a global $\text{SO}(8)$ symmetry. Even if $\text{SO}(8)$ is gaugeable, it is too small¹²⁾ to contain the observed gauge group of Eq. (1). Therefore this approach appeared to be a dead end until Cremmer and Julia discovered¹³⁾ that $\text{SO}(N)$ super-

gravity theories have a concealed, nonlinear, local unitary symmetry: $U(N)$ for $N = 4, 5, 6$ and $SU(8)$ for $N = 7$ or 8 . They conjectured that the local gauge symmetry may become dynamically realized.

A supporting example for this conjecture comes from CP^{n-1} models in two dimensions¹⁴⁾. This model contains n complex scalar fields with the normalization condition

$$\sum_{i=1}^n |A_i|^2 = 1 \quad (8)$$

so that it in fact contains $2n - 1$ real scalar fields. The Lagrangian is

$$\mathcal{L} = - \sum_{i=1}^n (\partial_\mu - iV_\mu) A_i^* (\partial_\mu + iV_\mu) A_i \quad (9)$$

Since the vector field V_μ has no kinetic energy term it is not a propagating field and is determined by the equations of motion in terms of the scalar fields:

$$\frac{\delta \mathcal{L}}{\delta V_\mu} = 0 = -2V_\mu + i \sum A_i^* \overleftrightarrow{\partial}_\mu A_i \quad (10)$$

The Lagrangian (9) satisfies a formal $U(1)$ gauge invariance:

$$A_i \rightarrow e^{i\Lambda} A_i, \quad V_\mu \rightarrow V_\mu - \partial_\mu \Lambda \quad (11)$$

which can be used to reduce the number of scalar degrees of freedom to $2n - 2$ by a particular choice of gauge. While the $U(1)$ gauge invariance of the Lagrangian (9) is apparently a formal artifact, investigation of the two dimensional theory in the $1/N$ expansion shows¹⁴⁾ that the propagator for the composite vector field operator, which is just the conserved $U(1)$ current, develops a pole. So also does the partner fermion in a supersymmetric version of the two dimensional CP^{n-1} model^{15), 12)}.

In $SO(8)$ supergravity, the supermultiplet of elementary fields is self conjugate and contains spins 0 to 2 with multiplicities determined by the binomial expansion coefficients¹⁶⁾: 1 spin 2, 8 spin 3/2, 28 vectors, 56 spin 1/2 and 70 scalars. The scalars transform according to the 70 representation of $SO(8)$ which is an explicit global invariance of the theory. Making use of the implicit $SU(8)$ local gauge invariance¹³⁾ one can introduce 63 additional scalar fields which correspond to the parameters of the gauge transformations. This gives a total of 133 scalar fields which form a representation of a non-compact E_7 which is

also a global invariance of the theory. The original $SO(8)$ global invariance group is a subgroup of both E_7 and $SU(8)$.

The 133 scalar fields can be represented by a 56×56 matrix

$$S = \begin{pmatrix} u_{[AB]}^{[MN]} & v_{[AB]}^{[MN]} \\ \bar{v}_{[AB]}^{[MN]} & \bar{u}_{[AB]}^{[MN]} \end{pmatrix} \quad (12)$$

where square brackets imply antisymmetrization of indices, $A, B = 1, \dots, 8$ transform under $SU(8)$, and $M, N = 1, \dots, 8$ transform under E_7 . (The elements of (12) satisfy additional constraints so that the number of independent components is indeed 133). One may construct an E_7 invariant composite vector operator from the matrix product

$$(\partial_\mu S) S^{-1} = \begin{pmatrix} 2Q_\mu \begin{bmatrix} C & \delta \\ A & B \end{bmatrix} & P_\mu [A B C D] \\ \bar{P}_\mu [A B C D] & 2\bar{Q}_\mu \begin{bmatrix} C & \delta \\ A & B \end{bmatrix} \end{pmatrix} \quad (13)$$

which is a nonlinear function of the elementary scalar fields; the P_μ contains a linear term which is the gradient of a scalar, while $Q_\mu^A_B$ is a traceless 8×8 matrix which transforms according to the adjoint representation of $SU(8)$ and is in fact the part of the conserved $SU(8)$ current matrix constructed from the scalar fields. Cremmer and Julia constructed explicitly only this part of the Q_μ , but their generalization to supercovariant operators should correspond to the full $SU(8)$ conserved currents, containing fermion field bilinears, etc.

We conjecture that the composite operators so constructed become propagating fields which correspond to the standard gauge fields of low energy phenomenology, the photon, colored gluons, the W, Z of weak interactions, as well as the X and Y of $SU(5)$ and additional (super heavy) gauge fields associated with the generation group. We further conjecture that the additional fields of the low energy theory, the observed quarks and leptons and the scalars needed for symmetry breaking, are dynamical realizations of composite field operators which belong to the same $N = 8$ supersymmetric multiplet as the conserved current. An objection to this conjecture might be that, contrary to the case for the CP^{n-1} models, the $N = 8$ supergravity theory is infrared finite in perturbation theory, invalidating the previous analogy. We would then appeal to some as yet undiscovered non-perturbative phenomenon as being responsible; the point is really that such a dynamical realization of composite operators appears at present as the only

hope for reconciling the particle spectrum of the low energy gauge theory with supergravity.

