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SUPERGRAVITY AND GAUGE SUPERSYMMETRY

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A B S T R A C T

The relationship of the Riemannian superspace geometry of gauge supersymmetry to the non-Riemannian superspace geometry of supergravity is discussed. It is shown that the geometrical aspects of supergravity such as the connections, torsions, and curvatures arise in a natural manner as contractions of the larger Riemannian geometry. These reductions are possible since the tangent space group $OSp(3,1/4N)$ of the Riemannian theory contains the tangent space group $O(3,1) \times O(N)$ of extended supergravity. This subgroup is also seen to be preserved in the contraction limit after spontaneous breakdown of gauge supersymmetry. Other properties of gauge supersymmetry are briefly reviewed.

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1. INTRODUCTION

I would like to talk about the work which Richard Arnowitt and I have done recently regarding the structure of gauge supersymmetry and the relationship of the Riemannian space geometry of gauge supersymmetry to the non-Riemannian superspace geometry of supergravity¹⁾. Rather than start directly with a discussion of the Riemannian geometry of gauge supersymmetry it is perhaps worth while motivating the discussion by recalling the great amount of progress made in the superspace formulations of supergravity²⁾ and the nature of the questions that arise in the framework of such formulations. What the superspace formulations of supergravity show is that supergravity obeys a non-Riemannian geometry in superspace with torsion. The problem which we should like to address ourselves to, however, concerns the amount of unification that occurs in supergravity.

While supergravity theories show a remarkable unification of particles of different spins and statistics within their framework, have well defined particle content and are manifestly free of ghosts, a unification of interactions is thus far lacking. The lack of unification of interactions in supergravity arises essentially from the fact that the tangent space group of supergravity is a product group $H = O(3,1) \times O(N)$. An example of this lack of unification is that in the superspace formulations of extended supergravity the Yang-Mills connections and the Lorentzian connections arise in the formalism in an unrelated fashion.

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There is a second novel aspect that appears in the superspace formulations of supergravity. This concerns the fact that the vacuum state of supergravity is postulated to obey the condition that the tangent space covariant derivative for the vacuum state obeys the graded algebra of $OSp(N/4)$, i.e.

$$\left[D_A^{(0)}, D_B^{(0)} \right] = D_C^{(0)} C_{AB}^C + \frac{1}{2} X^{mn} C_{mnAB}, \quad (1.1)$$

where C_{AB}^C and C_{mnAB} are the structure constants of $OSp(N/4)$. Equation (1.1) implies then that the vacuum expectation values of some of the components of torsion and curvature are non-vanishing. The assumption of Eq. (1.1) represents a novel aspect, because in the conventional gauge theory formulations one does not normally postulate non-vanishing vacuum expectation values. Rather, in conventional gauge theories one expects that the non-vanishing expectation values of dynamical objects would arise as a consequence of spontaneous or dynamical symmetry breaking. One may ask if such an approach is also possible in supergravity.

Since the tangent space group of extended supergravity is a product structure, it is natural to investigate if such a product structure is embedded in the unified tangent group of a larger theory. One might contemplate a situation analogous to the product group structure $SU(2) \times U(1) \times SU(3)_C$ of weak, electromagnetic, and strong interactions embedded in $SU(5)$, $SO(10)$ or some other grand unified theory. In analogy to the case of grand unified theories one may also expect that the spontaneous breakdown of the larger theory with a unified tangent space group $\mathcal{H}$ would result in the product group structure of supergravity $H = O(3,1) \times O(N)$. One may also hope that in the process of spontaneous breakdown, the vacuum expectation values of the various quantities such as supervielbeins, torsions and curvatures, postulated in the superspace formulations of supergravity would arise from a spontaneous breaking of such a theory.

It is from the viewpoint outlined above that we shall examine gauge supersymmetry, i.e. as a grand unified supergravity theory. In fact, we shall look in detail at the superspace geometry of supergravity as the contracted limit of the larger Riemannian theory whose tangent space group is the unified group $\mathcal{H} = OSp(3,1/4N)$ and contains as a subgroup the tangent space group $H = O(3,1) \times O(N)$ of supergravity. From the analysis of gauge supersymmetry and its spontaneous breakdown one arrives at a number of interesting results regarding supergravity. We list some of the more important results below.

First one finds that the vacuum state of supergravity is precisely deduced as a consequence of the spontaneous breakdown of gauge supersymmetry. One also finds that the Lorentzian and the Yang-Mills connections arise as two parts of a single unified connection of the $OSp(3,1/4N)$ tangent space group. Since for the Riemannian theory a relation exists between the connection and the vielbein in a closed form, one also obtains closed form expressions for the connections of supergravity in terms of the vielbeins as the Riemannian geometry is contracted to the supergravity geometry^{3,4)}. We exhibit first the Lorentzian part of the supergravity connections. The Lorentzian tangent space connection $h_{mn\Lambda}(z)$ is given by^{*)}

$$h_{mn\Lambda}(z) = h_{mnC} V_{\Lambda}^C (-1)^{C+\Lambda C}, \quad (1.2)$$

where the components h_{mns} and h_{mnai} of h_{mnC} are given by

$$h_{mns}(z) = \frac{1}{2} [\Omega_{mns}(z) + \Omega_{msn}(z) - \Omega_{nsm}(z)] \quad (1.3)$$

$$h_{mnai}(z) = \frac{1}{2} [\Omega_{main}(z) - \Omega_{naim}(z)]. \quad (1.4)$$

The components of $\Omega_{mBn}(z)$ appearing in Eqs. (1.3) and (1.4) are determined by the following relation in terms of the vielbein V_{Λ}^A :

$$\Omega_{mBn}(z) = V_m^{\Lambda} [V_{\Lambda,\Sigma}^r - (-1)^{\Lambda+\Sigma+\Lambda\Sigma} V_{\Sigma,\Lambda}^r] (-1)^{B\Sigma} V_B^{\Sigma} \eta_{rn}. \quad (1.5)$$

*) Our notation is as follows. Superspace coordinates are denoted by $z^A = (x^{\mu}, \theta^{\alpha i})$, where $[x^{\mu}, x^{\nu}] = 0 = \{\theta^{\alpha i}, \theta^{\beta j}\}$. Here $\alpha, \beta = 1, 2, 3, 4$ are the Majorana indices and $i, j = 1 \dots N$ are the internal symmetry indices. In general we shall use Greek letters to denote global coordinates and Latin letters to denote the local or tangent space coordinates. The capital letters are assumed to run over the full array of Bose and Fermi indices of superspace. Further, early letters, such as $\alpha, \beta, \dots$ or $a, b, \dots$ are used to label Fermi coordinates and late letters such as $\mu, \nu, \dots$ or $m, n, \dots$ are used to label the Bose coordinates. Factors such as $(-1)^{\Lambda}$ have values with $\Lambda = (0,1)$ corresponding to whether the index is Bose or Fermi. η_{mn} denotes the Lorentz metric and η_{ab} (the Fermi metric) is related to the charge conjugation matrix C by $\eta_{ab} = (-C^{-1})_{ab}$.