Once this viewpoint is adopted, we are led to a unique candidate theory for the embedding of a GUT in a supergravity theory, namely $SO(8)$ supergravity with an $SU(5)$ GUT. The argument is based on three points: a) the largest supergravity theory is $SO(8)$, so the largest dynamically realized gauge group is $SU(8)$, b) the minimal GUT is $SU(5)$, and c) since all observed particles are contained in a common supermultiplet, its algebra must contain a non-trivial generation group for which the minimal candidate is $SU(2)$. Since $U(5)$ and $U(6)$ do not contain $SU(5) \otimes SU(2)$, let alone anything larger, we are immediately led to $SO(7)$ or $SO(8)$ supergravity with

$$SU(8) \supseteq GUT \otimes \text{Generation group}.$$

Since $SU(8)$ has rank 7, and the minimal GUT has rank 4, inclusion of a generation group restricts us to GUTs with rank 4, 5 and 6. Georgi and Glashow²⁾ showed some time ago that the only acceptable rank 4 GUT is $SU(5)$. Similarly, the only viable⁸⁾ rank 5 GUT is $SO(10)$, but this is not contained in $SU(8)$. It is then easy to see that $SU(8)$ does not contain the product of $SU(2)$ with a suitable^{*)} rank 6 group. We are thus uniquely restricted to

$$SU(8) \rightarrow SU(5)_{GUT} \otimes SU(3)_{Gen} \otimes U(1).$$

The philosophy which we tentatively adopt is the following. We plead ignorance of the dynamics of the underlying fully supersymmetric and $SU(8)$ gauge invariant theory, but assume that composite states are generated dynamically, and that at, say, the Planck mass the full symmetry breaks by some unspecified dynamical mechanism down to $SU(5)$, possibly, but not necessarily, with an $SU(8)$ or $SU(5) \otimes SU(3)$ invariance at an intermediate stage above the mass scale of 10^{15} GeV where we believe $SU(5)$ to break through a conventional Higgs mechanism. We use the initial symmetries only to study the particle spectrum and constrain couplings; our strategy consists of two steps. First we look for the supermultiplet which contains the conserved currents of $SU(8)$, which

*) We are implicitly assuming that the GUT by definition unifies only the presently observed interactions and does not account for generation replication; the assumption that GUT commutes with the generation group, which will be relaxed in section 4, allows it to be made fully natural⁸⁾ in the Glashow-Weinberg¹⁷⁾ sense.

are assumed to be realized on mass shell as the corresponding gauge bosons. We further assume that this multiplet contains all the particles relevant to the "observed" low energy gauge theory (fermions and Higgs scalars), as well as many unobserved states including some of high spin. We ignore the fields contained in the original supergravity multiplet (preons) as being superconfined, with the exception of the graviton which is an overall singlet. Finally we adopt a conventional, purely group theoretical, symmetry breaking scenario according to which whatever states which can acquire invariant masses at each stage of symmetry breaking do so, and ask what states remain massless at each level. That is, do we end up with an acceptable low mass particle spectrum? We shall see in section 3 that the answer is negative, and in section 4 that assuming we misidentified the supermultiplet and/or the overall pattern of symmetry hierarchies does not alleviate the problem. In section 5 we consider an alternative conjecture¹⁸⁾, according to which the effective theory of the self-interacting composite fields breaks dynamically at the Planck mass to the maximal renormalizable sub theory, and conclude that this approach appears somewhat more promising.

2. THE SUPERMULTIPLYET CONTAINING THE $SU(8)$ VECTOR FIELDS

We take the point of view that the vector fields in the adjoint representation of $SU(8)$ are described by composite fields which are at least bilinear in the fundamental fields of the theory. The expressions given by Cremmer and Julia¹³⁾ are functions of the scalar fields alone, but it is clear that in a supersymmetric description there will be additional terms containing the other fundamental fields (which have spins up to 2), and that the vector fields are just the fields describing the conserved $SU(8)$ currents. Therefore, the problem of finding the other members of the supermultiplet is the same as that of finding the supermultiplet of the "supercurrent", to which belong, in particular, the spinor currents (spin $3/2$) and the (chiral) vector currents (spin 1). The corresponding particle states can then be obtained by putting on the mass shell of the composite particles the fields of the supercurrent multiplet.

The supercurrent multiplet is not known for extended supersymmetry for general N . However, it is known in great detail¹⁹⁾ for $N = 1$. For $N = 2$ it is known for a simple special case²⁰⁾; from the experience with $N = 1$, one can assume that the fields and the transformation formulas obtained in the special case are of general validity for $N = 2$. In both the $N = 1$ and $N = 2$ cases we take the supercurrent for a massless supermultiplet of basic fields. It is not difficult to see which is the on-mass shell supermultiplet for the composite fields. Since these two cases follow exactly the same pattern, one can extrapolate and make a "calculated" guess for higher N .

Consider for instance $N = 1$. For massless fundamental fields the supercurrent multiplet consists of the axial vector current $J_\mu(5)$, of the spinor current J_μ , and the symmetric energy momentum tensor $\Theta_{\mu\nu}$. They are all three conserved and

$$\gamma^\mu J_\mu = \Theta^\mu_\mu = 0.$$

As it is shown in the Appendix of Ref. 19), these fields can be used to describe a massive supermultiplet of spins $(2, 3/2, 3/2, 1)$. It suffices to put them on the mass shell, by requiring all fields to satisfy in addition the Klein-Gordon equation for mass m . As $m \rightarrow 0$, this massive supermultiplet decomposes into massless supermultiplets having the spins indicated: $(2, 3/2) + (3/2, 1) + (1, 1/2) + (1/2, 0, 0)$. There are two spin 1 fields, but it is easy to see that the axial current gives rise to the spin 1 field of the massless supermultiplet $(3/2, 1)$ (the other spin 1 field comes from the $m \rightarrow 0$ limit of the original massive spin 2).