For the $O(N)$ Yang-Mills connections $h_{ij\Lambda}$ of supergravity one obtains

$$h_{ij\Lambda}(z) = \frac{1}{4} \bar{\omega}_{aibj\Lambda} \eta^{ab} \quad (1.6)$$

$$\bar{\omega}_{aibjm} = \frac{1}{2} [\bar{\Omega}_{aimbj} + \bar{\Omega}_{bjmai}] \quad (1.7)$$

$$\bar{\omega}_{aibjck} = \frac{1}{2} [\bar{\Omega}_{aibjck} - \bar{\Omega}_{aickbj} - \bar{\Omega}_{bjckai}] , \quad (1.8)$$

where $\bar{\Omega}_{aibcj}$ is given in terms of the vielbein by the following relation

$$\bar{\Omega}_{aiCb j} = (-1)^{C+1} v_{ai}^{\Lambda} [v_{\Lambda}^{dj},_{\Sigma} - (-1)^{\Lambda\Sigma} v_{\Sigma}^{dj},_{\Lambda}] (-1)^{C\Sigma} v_C^{\Sigma} \eta_{db} . \quad (1.9)$$

The closed form solutions of the connections in terms of the vielbeins given by Eqs. (1.2)-(1.9) hold for arbitrary values of N . It is understood that in these closed form solutions of the connections in terms of the vielbeins one must only use a gauge complete vielbein with respect to the supergravity transformation laws of an extended supergravity multiplet*). It is in the formulation of a gauge complete vielbein that the restriction $N \leq 8$ is expected to arise. We also note that closed form solutions of the connections in terms of the vielbeins have been obtained through explicit calculations for the case $N = 1$ by Gates and Siegel⁵⁾, though up to now these do not exist for general N . In the solutions that one obtains from the contraction of the Riemannian theory, one finds that the Lorentzian and the $O(N)$ Yang-Mills connections arise from the different components of a single quantity Ω_{ABC} . We see here then a unification of these connections in the Riemannian geometry framework.

The reduction of the Riemannian geometry to the supergravity geometry leads to an apparent paradox. The apparent paradox arises from the fact that the Riemannian superspace geometry has a vanishing superspace torsion. Thus if the non-Riemannian supergravity geometry which has a vanishing torsion arises from the Riemannian geometry, one is led to ask where the non-vanishing

*) The technique of gauge completion was first introduced by P. Nath and R. Arnowitt in Ref. 2 in the superspace formulation of supergravity in the metric formalism. Extensive use of this technique has subsequently been made by many authors; see, for example, L. Brink et al., S. Ferrara and P. van Nieuwenhuizen, E. Cremmer and S. Ferrara in Ref. 2. See also S. Ferrara in these proceedings.

torsion of supergravity came from. The resolution of this apparent paradox is rather simple. While the torsion corresponding to the tangent space group $OSp(3,1/4N)$ does indeed vanish in the Riemannian formulation, the torsion corresponding to its subgroup $O(3,1) \times O(N)$ does not. It is the non-vanishing $O(3,1) \times O(N)$ subpiece of the $OSp(3,1/4N)$ torsion $\mathcal{T}_{AB}^C$ that goes over to the supergravity torsion in the limit^{*)} $k \rightarrow 0$. Thus one may decompose the vanishing $OSp(3,1/4N)$ torsion $\mathcal{T}_{AB}^C$ into two parts

$$\mathcal{T}_{AB}^C = \tilde{T}_{AB}^C + S_{AB}^C, \quad (1.10)$$

where $\tilde{T}_{AB}^C$ represents the $O(3,1) \times O(N)$ subpiece of the $OSp(3,1/4N)$ torsion and S_{AB}^C is the remainder. Equation (1.10) tells us that the vanishing torsion of the Riemannian theory polarizes into two equal and opposite parts. The supergravity torsion T_{AB}^C arises from $\tilde{T}_{AB}^C$ on contraction,

$$T_{AB}^C = \tilde{T}_{AB}^C \Big|_{k=0}. \quad (1.11)$$

The topics which we should like to discuss in greater detail are the following. First we shall discuss the derivation of the Riemannian geometry in superspace in the supervielbein language. It is the vielbein rather than the metric formulation which is more appropriate for making contact with the supergravity geometry. We shall also discuss briefly how the vacuum state of supergravity arises as a consequence of the spontaneous breaking of gauge supersymmetry. The second topic will concern the reduction of the Riemannian space geometry to the supergravity geometry. In particular, we shall use the reduction of the Riemannian theory to derive the closed form solutions for the supergravity connections exhibited in Eqs. (1.2)-(1.9). Finally we shall discuss properties of gauge supersymmetry as such. These will include the ultraviolet finiteness of the S-matrix of gauge supersymmetry to all quantum loop orders and a discussion of the particle content of some prototype models of gauge supersymmetry.

2. SUPERSPACE GEOMETRY WITH TANGENT GROUP $OSp(3,1/4N)$

Superspace offers a natural framework for a discussion of the geometrical aspects of local supersymmetry. [For a discussion of superspace, see Zumino⁷⁾.] Here we want to discuss the superspace

*) This point was first made in Arnowitt and Nath⁶⁾. (The parameter k arises in the spontaneous breaking solutions of the vacuum of gauge supersymmetry.)

geometry with the tangent space group $OSp(3,1/4N)$, since the full discussion of this geometry represents the general case. We begin by introducing the supervielbein $V_\Lambda^A(z)$ which we assume possesses an inverse so that $V_B^\Lambda V_\Lambda^A = \delta_B^A$. Under the global coordinate space transformations $Z^\Lambda \rightarrow Z^\Lambda + \xi^\Lambda(z)$ and the local tangent space transformations parametrized by ε_B^A , V_Λ^A transforms so that^{*)}

$$\delta V_\Lambda^A = V_\Lambda^A{}_{,\Sigma} \xi^\Sigma + (-1)^{(\Lambda+\Sigma)(A+1)} V_\Sigma^A \xi_{,\Lambda}^\Sigma + V_\Lambda^B \varepsilon_B^A. \quad (2.1)$$

One may also introduce the covariant derivative of V_Λ^A by

$$V_\Lambda^A{}_{;\Sigma} = V_\Lambda^A{}_{,\Sigma} - (-1)^{\Sigma(A+\Delta)} \Gamma_\Lambda^\Delta{}_\Sigma V_\Delta^A + V_\Lambda^B \omega_B^A{}_\Sigma. \quad (2.2)$$

In Eq. (2.2) $\omega_B^A{}_\Sigma$ denotes the supervielbein connection and $\Gamma_\Lambda^\Delta{}_\Sigma$ stands for the global space affinity. From Eqs. (2.1) and (2.2) one finds that $\omega_A^B{}_\Sigma$ obeys the transformation law

$$\delta \omega_A^B{}_\Sigma = \varepsilon_{A,\Sigma}^B - \varepsilon_A^C \omega_C^B{}_\Sigma + (-1)^{\Sigma(B+C)} \omega_A^C{}_\Sigma \varepsilon_C^B. \quad (2.3)$$

Further, one may always define a metric tensor $g_{\Lambda\Sigma}(z)$ of the global space given a vielbein V_Λ^A by the introduction of a tangent space metric^{**) η_{AB}} . Thus one defines

$$g_{\Lambda\Sigma}(z) = V_\Lambda^A \eta_{AB} (-1)^{\Sigma(1+B)} V_\Sigma^B. \quad (2.4)$$

It follows from Eq. (2.1) and the assumption of a constant tangent space metric η_{AB} that in order that $g_{\Lambda\Sigma}(z)$ obeys the correct transformation law under global coordinate transformations, ε_A^B must obey the condition

$$\varepsilon_A^C \eta_{CB} + (-1)^{A+B+AB} \varepsilon_B^C \eta_{CA} = 0. \quad (2.5)$$

Further, the general form of a constant tangent space metric which maintains Lorentz covariance and is consistent with its symmetry properties is

^{*)} We use right derivatives as in Refs. 3 and 4.

^{**) The tangent space metric η_{AB} possesses the symmetry property $\eta_{AB} = (-1)^{A+B+AB} \eta_{BA}$.}

$$\eta_{AB} = \begin{pmatrix} \eta_{mn} & 0 \\ 0 & k\eta_{ab} \end{pmatrix}. \quad (2.6)$$

We note that the transformation parameters ε_B^A which describe the local tangent space transformations in Eq. (2.1) are not all arbitrary but must satisfy the conditions of Eq. (2.5). Actually the ε_B^A as constrained by Eq. (2.5) generate the tangent space group transformations corresponding to the group $OSp(3,1/4N)$. To see this more clearly, we consider the transformation of a contravariant vector $\phi^A(z)$ so that^{*)}

$$\delta\phi^A = \phi^B \varepsilon_B^A. \quad (2.7)$$

We may rewrite Eq. (2.8) so that

$$\delta\phi^A = \phi^B \frac{1}{2} (Z^{MN})_B^A \varepsilon_{MN}, \quad (2.8)$$

where $(Z^{MN})_B^A$ is the contravariant vector representation of $OSp(3,1/4N)$, i.e.

$$(Z^{MN})_B^A = (-1)^N \delta_B^M \eta^{NA} - (-1)^{M+MN} \delta_B^N \eta^{MA} \quad (2.9)$$

and $\varepsilon_{MN} \equiv \varepsilon_M^A \eta_{NA}$. Further, the Z^{MN} obey the graded algebra⁸⁾,

$$\left[Z^{MN}, Z^{PQ} \right] = f_{RS}^{MNPQ} Z^{RS}, \quad (2.10)$$

where f_{RS}^{MNPQ} represent the structure constants of $OSp(3,1/4N)$:

$$\begin{aligned} f_{RS}^{MNPQ} = & \left[\eta^{PQ} \frac{1}{2} (\delta_R^M \delta_S^Q - (-1)^{R+S+RS} \delta_S^M \delta_R^Q) (-1)^{N+Q+S} \right. \\ & \left. - (-1)^{MN} (M \leftrightarrow N) \right] - (-1)^{PQ} [P \leftrightarrow Q]. \end{aligned} \quad (2.11)$$

*) We note here that while there is only one contravariant vector $\phi^A(z)$ there are two types of covariant vectors $\chi_A = \eta_{AB} \chi^B$ and $\hat{\chi}_A = \chi^B \eta_{BA}$ depending on how the indices are lowered. The two types of covariant vectors are characterized by their different transformation properties, i.e. $\delta\chi_A = -\varepsilon_A^B \chi_B$ and $\delta\hat{\chi}_A = -(-1)^{A+AB} \hat{\chi}_B \varepsilon_A^B$.

Here η^{PQ} appearing in Eq. (2.11) is the inverse of η_{AB} defined in Eq. (2.6) so that

$$\eta^{PQ} = \begin{pmatrix} \eta^{mn} & 0 \\ 0 & \frac{1}{k} \eta^{ab} \end{pmatrix}. \quad (2.12)$$

We have here exhibited η^{PQ} explicitly to emphasize the appearance of $O(1/k)$ factors in the structure constants of the group $OSp(3,1/4N)$ of Eq. (2.11). This aspect of the structure constants will lead us to choose the correct basis in the Lie algebra of the tangent space group, so that we will take a smooth $k \rightarrow 0$ limit in order to make contact with the non-Riemannian geometry of supergravity.

In order to develop the geometry of superspace we need to introduce the definition of the tangent space covariant derivative. First let us introduce the covariant derivative $\mathcal{D}_\Lambda$ of a tangent space vector ϕ^A . One has *

$$\phi^A \overleftarrow{\mathcal{D}}_\Lambda = \phi^A_{,\Lambda} - \phi^B \omega_{B\Lambda}^A. \quad (2.13)$$

A more useful derivative for the definition of the geometry is obtained by projecting $\overleftarrow{\mathcal{D}}_\Lambda$ into the tangent space as follows

$$\overleftarrow{\mathcal{D}}_A = \overleftarrow{\mathcal{D}}_\Lambda V^\Lambda_A. \quad (2.14)$$

The quantity V^Λ_A appearing in Eq. (2.14) may be constructed explicitly by use of the inverse metric*)

$$V^\Lambda_A = g^{\Lambda\Sigma} V_\Sigma^B \eta_{BA}. \quad (2.15)$$

The reason that the tangent space projection $\overleftarrow{\mathcal{D}}_A$ of Eq. (2.14) is the useful quantity is that it is in terms of this projection that one defines the curvature and torsion of the space. Thus the curvatures and torsions are introduced through the action of a graded commutator on an arbitrary tensor ϕ as follows:

*) The definition of the inverse metric tensor easily follows from the existence of the inverse tangent space metric so that $g^{\Lambda\Pi} = V^\Lambda_{\eta AB} (-1)^{B(1+\Pi)} V_B^\Pi$, where V_A^Λ is the inverse vierbein satisfying $V_A^\Lambda V_\Lambda^B = \delta_A^B$.

$$\phi \left[\overset{\leftarrow}{\mathfrak{D}}_A, \overset{\leftarrow}{\mathfrak{D}}_B \right] = \phi \overset{\leftarrow}{\mathfrak{D}}_C \mathfrak{T}_{AB}^C + \frac{1}{2} \phi Z^{MN} \mathcal{R}_{MNAB} , \quad (2.16)$$

where Z^{MN} is the representation of the $OSp(3,1/4N)$ generators for the tensor ϕ . Further, computation of the graded commutator on the left-hand side of Eq. (2.16) allows one to determine the curvatures and torsions explicitly and one has

$$\mathfrak{T}_{AB}^C = \left[V_{\Pi}^C \overset{\leftarrow}{\mathfrak{D}}_{\Lambda} - (-1)^{\Pi\Lambda} V_{\Lambda}^C \overset{\leftarrow}{\mathfrak{D}}_{\Pi} \right] (-1)^{\Pi A} V_{\Lambda}^{\Lambda} V_{\Pi}^{\Pi} \quad (2.17)$$

$$\begin{aligned} R_{MNAB} = & \left[\omega_{MN\Pi, \Lambda} - (-1)^{\Pi\Lambda} \omega_{M\Lambda, \Pi} \right. \\ & \left. + \frac{1}{2} (-1)^{\Lambda(\Pi+R+S)} f_{MN}^{PQRS} \omega_{PQ\Lambda} \omega_{RS\Pi} \right] V_{\Lambda}^{\Lambda} (-1)^{\Pi A} V_{\Pi}^{\Pi} , \quad (2.18) \end{aligned}$$

where we have defined

$$\omega_{M\Lambda}^P \equiv \omega_M^P \wedge \eta_{NP} . \quad (2.19)$$

In the analysis carried out thus far we have treated the vielbein and the connections as independent quantities. However, in the Riemannian space the vierbein covariant derivative $V_{\Lambda}^A{}_{;\Sigma}$ vanishes and a relation between these quantities is thus obtained. Using Eq. (2.2) and the symmetry property of the affinity^{*)} $\Gamma_{\Lambda}^{\Delta}{}_{\Sigma}$ one may obtain the following equation for $\omega_B^A{}_C$:

$$\begin{aligned} \omega_B^A{}_C - (-1)^{AB+BC+AC} \omega_C^A{}_B = \\ = V_B^{\Lambda} \left[V_{\Lambda}^A{}_{;\Sigma} - (-1)^{A(\Lambda+\Sigma)+\Lambda+\Sigma+\Lambda\Sigma} V_{\Sigma}^A{}_{;\Lambda} \right] (-1)^{C\Sigma} V_C^{\Sigma} , \quad (2.20) \end{aligned}$$

where $\omega_B^A{}_C \equiv \omega_B^A{}_{\Lambda} V_{\Lambda}^{\Lambda}{}_C$. Now $\omega_B^A{}_C$, etc., possess no symmetry properties in their indices and consequently one cannot directly solve

*) The global space affinity $\Gamma_{\Lambda}^{\Delta}{}_{\Sigma}$ obeys the following symmetry property

$$\Gamma_{\Lambda}^{\Delta}{}_{\Sigma} = (-1)^{\Lambda+\Sigma+\Lambda\Sigma+\Delta(\Lambda+\Sigma)} \Gamma_{\Sigma}^{\Delta}{}_{\Lambda} .$$

On using this symmetry to eliminate the affinity from Eq. (2.2), one arrives at Eq. (2.20).

for $\omega_B^A C$ from Eq. (2.20). The essential step in the solution of ω 's requires that one lower the upper middle index A by η_{DA} and use the symmetry property^{*)}

$$\omega_{ABC} = -(-1)^{AB} \omega_{BAC} \quad (2.21)$$

to solve for ω_{ABC} from Eq. (2.20). One arrives then at the following expression for ω_{ABC} in terms of the vielbein V_Λ^A :

$$\omega_{ABC} = \frac{1}{2} \left[\Omega_{ABC} + (-1)^{BC} \Omega_{ACB} - (-1)^{A(B+C)} \Omega_{BCA} \right], \quad (2.22)$$

where

$$\Omega_{ABC} = V_A^\Lambda (-1)^{BC+C} \left[V_{\Lambda C, \Sigma} - (-1)^{C(\Lambda+\Sigma)+\Lambda+\Sigma+\Lambda\Sigma} V_{\Sigma C, \Lambda} \right] (-1)^{B\Sigma} V_B^\Sigma. \quad (2.23)$$

One may note that the expression for Ω_{ABC} involves $V_{\Lambda C}$, which has its local index lowered by η_{DC} , i.e.

$$V_{\Lambda C} = V_\Lambda^D \eta_{DC}. \quad (2.24)$$

Because of the η_{DC} factor, $V_{\Lambda C}$ has now an explicit k dependence and thus Eq. (2.24) will play an important role when we later discuss the reduction of the Riemannian geometry to the supergravity geometry in the limit $k \rightarrow 0$.

Equations (2.22) and (2.23) represent the connections of a superspace Riemannian geometry. As in the Bose space Riemannian geometry, the connections involve three permutations of a single Ω_{ABC} . In order to make contact with the non-Riemannian geometry of supergravity, one must consider a contraction of the Riemannian space geometry. Before one can carry out the contraction, however, one must affect a change of basis in the Lie algebra of the Riemannian tangent space group $\mathcal{H} = \text{OSp}(3, 1/4N)$. The reason for this is that it is only on the new basis that one achieves a smooth $k \rightarrow 0$ limit in the subsectors that connect to the supergravity geometry. The new basis X^{MN} is defined by

$$X^{MN} = C_{RS}^{MN} Z^{RS}. \quad (2.25)$$

*) Equation (2.21) follows from the requirement that Eq. (2.4) reproduces the correct formula for the covariant derivative for $g_{\Lambda\Sigma}$ when Eq. (2.2) is employed. The symmetries of Eq. (2.21) are consistent with those implied by Eqs. (2.3) and (2.5).