Similarly, for $N = 2$, the fields of the supercurrent multiplet given in Ref. 3) can be used to describe a massive supermultiplet having spins $2, 4 \times (3/2), 6 \times (1), 4 \times (1/2), 0$. (We take the reduced multiplet corresponding to vanishing central charge). As $m \rightarrow 0$ this massive supermultiplet decomposes. Among the resulting massless supermultiplets one finds an $N = 2$ supergravity multiplet with spins $2, 2 \times (3/2), 1$, and a supermultiplet with spins $2 \times (3/2), 4 \times 1, 2 \times (1/2)$. The latter is the only one which has 3 spin 1 fields in the adjoint representation of $SU(2)$, plus a spin 1 singlet. The additional massive supermultiplets have spin 1 or lower, but the spin 1 are not in the adjoint representation.

The cases $N = 1$ and $N = 2$ indicate that, for general N , the supermultiplet of states with the vectors in the adjoint representation can be constructed as follows. One starts with states $(+3/2)^A$, with $A = 1, \dots, N$, of positive helicity $3/2$, belonging to the fundamental representation N of $SU(N)$. Applying to them the N spinor charges*), one obtains states $(+1)^A_B$ belonging to the $N \otimes \bar{N}$. This decomposes into the adjoint representation plus a singlet. Continuing this operation one obtains states $(+1/2)^A_{[BC]}$, $(0)^A_{[BCD]}$, $(-1/2)^A_{[BCDE]}$ etc. These tensors are totally antisymmetrized in the lower indices, which all take values $1, \dots, N$. The procedure stops when one reaches helicity $3/2 - N/2$. The states of this last helicity are in the same representation $(3/2 - N/2)^A$ as the original helicity $3/2$ states, since a totally antisymmetric tensor with N indices is an $SU(N)$ singlet. To all the above states one must add the TCP conjugate states $(-3/2)_A$, $(-1)_A^B$ etc.

*) This procedure for constructing multiplets of states is standard. See, for instance Ref. 21).

We have constructed the supermultiplet for general N by extrapolating from the known cases for $N = 1$ and 2 . We can further argue that the general result must be correct on the grounds that the supergravity current is unique since it is the source of supergravity. It is also worth pointing out that the massless multiplets so obtained are precisely the particle states of superconformal gravity (but, hopefully, in the present case, without the accompanying ghosts). We may believe the algebraic structure of the supermultiplet which we are studying is more general than the specific $SO(8)$ model which was our original motivation. Therefore some of our subsequent analysis may have a wider validity than this specific model.

3. THE PARTICLE CONTENT OF THE COMPOSITE MULTIPLY

Applying the construction obtained in the preceding section for the multiplet of the supercurrent on zero mass shell, we obtain the states of Table 1 and their TCP conjugates. (We start with negative rather than positive helicity because we want to embed left-handed 10-plets and $\bar{5}$ -plets in the usual $SU(5)$ gauge theory; this simply means that we define the spinor operator of supersymmetry transformations which is an 8 of $SU(8)$ to be left-handed). Except for the first and last entries, each helicity state transforms according to a reducible multiplet of $SU(8)$, $(h)^a_{[bc...]}$, which decomposes into its trace, $(h)^a_{[ab...]}$, and a traceless part which is the larger irreducible representation in each case. In addition to the spin 1 63-plet which we identify with the $SU(8)$ gauge bosons, we are interested in fermions and scalars. The left-handed spin $1/2$ traceless multiplets decompose under $SU(5) \otimes SU(3)_{\text{Gen.}}$ as

$$\begin{aligned}
 216 &= (10, \bar{3}) + (\bar{5}, \bar{3}) + (\bar{45}, 1) + (1, 3) + (5, 1) \\
 &\quad + (1, \bar{6}) + (24, 3) + (5, 8). \\
 \bar{504} &= (45, 3) + (40, \bar{3}) + (24, 1) + (15, 1) + (10, 1) \\
 &\quad + (10, 8) + (\bar{10}, 6) + (\bar{10}, \bar{3}) + (5, 3) + (\bar{5}, \bar{3})
 \end{aligned} \tag{14}$$

The first two terms represent obvious candidates for the three generations of light fermions. The scalar 420-plet decomposes according to

$$\begin{aligned}
 420 &= (24, \bar{3}) + (5, 3) + (5, \bar{6}) + (45, 3) + (\bar{40}, 1) \\
 &\quad + (\bar{10}, \bar{3}) + (\bar{5}, 1) + (10, 1) + (10, 8) + (1, \bar{3})
 \end{aligned} \tag{15}$$

Helicity	-3/2	-1	-1/2	0	+1/2	+1	+3/2	+2	+5/2
SU(8) content	$\bar{8}$	1, 63	8, 216	28, 420	56, 504	70, $\overline{378}$	$\overline{56}$, $\overline{168}$	$\overline{28}$, $\overline{36}$	$\bar{8}$

Table 1. Helicity states of the composite multiplet of SU(8) gauge vectors.

If the generation triplet of 24's acquires a vacuum expectation value at a scale of 10^{15} GeV, it will serve to break^{2),4)} SU(5) down to $SU(3)_C \otimes SU(2)_L \otimes U(1)$, and any combination of 5's and 45's can be used^{2),4)} to break $SU(2)_L \otimes U(1)$ down to $U(1)_{e.m.}$ at the second stage of the usual GUT hierarchy. One would like the same objects to generate fermion masses. The (5,3), (5, $\bar{6}$) and (45,3) all have $SU(5) \otimes SU(3)_{Gen}$ invariant coupling to the fermion bilinear

$$(10, \bar{3})_L (\bar{5}, \bar{3})_L$$

which is the conventional source of masses for charge -1/3 quarks and charged leptons. However, in the fermion bilinear which can give masses to charge 2/3 quarks

$$(10, \bar{3})_L (10, \bar{3})_L = (\bar{5} + \bar{45} + 50, 3 + \bar{6})$$

only the symmetric part, $(\bar{5}, \bar{6}) + (\bar{45}, 3) + (50, \bar{6})$, can contribute, and none of these representations is contained in the multiplet (15) nor in the scalar 28-plet. Retaining the aesthetic and simple identification of the observed fermions with the $(10, \bar{3}) + (\bar{5}, \bar{3})$ would then imply that the top quark mass arises from radiative corrections, which seems somewhat hard to swallow even if generation SU(3) is already broken at the Planck mass. However, there is also a $(10, 1)_L + (10, 8)_L$ in the conjugate of the helicity +1/2 504, some of which might also be identified with the three 10's of SU(5), if one is willing to abandon a simple interpretation of the generation group. We will see in section 5 that this complication may indeed be what happens.