The $O(3,1) \times O(N)$ generators in the new basis are related to the components of Z^{RS} as follows^{*)}

$$X^{mn} = Z^{mn} - \frac{i}{4} (k\eta\sigma^{mn})_{cd} Z^{ckdk} \quad (2.26)$$

$$Y^{ij} = Z^{aibj} k\eta_{ba} , \quad (2.27)$$

where X^{mn} present the generators of the $O(3,1)$ Lorentzian group and Y^{ij} represent generators of the $O(N)$ internal symmetry group. The connections in the new basis may also be defined so that

$$X^{MN\sim} \omega_{MN\Lambda} = Z^{MN} \omega_{MN\Lambda} . \quad (2.28)$$

Hence the $O(3,1) \times O(N)$ connections in the new basis are^{**)}

$$\tilde{\omega}_{mn\Lambda} = \omega_{mn\Lambda} , \quad \tilde{\omega}_{ij\Lambda} = \frac{1}{4} \omega_{aibj\Lambda} \frac{1}{k} \eta^{ab} . \quad (2.29)$$

Thus one may express the covariant derivative $\mathcal{D}_\Lambda$ on the new basis to exhibit explicitly the $O(3,1) \times O(N)$ part of the $OSp(3,1/4N)$ connections^{**)}

$$\begin{aligned} \mathcal{D}_\Lambda = \partial_\Lambda - \frac{1}{2} \left[X^{mn\sim} \omega_{mn\Lambda} + Y^{ij\sim} \omega_{ij\Lambda} \right] \\ + \left[X^{mai\sim} \omega_{mai\Lambda} + \frac{1}{2} \bar{X}^{aibj\sim} \omega_{aibj\Lambda} \right] . \end{aligned} \quad (2.30)$$

In a similar manner the curvature components on the new basis are defined by

$$X^{MN\sim} R_{MNAB} = Z^{MN} R_{MNAB} . \quad (2.31)$$

*) The remaining set of generators of $OSp(3,1/4N)$ on the new basis are given by

$$\begin{aligned} X^{mai} &= Z^{mai} \\ X^{aibj} &= \bar{X}^{aibj} + \frac{1}{4} \frac{1}{k} \eta^{ab} Y^{ij} \\ \bar{X}^{aibj} &= Z^{aibj} - \frac{1}{4} Z^{cidj} \eta_{dc} \eta^{ab} . \end{aligned}$$

**) The remaining set of connections of $OSp(3,1/4N)$ on the new basis are

$$\begin{aligned} \tilde{\omega}_{mai\Lambda} &= \omega_{mai\Lambda} \\ \tilde{\omega}_{aibj\Lambda} &= \omega_{aibj\Lambda} - \frac{1}{4} \eta_{ab} \omega_{cidj\Lambda} \eta^{dc} \\ &\quad + \frac{i}{4} (k\eta\sigma^{mn})_{ab} \delta_{ij} \omega_{mn\Lambda} . \end{aligned}$$

The curvature components corresponding to $O(3,1) \times O(N)$ parts are then

$$\tilde{R}_{mnAB} = \mathcal{R}_{mnAB} \quad (2.32)$$

$$\tilde{R}_{ijAB} = \frac{1}{4} \mathcal{R}_{aibjAB} \frac{1}{k} \eta^{AB} . \quad (2.33)$$

Here $\tilde{R}_{ijAB}$ are the Yang-Mills superfield strengths. From Eqs. (2.29), (2.32), and (2.33) we see that the Lorentzian and the Yang-Mills parts of connections as well as of curvatures are unified in the larger Riemannian geometry framework.

3. VACUUM SYMMETRIES

We now discuss how gauge supersymmetry deduces the vacuum state of supergravity as a consequence of spontaneous breakdown, in the limit $k \rightarrow 0$. Thus in this limit gauge supersymmetry and supergravity possess exactly the same vacuum state. The vacuum metric that results from the spontaneous breakdown of gauge supersymmetry has the form^{4)*)}

$$\begin{aligned} g^{(0)}_{\mu\nu}(z) &= \eta_{\mu\nu} , & g^{(0)}_{\mu\alpha} &= -i(\bar{\theta}\Gamma_{\mu})_{\alpha} \\ g^{(0)}_{\alpha\beta} &= k\eta_{\alpha\beta} + (\bar{\theta}\Gamma^{\mu})_{\alpha}(\bar{\theta}\Gamma_{\mu})_{\beta} , \end{aligned} \quad (3.1)$$

where Γ_{μ} obeys the condition $(\eta\Gamma_{\mu})^T = (\eta\Gamma_{\mu})$. The full set of transformations that leave invariant the vacuum metric may now be obtained from the killing vectors $\xi^{\Lambda(0)}$ of the space defined by $g^{(0)}_{\Lambda\Pi}$, i.e.

$$0 = \delta g^{(0)}_{\Lambda\Pi} = \xi^{\Lambda(0)}_{\Lambda|\Pi} + \xi^{\Lambda(0)}_{\Pi|\Lambda} , \quad (3.2)$$

where " $|$ " means the covariant derivative with respect to the vacuum metric $g^{(0)}_{\Lambda\Pi}$. The full set of $\xi^{\Lambda(0)}$ which obey Eq. (3.2) are⁴⁾

*) Γ_{μ} are matrices in the Dirac and internal symmetry space and obey the relations

$$k^{-1}\Gamma_{\mu}\Gamma^{\mu} = -\frac{1}{2}(\lambda - K') , \quad k^{-1}\text{Tr}(\Gamma_{\mu}\Gamma^{\mu}) = -4(\lambda - \lambda') ,$$

where the parameter λ enters in the action of gauge supersymmetry and λ' and K' arise from quantum loop corrections to the effective potential. The above equations assume a preserved global supersymmetry at the quantum loop level. Explicit solutions to spontaneous breakdown of gauge supersymmetry exist in the tree approximation and one hopes that the solutions to spontaneous symmetry breaking with quantum corrections included also exist.

$$\xi^\mu(0) = [\varepsilon^\mu + \varepsilon^{\mu\nu} x_\nu] + \{i\bar{\lambda}\Gamma^\mu\theta\} .$$

$$\xi^{\alpha(0)} = \left[\frac{i}{4} \varepsilon^{\mu\nu} (\sigma_{\mu\nu} \theta)^{\alpha i} \right] + \{\lambda^{\alpha i}\} - (M\theta)^{\alpha i} \quad (3.3)$$

$$[\Gamma_\mu, M] = 0 ; \quad M^T = -\eta M \eta^{-1} . \quad (3.4)$$

From Eqs. (3.3) we see that the preserved symmetries consist of the Poincaré transformations parametrized by ε^μ and $\varepsilon^{\mu\nu} = -\varepsilon^{\nu\mu}$, global supersymmetry transformations parametrized by $\lambda^{\alpha i}$, and a set of internal symmetry transformations generated by M obeying the conditions of Eq. (3.4). Further, in the vielbein language the spontaneous symmetry-breaking solutions may be expressed in the form of a vacuum vielbein^{*})

$$\begin{aligned} V_\mu^{m(0)} &= \delta_\mu^m , & V_\alpha^{m(0)} &= i(\bar{\theta}\Gamma^m)_\alpha \\ V_\mu^{a(0)} &= 0 , & V_\alpha^{a(0)} &= \delta_\alpha^a . \end{aligned} \quad (3.5)$$