There is also, of course, a lot of unwanted garbage in the states of Table 1. We should remark at this point that we checked that the SU(8) gauge currents are not anomaly free at the preon level, nor at the composite level either, and presumably the spinor currents coupled to higher spin states also have anomalies. The singularities arising from anomalies could induce large masses in

some states and effectively remove them from the theory below the Planck mass. However at this stage we simply ignore the question of what mechanisms induce masses (and also the question of what keeps some scalars effectively massless so that the usual $SU(5)$ gauge hierarchy can go through), and take it as an empirical "fact" that $SU(5)$ is a good gauge symmetry above 10^{15} GeV and $SU(3)_C \otimes SU(2)_L \otimes U(1)$ is good above 10^2 GeV. Then, as in conventional Higgs and super-Higgs symmetry breaking mechanisms, those states which can acquire $SU(5)$ gauge invariant masses will have $m \approx m_p$, those masses which are $SU(3) \otimes SU(2) \otimes U(1)$ invariant will be $m \approx 10^{15}$ GeV, and those which break $SU(2)_L$ will be $m \approx 10^2$ GeV (with the very light fermion masses possibly corresponding to zeros in the original mass matrix). A spin $1/2$ fermion, for example, in a representation r of $SU(5)$ acquires a mass $O(m_p)$ if the reducible representation $(8)_L + (216)_L + (\overline{56})_L + (\overline{504})_L$ contains the conjugate representation $\bar{r}$ as well as r . For a massive higher spin fermion, rotational invariance in its rest frame demands that all helicity states $n, n-1, \dots, -n$ occur. Therefore elimination of a spin $3/2$ object in representation r requires that $r + \bar{r}$ be contained in the $-3/2$ representation $8 + 56 + 168$ and also in the $-1/2$ representation given above. One sees immediately that the charged members of the $SU(5)$ $\bar{5}$ -plet contained in the $\bar{8}$ -plet of $-5/2$ states will never acquire a mass if conventional $SU(5)$ charge assignments are used and if electromagnetism remains unbroken. One might try to argue that the confined color triplet of spin $5/2$ has such highly non-renormalizable interactions that its bound states acquire masses beyond the reach of present experiment, but we certainly cannot tolerate a massless charged lepton, which must remain forever massless as it has no helicity partner.

Since the program described up to here has obviously failed, we tried several alternatives which we outline in the next section.

4. ALTERNATIVE SCENARIOS

A possibility might be that we misidentified the multiplet containing the conserved vector current. We also tried a multiplet with the structure

$$(-2) \begin{smallmatrix} [AB] \\ [C] \end{smallmatrix}, (-3/2) \begin{smallmatrix} [AB] \\ [CD] \end{smallmatrix}, (-1) \begin{smallmatrix} [AB] \\ [CDE] \end{smallmatrix}, (-1/2) \begin{smallmatrix} [AB] \\ [CDE] \end{smallmatrix}, \dots, (+2) \begin{smallmatrix} [AB] \\ [CDE] \end{smallmatrix}. \quad (16)$$

The supermultiplet (16) contains $3/2$ as its highest half-integral spin, and this falls in a complicated reducible multiplet where we might hope to find at least opposite charge partners for everyone. This turns out not to be the case.

A second possibility is that we have been taking our conventional view of GUT too seriously, and that we should bypass $SU(5)$

as an intermediate, physically relevant symmetry. Our previous difficulty was that the electromagnetic theory was not vector-like. We can cure this by imposing charge symmetry on the basic supermultiplet. The only charge assignment which does that and which preserves the factorization and observed charge spectrum of the observed group (1) is

$$8 = [\bar{3}_{\text{colour}}(Q = 0), 2_{\text{SU}(2)_L}(Q, Q - 1), -Q, -Q + 1, 0].$$

We found that in order to give masses to all spin $3/2$ states we had to impose an additional and amusing charge quantization condition, $Q = n/3$, which gave us quarks and leptons with the conventional charges as well as integrally charged coloured quarks and fractionally charged leptons (which might possibly be found²²) on Niobium balls). However, even in this scenario we found it necessary to abandon partially the principle of attributing all possible invariant masses at each stage of breaking; some spin $1/2$ self-conjugate $\text{SU}(2)$ triplets had to remain massless until after $\text{SU}(2)$ breaking to feed left over $3/2$ states which would otherwise have eaten desirable fermions.

A much worse result was that we were left with many massless unpaired helicity states which were non-singlet and non-self-conjugate under colour $\text{SU}(3)$. The previous argument wishing away bound states of high spin quarks would not seem applicable to spin $1/2$ states. In addition, even if one could wish away all the bound states of such quarks, it would seem difficult to wish away their loop contributions to gluon self-couplings at low energies, and we would expect them to generate parity violation in strong interactions. We could try to impose that the basic supermultiplet be self conjugate, and thus vector-like, with respect to both charge and colour. For $N \leq 8$, the only such possibility is $\text{SU}(8)$ with

$$8 = [3_c(Q), \bar{3}_c(-Q), 2_L(Q', -Q')]. \quad (17)$$

The observed $\text{SU}(2)$ charge structure dictates $-Q' = Q' - 1$, so $Q' = 1/2$, and we found that there is no choice of Q which reproduces the observed particle charge spectrum. (Another possibility, not a priori vector-like, for embedding $\text{SU}(3)_c \otimes \text{SU}(2)_L$ in $\text{SU}(8)$ would be with a fundamental 6-plet transforming like a colour triplet and an $\text{SU}(2)$ singlet; this does not work either.)

The major difficulty is simply that the composite multiplets of $\text{SO}(8)$ supergravity appear as chiral multiplets of dynamical $\text{SU}(8)$. This chirality property is a priori appealing as it may be relevant to the apparent masslessness of the neutrino and the observed chirality of weak interactions. Our problem is one of too much chirality; we are unable to make any interactions vector-like. Is there any way to salvage the marriage of low energy gauge

theories with supergravity? One might argue that any composite supermultiplet containing vectors which transform according to the adjoint representation of the unitary group might be the one containing dynamically realized physical states, on the grounds that the vector interactions have to be those of a Yang-Mills theory if they are to result in an effective renormalizable theory. One could further argue that the condition $N = 8$ is not necessary. Since the supermultiplets contain large irreducible representations of helicity $1/2$ states, containing repeated representations of subgroups, it may be that an explicit non trivial generation group is not a necessary condition for a realistic theory. Therefore we asked the question: is there any suitable irreducible supermultiplet of $SO(N)$ supersymmetry, with the basic N -plet transforming under $SU(N) \rightarrow SU(3)_{\text{colour}}$ according to

$$N = [3_{\text{colour}}, (N - 3)_{\text{colour singlets}}],$$

and which contains spin 1 states which transform according to the adjoint representation of $SU(N)$? A general irreducible multiplet, is constructed²¹⁾ by assigning helicity h to some irreducible representation R_0 and applying the (automatically totally antisymmetrized) N spinor charges to obtain states of helicity decreasing (or increasing) successively by one half unit. Thus we consider a multiplet of the form

$$S = [R_0(h), R_{0A}(h-1/2), \dots, R_0[A, \dots A_k] (h-k/2 \equiv 1), \dots, R_0(h-N/2)] \quad (18)$$

and impose the condition that states of given half-integer helicity in $S + S^\dagger$ (the TCP conjugate) be vector-like under $SU(3)$, that is, that each set of helicity states contain equal numbers of r and $\bar{r}$ where r is any non-self-conjugate representation of $SU(3)$. Our analysis of this general case led to the conclusion that no such irreducible multiplet can be found, at least for $5 \leq N \leq 8$, which are the limiting values imposed by the minimal rank incorporating the observed gauge interactions and the maximal supergravity theory.

Perhaps even the restriction to an irreducible composite supermultiplet is too strong, and it might be possible to construct a viable theory using some superposition of multiplets. There is no obvious way to see that this is possible, and it would require a better group theoretical technology than we have found to either construct an example or prove that it is not possible. Another escape route is to conjecture that a supergravity theory with $N > 8$ might be constructed by means of a non-linear realization of the supersymmetry. For $N \geq 10$ it is trivial to impose the required vector-like conditions by choosing, for example, a

decomposition of $SU(10) \rightarrow SU(5)$ according to

$$10 = (5, \bar{5}).$$

In this case even $SU(5)$ is initially vector-like, and the observed chirality would have to be attributed to symmetry breaking effects.

There is yet an alternative possibility, which seems to us at present the most plausible one, which is that the initial philosophy was misguided. We have assumed that composite states of high spin must necessarily acquire group invariant masses in the way we conventionally attribute masses to elementary fields described by an elementary field theory. In fact we have no notion of how dynamical symmetry breaking may occur in supergravity theories, nor of the dynamics of a self-coupled composite supermultiplet, and in particular no indication that its effective interactions could ever be made renormalizable even if the underlying supergravity theory were shown to be renormalizable on the preon level. In the following section, we outline an alternative program¹⁸⁾ based largely on intuition and experimental evidence for the reality of renormalizable gauge theories.

5. AN EFFECTIVE RENORMALIZABLE SUB-THEORY

Present experimental data strongly support the belief that low energy physics is described by renormalizable field theory, which as far as we know could be valid up to energies not much below the Planck mass; this is precisely the assumption made in obtaining the GUTs predictions (2), (4) and (5). The renormalizable theories that we know consist solely of Yang-Mills vector fields, spin 1/2 fermions, and scalar fields.

There is an unpublished argument of Veltman²³⁾, which we understand as follows: if composite states with masses that are much smaller than their inverse size have effective interactions which can be described by perturbation theory at energy scales much lower than the inverse size, then these interactions must be renormalizable. Otherwise singularities would arise in the computation of vertex functions for which the only available cut-off is the inverse size.

The above two points encourage us to exploit the following conjecture¹⁸⁾: the would-be supersymmetry of the effective Lagrangian describing the self-interaction of the dynamically realized composite supermultiplet in the $SO(8)$ supergravity theory is dynamically broken down to a sub-theory containing only renormalizable interactions. Those states which do not have renormalizable interactions may acquire masses $\sim (\text{size})^{-1} \sim m_p$ and decouple from the effective "low" ($\lesssim m_p$) energy theory. Within

this philosophy we are simply ignoring the previous question as to where all the states in the original supermultiplet find helicity partners with the same $SU(3)_c \otimes U(1)_{e.m.}$ transformation properties to form a rotationally invariant massive state. Since we are dealing with composite states, they could simply become unbound at the scale m_p or possibly marry helicity components of some other composite supermultiplet in a non-supercovariant way.