One may now proceed to determine the set of vacuum and tangent space transformations that leave the vacuum vielbein invariant. For $\xi^{\Lambda(0)}$ one obtains the same result as in Eq. (3.3), while for $\varepsilon_A^{B(0)}$ one obtains

$$\begin{aligned} \varepsilon_m^{n(0)} &= \varepsilon_m^n , & \varepsilon_{ai}^{m(0)} &= 0 = \varepsilon_m^{ai(0)} \\ \varepsilon_{ai}^{bj(0)} &= -\frac{i}{4} \varepsilon_{mn} (\sigma^{mn})^b{}_a \delta_i^j - M^{bj}{}_{ai} . \end{aligned} \quad (3.6)$$

From Eq. (3.6) we see that after spontaneous breaking the tangent space group invariance $O\text{Sp}(3,1/4N)$ is broken and the residual symmetries consist only of the set corresponding to $O(3,1) \times \mathfrak{G}(N)$, where the group $\mathfrak{G}(N)$ corresponds to the internal symmetries generated by M of Eq. (3.4). It can also be easily shown⁴⁾ that as one takes the limit $k \rightarrow 0$, the group $\mathfrak{G}(N)$ expands to the group $O(N)$, so that the residual tangent space symmetries in the limit $k \rightarrow 0$ consist of $O(3,1) \times O(N)$. These are precisely the tangent space symmetries of extended supergravity.

*) $\Gamma^{m'}$ plays the same role for the vielbein case that Γ^m does for the metric formulation.

4. REDUCTION OF RIEMANNIAN GEOMETRY

We discuss next the actual reduction of the non-Riemannian supergravity geometry. We have seen already that the unified tangent space group $OSp(3,1/4N)$ of the Riemannian theory contains as subgroups the tangent space group $O(3,1) \times O(N)$ of extended supersymmetry, which turns out to be indeed the only preserved group of the tangent space of the Riemannian theory after spontaneous breaking in the limit $k \rightarrow 0$. The reduction of the Riemannian geometry to the supergravity geometry thus involves both the extraction of the correct $O(3,1) \times O(N)$ subgroup of the Riemannian tangent space as well as the limit $k \rightarrow 0$. To exhibit this more clearly we first carry out the decomposition of $\mathcal{D}_\Lambda$ into its $O(3,1) \times O(N)$ subpart and a remainder

$$\mathcal{D}_\Lambda = \tilde{\mathcal{D}}_\Lambda + K_\Lambda \quad (4.1)$$

$$\tilde{\mathcal{D}}_\Lambda = \tilde{\partial}_\Lambda - \frac{1}{2} \left[X^{mn} \tilde{\omega}_{mn\Lambda} + Y^{ij} \tilde{\omega}_{ij\Lambda} \right] \quad (4.2)$$

$$K_\Lambda = - \left[X^{mai} \tilde{\omega}_{mai\Lambda} + \frac{1}{2} \bar{X}^{aibj} \tilde{\omega}_{aibj\Lambda} \right], \quad (4.3)$$

where both $\tilde{\mathcal{D}}_\Lambda$ and K_Λ are $O(3,1) \times O(N)$ covariant quantities. Thus in the formulation of a geometry with an $O(3,1) \times O(N)$ tangent space one may use either or both in a linear combination. The formulation of supergravity geometry however, requires that one uses only the reduced $\mathcal{D}_\Lambda$ in formulating this theory, i.e. the quantity

$$\mathcal{D}_\Lambda = \tilde{\mathcal{D}}_\Lambda \Big|_{k \rightarrow 0}. \quad (4.4)$$

$\mathcal{D}_\Lambda$ represents the tangent space covariant derivative for supergravity involving the Lorentzian connection $h_{mn\Lambda}$ and the Yang-Mills connection $h_{ij\Lambda}$:

$$\mathcal{D}_\Lambda = \partial_\Lambda - \frac{1}{2} \left[X^{mn} h_{mn\Lambda}(z) + Y^{ij} h_{ij\Lambda}(z) \right], \quad (4.5)$$

where $h_{mn\Lambda}$ and $h_{ij\Lambda}$ are to be calculated. Since the connections for Riemannian space are known in terms of the vielbeins in a closed form, one also expects closed form solutions for the supergravity connections through the reduction of Eq. (4.4). The results of this reduction were displayed in Section 1. However, before we discuss the actual deduction of the closed forms, we should like to mention briefly the direct technique that one may always use for the construction of these connections. This

technique is the gauge completion method and we illustrate it first for the construction of the vielbein V_{Λ}^A that enters in the superspace formulations of supergravity.

The gauge completion method for the construction of the vielbein V_{Λ}^A of supergravity consists in computing V_{Λ}^A order by order in θ^{α} , so that the transformation laws of superspace Eq. (2.1) are consistent with the supergravity transformation laws^{*)}

$$\delta e_{\mu}^m = i\bar{\psi}_{\mu}\gamma^m\lambda, \quad \frac{1}{2}\delta\psi_{\mu} = D_{\mu}\lambda. \quad (4.6)$$

Thus one starts with the boundary conditions on V_{Λ}^A which are just the zeroth order terms in θ^{α} , i.e.

$$\begin{aligned} V_{\mu}^m(\theta=0)^{(0)} &= e_{\mu}^m(x), & V_{\mu}^a(\theta=0)^{(0)} &= \frac{1}{2}\psi_{\mu}^a \\ V_{\alpha}^m(\theta=0)^{(0)} &= 0, & V_{\alpha}^a(\theta=0)^{(0)} &= \delta_{\alpha}^a. \end{aligned} \quad (4.7)$$

Similarly one assumes an expansion for $\xi^{\Lambda}(z)$ starting with

$$\xi^{\mu}(z) = i\bar{\lambda}\gamma^{\mu}\theta, \quad \xi^{\alpha}(z) = \lambda^{\alpha} \quad (4.8)$$

and allowing λ^{α} to become a function of x so that $\lambda^{\alpha} = \lambda^{\alpha}(x)$. Consistency of Eqs. (4.7) and (4.8) then demands specific higher order θ^{α} field dependent terms^{**)}.