We thus arbitrarily reduce the original composite supermultiplet to its renormalizable sector which to our knowledge can contain only the Yang-Mills vectors, spin $1/2$ fermions and scalars of a renormalizable gauge theory which could be as large as $SU(8)$. Aside from restrictions on the multiplet structure, are we left with any predictive power at all? An initial working hypothesis¹⁸⁾ might be that a remnant of the underlying supersymmetry of the interactions survives in the coupling constants of the residual renormalizable theory as specified at the Planck mass. Since perturbation theory is by assumption applicable below the Planck mass, the low energy theory becomes in principle completely determined.

Returning to our original arguments that the only viable embedding of a GUT in a supergravity theory requires $SO(8)$ supergravity, and that the dynamically realized supermultiplet is the one of Table 1, we reduce the theory to that part which contains only the 63 gauge bosons of $SU(8)$, the 216 (+ 8?) left-handed spinors, the 504 (+ 56?) right-handed spinors, the 420 (+ 28?) scalars, and the TCP conjugates of these states. At this stage there is a certain amount of ambiguity as to how much of the fermion and scalar sector we should actually retain. Our criterion of renormalizability requires us to make the theory anomaly free which implies the elimination of some fermions. It is a priori unclear as to at what stage and in what way this should be done. Also, in addition to the non-Yang-Mills $378 + 70$ of vector bosons, we have discarded the $SU(8)$ singlet accompanying the 63 as it does not couple to a conserved current (this is because of the self-conjugate nature of the fundamental supermultiplet which is unique to the $SO(8)$ supergravity theory). It could be that the trace parts $(8 + \overline{56}, 28)$ of the fermion and scalar reducible $SU(8)$ multiplets go out in the wash with the vector singlet, but this cannot be inferred from the renormalizability criterion alone. In either case, the group theoretical structure of our renormalizable sub-theory is much more general than our starting point which was $SO(8)$ supergravity; starting with vector fields in the adjoint $SU(N)$ representation, application of N supersymmetry transformation operators will generate the same fermion and scalar states although they can presumably not be made to form a closed multiplet.

The next step in our program is to extract a candidate effective Lagrangian for the surviving fields. To this effect we assume

that there exists an effective Lagrangian describing the self-coupling of the entire composite supermultiplet which is superficially invariant under global $SO(8)$ supersymmetry and local $SU(8)$ gauge symmetry. We then extract that part which corresponds to a renormalizable gauge theory describing the interactions of the above subset of fields. There are several methods which we can appeal to. Relations have been derived²⁴⁾ for S matrix elements in an unbroken extended supersymmetric theory. By imposing these constraints on the amplitudes calculated in the tree approximation in perturbation theory, one can, in principle, reconstruct an effective Lagrangian which has the required invariance properties. In practice this approach is lengthy, and since we are only interested in a subset of couplings, we can make use of supersymmetric Lagrangians in the existing literature by noticing that the full symmetry contains many smaller symmetries. The eight indices of the basic $SO(8)$ $[SU(8)]$ vector (spinor) play a double role: they label the eight supersymmetries of the theory and the eight fundamental gauge transformation properties. We may consider a sub-symmetry which is the product of $N < 8$ supersymmetries with $8 - N$ variations under unitary gauge transformations:

$$SO(N)_{\text{supersymmetry}} \otimes SU(8 - N)_{\text{Yang-Mills}} \quad (19)$$

Lagrangians for theories invariant under the product group (19) are known²⁵⁾ for $N \leq 4$. The $N = 4$ case in fact describes a real supermultiplet, but is equivalent to a complex $N = 3$ theory which is relevant for us since the right-handed fermions in Table 1 are not the conjugates of the left-handed fermions. The use of known Lagrangians symmetric under (19), with the additional constraint of a full local $SU(8)$ symmetry allows us to partially constraint some couplings, namely:

$$\mathcal{L}[(420)^4] + \mathcal{L}[(216)^2 \times 420] + \mathcal{L}(216 \times \overline{504} \times 420) \quad (20)$$

together with the $SU(8)$ gauge couplings. Even this limited sector of the effective Lagrangian displays some interesting features¹⁸⁾, which we summarize.

a) The quadratic coupling of the 420 vanishes for certain non trivial scalar field configurations, so the local $SU(8)$ gauge symmetry can be spontaneously broken at the tree level. If we may neglect non-perturbative effects, the minimum of the potential is then determined by radiative corrections, which in the one-loop approximation take the form²⁶⁾

$$V(\phi) = \sum_i \eta_i M_i^4(\phi) \ln [M_i^2(\phi)/u^2], \quad (21)$$

where u is the field strength for which the tree Lagrangian

couplings are specified, here assumed to be the Planck mass, $M_i(\emptyset)$ is the mass acquired by the i th particle in the tree approximation when $\emptyset$ is the scalar vacuum expectation value, and $\eta_i = +3, +1$ and -4 for vectors, scalars and fermions, respectively. At this stage it is essential that the high spin states have decoupled from the effective Lagrangian; if the full supermultiplet were retained, the vacuum degeneracy would persist to all orders in perturbation theory (if it could be defined!).

b) By inspection of the $SO(2) \otimes SU(6)$ and $SO(3) \otimes SU(5)$ sub-Lagrangians which were used to construct the fully $SU(8)$ invariant quartic coupling, it is easy to show that the tree potential vanishes for certain field configurations, where the only non-vanishing components of the 420-plet $\psi^i_{[jkl]}$ have one lower index equal to the upper index:

$$\langle \psi^i_{[ikl]} \rangle \neq 0 \quad (22)$$

One can see immediately (excluding the unlikely possibility that the true minimum occurs for zero field strength), that the largest simple unitary group which can remain unbroken at the Planck mass is $SU(5)$. This is because the tensor $\psi^i_{[jkl]}$ contains no singlets of $SU(6)$ or of $SU(7)$, but the diagonal form (22) does contain an $SU(5)$ singlet.

To pursue our preliminary analysis further we make the simplifying assumption that the true vacuum expectation value has non-vanishing elements only for fixed values of (k,l) in (22), i.e. that it transforms according to the adjoint representation of an $SU(6)$. Then we can make the following further observations.

c) The surviving group will be a rank 5 subgroup of $SU(6)$. From the study¹¹⁾ of adjoint breaking in $SU(5)$, we can make some guesses as to how the radiative corrections will determine the preferred direction of symmetry breaking in $SU(6)$ space. For positive η , the lowest vacuum is generally that with the greatest degree of (non-vanishing) mass degeneracy. Thus the contribution of gauge boson loops alone would choose a symmetry breaking to $SU(3) \otimes SU(3) \otimes U(1)$. The scalar loop contribution is more complicated to analyse, but is likely to go in the same direction. However, since the fermion loop contribution has a negative coefficient, it will prefer the solution with maximum mass degeneracy, namely $SU(5) \otimes U(1)$. Since we have more fermions (with weight factor 4) than bosons, they may turn out to play a determining

role. In the subsequent remarks we assume that the surviving symmetry at the Planck mass is $SU(5) \otimes U(1)$.

d) Keeping only the 420-plet of scalars, at least two adjoints and one 5-plet of $SU(5)$ remain massless at the tree level. Such massless scalar multiplets are precisely what is needed to implement further symmetry breakdown at lower mass scales through radiative corrections^{27),11)}. In conventional perturbation theory, in fact, the massless 5-plet would have been eaten by a 5-plet of ultra-heavy gauge bosons of $SU(8)$. However, the analysis of anomalies discussed below shows that this vector multiplet should already have acquired a mass through its anomalous current divergence. It remains to be seen whether the masslessness of the $SU(2)$ doublet of the 5-plet will survive $SU(5)$ breaking. (There are of course other 5-plets in the 28, some of which may remain massless if their couplings are included in the effective Lagrangian.)

e) If we examine the Yukawa couplings contained in the partial Lagrangian (20), we find that the $(\bar{5}, 3)_L$ of 216 [see Eq. (14)] remains massless, but the $(10, \bar{3})_L$ acquires masses through its Yukawa couplings to combinations of $(10, 6)_L$ and $(10, \bar{3})_L$ contained in $\bar{504}$. It therefore cannot be identified with the three generations of "massless" 10's in the conventional $SU(5)$ theory. Since the set $(216)_L + (\bar{504})_L$ contain altogether 12 10's and 9 $\bar{10}$'s, including the self-couplings of $\bar{504}$ to 420 will necessarily leave at least three 10's massless. These surviving states can have $SU(8)$ allowed tree-level couplings to the $(\bar{5}, 1)$ of the scalar 420, providing a zeroth order t quark mass upon $SU(2)$ breaking. However, the set $(216)_L + (\bar{504})_L$ also contains 6 $\bar{5}$'s and 12 5's, which would leave us with at least 6 massless 5's (and not necessarily any $\bar{5}$'s). What actually survives in the low energy effective theory depends on how the initial fermion content is reduced to an anomaly free subset.

From an analysis of $SU(N)$ anomalies we learn the following. The effective $SU(8)$ gauge theory cannot be made anomaly free. Sub-theories based on $SU(7)$ or $SU(6)$ which are anomaly free cannot be made vector-like under $SU(3)_{\text{colour}} \otimes U(1)_{\text{e.m.}}$ unless they are completely vector-like. On the other hand there is a set of fermions contained in $(216)_L + (\bar{504})_L$ which has no $SU(5)$ anomalies, is chiral, and is vector-like under $SU(3)_{\text{colour}} \otimes U(1)_{\text{e.m.}}$, namely:

$$\begin{aligned} & (45 + \bar{45})_L + 4(24)_L + 9(10 + \bar{10})_L + \\ & + 3(5 + \bar{5})_L + 9(1)_L + 3(\bar{5} + 10)_L. \end{aligned} \quad (23)$$

This choice is maximal in the sense that it contains the largest number of anomaly free helicity states which can be found in

$(216)_L + (\overline{504})_L$. (There is another subset with the same number of states $_L$ which is excluded phenomenologically because it is not vector-like under $SU(3)_{\text{colour}}$.) We note with satisfaction that it contains precisely three generations of $(\overline{5} + 10)_L$ which is the minimal number compatible with observation, and the preferred number for giving a satisfactory calculation of the b quark mass^{4),9)}. The remaining content of (23) is vector-like under $SU(5)$ and can acquire $SU(5)$ invariant (Dirac or Majorana) masses $O(m_p)$. We note also that the content of (23) is too large to fit into one of the vector-like anomaly free $SU(6)$ or $SU(7)$ sub-theories of $SU(8)$.

A final feature of our effective Lagrangian is that the β function is extremely positive when the full fermion and scalar multiplets are included with the Yang-Mills fields of $SU(8)$ ($\beta = (147.5) \frac{g}{48\pi^2}$ for the traceless parts). If there are a sufficient number of superheavy fermions which acquire masses only at the $SU(5)$ breaking scale of 10^{15} GeV where $\alpha_{\text{GUT}} \approx 1/40$, the coupling may become strong²⁸⁾ at m_p , which is desirable if we wish to attribute the primary dynamical breakdown of supersymmetry to a non-perturbative strong coupling effect.