A similar procedure can be carried out for the construction of the superconnections of supergravity through the gauge completion method. Thus in the analysis of Brink et al.²⁾ one uses the transformation equations for $h_{mn\Lambda}(z)$, i.e.

$$\begin{aligned} \delta h_{mn\Lambda}(z) &= \varepsilon_{mn}(z)_{,\Lambda} + (-1)^{\Lambda+\Sigma}\xi_{\Sigma}^{\Lambda},_{\Lambda} h_{mn\Sigma} \\ &\quad + h_{mn,\Sigma}\xi_{\Sigma}^{\Sigma} \end{aligned} \quad (4.9)$$

*) D_{μ} appearing in Eq. (4.6) is defined by

$$D_{\mu} = \partial_{\mu} + \frac{i}{4}\sigma^{mn}\omega_{mn\mu}(x),$$

where $\omega_{mn\mu}(x)$ is the sum of the vierbein affinity and a spin $\frac{3}{2}$ torsion (see Ref. 1).

***) We note also that gauge completion in the higher order in θ sectors without use of field equations would also require introduction of auxiliary fields.

and the boundary conditions at $\theta^\alpha = 0$, i.e.

$$h_{mn\mu}(x; \theta^\alpha = 0) = \omega_{mn\mu}(x) \quad (4.10)$$

to construct $h_{mn\Lambda}(z)$ in a manner similar to the one described above to construct the vielbein. A similar procedure can also be used to construct the Yang-Mills connection $h_{ij\Lambda}(z)$, where the boundary conditions chosen are

$$h_{ij\mu}(x; \theta^{\alpha i} = 0) = e A_\mu^{ij}(x) , \quad (4.11)$$

where $A_\mu^{ij}(x)$ are the Yang-Mills potentials and e is the parameter that gauges the internal symmetry in $SO(N)$ supergravity models⁹⁾. Next we discuss how the contraction procedure generates the closed form solutions for the supergravity connections.

From Eqs. (4.2), (4.4), and (4.5) we have the following expressions for the supergravity connections $h_{mn\Lambda}$ and $h_{ij\Lambda}$:

$$h_{mn\Lambda}(z) = [\tilde{\omega}_{mn\Lambda}(z)]_{k \rightarrow 0} \quad (4.12)$$

$$h_{ij\Lambda}(z) = \left[\frac{1}{4} \tilde{\omega}_{aibj\Lambda} \frac{1}{k} \eta^{ba} \right]_{k \rightarrow 0} . \quad (4.13)$$

From Eqs. (2.23) and (2.24) we find that

$$\Omega_{ABci} \sim O(k) , \quad \Omega_{ABm} \sim O(1) . \quad (4.14)$$

Equations (4.12) and (4.14) then immediately lead to Eqs. (1.3) and (1.4). To verify that the connections of Eq. (1.3) and (1.4) are indeed the correct Lorentzian parts of the supergravity, we note that the $h_{mn\Lambda}$ of Eq. (4.12) obey the transformation law of Eq. (4.9). Further by a direct substitution of the gauge complete vielbein one can also verify that the boundary conditions of Brink et al.²⁾ for the cases $N = 1, 2$ of Eq. (4.10) are also reproduced.

In a similar fashion Eqs. (4.13) and (4.14) lead to Eqs. (1.6)-(1.8), though the limit is somewhat more subtle owing to the $1/k$ factor appearing in Eq. (4.13). Since one has a Majorana trace appearing in Eq. (4.13), the singular $1/k$ term actually cancels out at $\theta^\alpha = 0$ and thus the limit is smooth as $k \rightarrow 0$. Again the $h_{ij\Lambda}$ of Eqs. (4.13) or (1.6) satisfies the correct transformation laws appropriate for a Yang-Mills connection and the set of Eqs. (1.6)-(1.8) yield the correct boundary

conditions*). Finally, a similar reduction procedure can also be employed to obtain the curvatures of supergravity. Thus the supergravity curvature R_{mnAB} and the Yang-Mills superfield strengths of supergravity are obtained directly from Eqs. (2.32) and (2.33). One has

$$R_{mnAB} = \tilde{R}_{mnAB} \Big|_{k \rightarrow 0}, \quad F_{ijAB} = \tilde{R}_{ijAb} \Big|_{k \rightarrow 0}. \quad (4.15)$$

5. PROPERTIES OF GAUGE SUPERSYMMETRY

The preceding discussion shows that the geometrical aspects of the supergravity theory arise in a natural and unified way from the reduction of the Riemannian space geometry of gauge supersymmetry. The Riemannian dynamics of gauge supersymmetry is essentially unique except for the presence of the parameter λ , which plays the role of a supercosmological constant in the theory (the theory is actually free of a true cosmological constant). A remarkable feature of the Riemannian dynamics of gauge supersymmetry is the ultraviolet finiteness of the theory. What actually has been shown is that the off-shell Green's functions of gauge supersymmetry for $N \geq 2$ are ultraviolet finite to all orders in the quantum loop in the linearized harmonic gauge¹⁰⁾. This result is then sufficient to establish the ultraviolet finiteness of the S-matrix of gauge supersymmetry.

The proof of the ultraviolet finiteness property of the Green's functions consists first in establishing that the zeroth order tensor propagator $\Delta_{\Pi\Lambda\Sigma\Delta}(z_1, z_2) = \langle T[h_{\Pi\Lambda}(z_1)h_{\Sigma\Delta}(z_2)] \rangle$, defined in the background field $g_{\Pi\Lambda}^{(0)}$ with $g_{\Pi\Lambda} = g_{\Pi\Lambda}^{(0)} + h_{\Pi\Lambda}$, has the decomposition in momentum space of the form^{**)}

$$\Delta_{\Pi\Lambda\Sigma\Delta}(p; \theta_1, \theta_2) = e^{\bar{\omega} p_\mu \Gamma^\mu \xi} \sum_{n=1}^4 F^{(n)}_{\Pi\Lambda\Sigma\Delta}(p; \omega^\alpha) P_n(\xi^\alpha), \quad (5.1)$$

when the quantization of the theory is carried out in a manifestly globally supersymmetric manner. Here ω^α and ξ^α are the relative and centre-of-mass Fermi coordinates, i.e.

$$\omega^\alpha = (\theta_1^\alpha - \theta_2^\alpha), \quad \xi^\alpha = \frac{1}{2} (\theta_1^\alpha + \theta_2^\alpha). \quad (5.2)$$

*) As one goes to higher orders in θ^α , possible non-zero $1/k$ terms can be omitted since they represent irrelevant constants of integration.