In summary, arguments based on the form of the Higgs potential as determined from the conjectured underlying symmetry, and on the requirement of an anomaly free but non vector-like fermion sector, both point to $SU(5)$ as the largest gauge group which can remain unbroken after the conjectured dynamical breakdown of the $SO(3)$ supersymmetry at the Planck mass. Superficially, at least, the content of the residual gauge theory contains the elements necessary to populate the low energy particle spectrum and to permit a perturbative $SU(5)$ theory to emerge from a strong coupling theory at the Planck mass, with the further breakdown of $SU(5)$ at lower energies arising from radiative corrections. It remains to be seen whether such a program can be made to work in detail.

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REFERENCES

1. J.C. Pati and A. Salam, Phys. Rev. D8:1240 (1973).
2. H. Georgi and S.L. Glashow, Phys. Rev. Letters 32:438 (1974).
3. H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Letters 33:451 (1974).
4. A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135:66 (1978).
5. K.T. Mahanthappa and M.A. Sher, Univ. of Colorado preprint COLO-HEP 13 (1979).
6. P. Langacker et al., U. of Penn. preprint COO-3071-243 (1979).
7. M. Roos and I. Liede, Helsinki Univ. preprint HU-TFT 79-27 (1979).
8. M.S. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B128:506 (1977).
9. D. Ross and D.V. Nanopoulos, Nucl. Phys. B157:273 (1979).
P. Binétruy and T. Schücker, CERN preprint TH-2802 (1980), submitted to Nucl. Phys. B.
10. See, e.g.:
W.J. Marciano, Rockefeller Univ. preprint COO-2232B-195 (1980);
T. Goldman and D.A. Ross, Caltech preprint CALT-68-759 (1980);
J. Ellis, M.K. Gaillard, D.V. Nanopoulos and S. Rudaz, preprint LAPP-TH-14/CERN-TH-2833 (1980);
and references contained in the above.
11. J. Ellis, M.K. Gaillard, A. Peterman and C.T. Sachrajda, Nucl. Phys. B164:253 (1980).
12. See e.g.:
B. Zumino in Proc. Einstein Symposium Berlin, Lecture Notes in Physics 100:114, Ed. H. Nelkowski, A. Herman, H. Poser, R. Schrader and R. Seiler (Springer-Verlag, Berlin 1979) and references therein. The first attempt to embed low energy gauge theories in $SO(8)$ supergravity using the basic supermultiplet was reported by M. Gell-Mann, Talk at the 1977 Washington Meeting of the American Physical Society.
13. E. Cremmer and B. Julia, Phys. Letters 80B:48 (1978) and Nucl. Phys. B 159:141 (1979). A previous attempt to relate this work to phenomenology has been made by T. Curtright and P. Freund, in "Super Gravity", Proc. of the Supergravity workshop at Stony Brook, Sept. 1979, Ed. P. Van Nieuwenhuizen and D.Z. Freedman, (North Holland, Amsterdam, 1979), p. 197. Telegraphic accounts of our work on embedding GUTs in supergravity have been reported in:
J. Ellis, Proc. E.P.S. Int. Conf. on High Energy Physics, Geneva June 1979 (CERN, 1979), p. 940,
M.K. Gaillard and L. Maiani, 1979 Cargèse Summer Institute Lectures, LAPP preprint TH-09 (1979).
14. A. d'Adda, P. Di Vecchia and M. Lüscher, Nucl. Phys. B146:63 (1978);
E. Witten, Nucl. Phys. B149:285 (1979).

15. A. d'Adda, P. Di Vecchia and M. Lüscher, Nucl. Phys. B152:125 (1979).
16. B. De Wit and D.Z. Freedman, Nucl. Phys. B130:150 (1977).
17. S.L. Glashow and S. Weinberg, Phys. Rev. D15:1958 (1977).
18. J. Ellis, M.K. Gaillard and B. Zumino, CERN preprint TH-2842/LAPP preprint TH-16 (1980).
19. S. Ferrara and B. Zumino, Nucl. Phys. B87:207 (1975).
20. M. Sohnius, Phys. Letters 81B:8 (1979).
21. D.Z. Freedman, in "Recent Results in Gravitation, Cargèse 1978", Ed. M. Levy and S. Deser (Plenum Press, New York, 1978), p. 549.
22. G.S. La Rue, W. M. Fairbank and A.F. Hebard, Phys. Rev. Letters 38:1011 (1977);
G.S. La Rue, W. M. Fairbank and J.D. Phillips, Phys. Rev. Letters 42:142 (1979).
23. M. Veltman, private communication; arguments similar in spirit have probably occurred to many people, for example, G. t'Hooft, Cargèse Lecture Notes (1979);
K. Wilson, G. Parisi and L. Susskind, private communications.
24. A. Salam and J. Strathdee, Nucl. Phys. B80:499 (1974);
M.T. Grisaru, H.N. Pendleton and P. Van Nieuwenhuizen, Phys. Rev. D15:996 (1976);
M.T. Grisaru and H.N. Pendleton, Nucl. Phys. B124:81 (1977).
25. S. Ferrara and B. Zumino, Nucl. Phys. B79:413 (1974)
P. Fayet, Nucl. Phys. B13:135 (1976);
L. Brink, J.H. Schwarz and J. Scherk, Nucl. Phys. B121:77 (1977);
F. Gliozzi, J. Scherk and D. Olive, Nucl. Phys. B122:253 (1977).
26. S. Coleman and E. Weinberg, Phys. Rev. D7:1888 (1977).
27. S. Weinberg, Phys. Letters 82B:387 (1979).
28. L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B 36:115 (1978) and references therein.