**) A similar analysis holds for the ghost propagators.

Equation (5.1) contains the full content of the constraints of global supersymmetry on the propagators. The dynamics of gauge supersymmetry further requires that the zeroth order propagators of Eq. (5.1) obey superspace wave equations. An iteration procedure in θ can be used on these wave equations to compute the large Euclidean behaviour of the form factors $F^{(n)}_{\Pi\Lambda\Sigma\Delta}$. In the linearized harmonic gauge one finds for large Euclidean momenta the form

$$F^{(m)}(p; \omega^\alpha) \simeq \Sigma A_n p^{-(2+4N-n)} \omega^{\alpha_1} \dots \omega^{\alpha_n} . \quad (5.3)$$

From Eq. (5.3) one notes that the form factors exhibit a dimensional rule where for every one less factor of ω^α the form factor falls off faster by a power of momentum.

Using the large Euclidean momentum behaviour of Eq. (5.3) one may next establish that the degree of divergence of an n -sided one-loop supergraph obeys the condition

$$d_{n,m} \leq m(4 - 4N) - 2(m - 1) . \quad (5.4)$$

Then for $N \geq 2$ one has that the over-all degree of divergence is negative and all the subintegrations are finite. Finiteness of the off-shell Green's function in the linearized harmonic gauge then result from an application of Weinberg's theorem¹¹⁾.

A question which naturally arises as a result of the ultra-violet finiteness of gauge supersymmetry concerns whether this theory possesses ghosts. A definitive answer to this question as well as to the broader question of the full particle content of gauge supersymmetry is still lacking. However, some insights have been gained through study of some prototype models which possess many of the properties exhibited by gauge supersymmetry. Thus the models investigated as prototypes share with gauge supersymmetry the properties that the quadratic part of their action is initially non-diagonal. They are invariant under global supersymmetry transformations and possess a mass term or a mass parameter. Here we shall discuss two such models briefly.

The first model investigated is a scalar model involving only spin-zero and spin-half fields. It is described by the Lagrangian density

$$\begin{aligned} \mathcal{L} = & - \left(\partial_\mu A \partial^\mu A_1 + \frac{1}{2} A_1^2 + \partial_\mu B \partial^\mu B_1 + \frac{1}{2} B_1^2 \right. \\ & \left. + \frac{1}{2} \partial_\mu F \partial^\mu F + \frac{1}{2} \partial_\mu G \partial^\mu G \right) \\ & + \bar{\psi} \frac{1}{i} \gamma^\mu \partial_\mu \psi_2 - \frac{1}{2} \bar{\psi}_1 \frac{1}{i} \gamma^\mu \partial_\mu \psi_1 + \bar{\psi}_2 \psi_1 \\ & - m^3 \left(AF + BG + \frac{1}{2} \bar{\psi} \psi \right) . \end{aligned} \quad (5.5)$$

Actually the model of Eq. (5.5) for the case $m = 0$ was investigated by Ferrara and Zumino¹²⁾ as a prototype of conformal supergravity¹³⁾. In their analysis, Ferrara and Zumino found a set of dipole and tripole ghost states appearing in the theory. To exhibit their result we consider the A, A_1 sector of Eq. (5.5) when $m = 0$. Here one can introduce two creation operators $a^*(p)$ and $a_1^*(p)$ and the set of one-particle states $|p\rangle = a^*(p)|0\rangle$ and $|p\rangle_1 = a_1^*(p)|0\rangle$ obeying the normalization

$$\langle p|p'\rangle = 0 = {}_1\langle p|p'\rangle_1, \quad \langle p|p'\rangle_1 = \delta^3(p - p'). \quad (5.6)$$

However, one cannot diagonalize the Hamiltonian in this case, as can be seen by noting that only one of the states introduced above is an eigenstate of energy. Thus defining $\omega \equiv |\vec{p}|$ one has

$$P^0|p\rangle_1 = \omega|p\rangle_1, \quad P^0|p\rangle = \frac{1}{2\omega}|p\rangle_1 + \omega|p\rangle. \quad (5.7)$$

This is a feature considered characteristic of dipole ghosts. A similar phenomena appears in the B, B_1 sector, while in the ψ, ψ_1 , and ψ_2 sector one has a system of tripole ghost states appearing.

On the other hand, in gauge supersymmetry one has a mass parameter appearing in the theory after spontaneous breakdown, so that the situation characteristic of gauge supersymmetry in Eq. (5.5) would correspond to the case when $m \neq 0$. When $m \neq 0$, one finds that the A, A_1 sector is now coupled to the field F and one must diagonalize the quadratic action involving the fields A, A_1 and F in Eq. (5.5). The eigenmass spectrum of the system now arises from the condition

$$\det(K_{ij}p^2 + M_{ij}) = 0, \quad (5.8)$$

where K_{ij} and M_{ij} represent elements of the kinetic energy and mass matrices of the A, A_1 , and F system. Solutions to Eq. (5.8) show that the eigenmasses are given by $p^2 = \mu_\lambda^2$, where

$$\mu_{\lambda=1}^2 = m^2, \quad \mu_{\lambda=2,3}^2 = \mu_\pm^2 = -m^2 e^{\pm i\pi/3}. \quad (5.9)$$

Equation (5.9) exhibits a set of complex conjugate poles rather than the dipole ghost structure that emerged when $m = 0$. Further, one may carry out a creation and annihilation operator expansion of the field $\phi_i = (A, A_1, F)$ allowing for the complex energy

eigensolutions in the expansion^{†)}. One may then establish that a set of creation operators $a^{\lambda*}$ and annihilation operators a^λ which exist for the coupled system obey the commutation relations

$$[a^\lambda(p), a^{\lambda'}(p')] = 0, \quad [a^\lambda(p), a^{\lambda'}(p')^*] = \delta_{pp'} K^{-1}_{\lambda\lambda'} \quad (5.10)$$

It is possible now to diagonalize the Hamiltonian of the A , A_1 , and F system unlike the case of Ferrara and Zumino¹²⁾ and one has

$$H = \sum_{p, \lambda} \omega_\lambda(p) \left[N_\lambda(p) + \frac{1}{2} \right], \quad (5.11)$$

where N_λ are a set of commuting operators such that

$$N_1 = a_1^* a_1, \quad N_2 = a^{(-)*} a^{(+)}, \quad N_3 = a^{(+)*} a^{(-)}. \quad (5.12)$$

In fact the entire analysis now is equivalent to that of Lee and Wick¹⁴⁾. An exactly identical phenomenon occurs in the remaining sectors of Eq. (5.5).

Several other prototype models have also been investigated involving features not contained in the simple model of Eq. (5.5) such as inclusion of vector particles and existence of residual gauge symmetries in the theory in addition to global supersymmetry. We mention here briefly only one such model described by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} - \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \\ & - \frac{1}{4a^2} \square^2 V_{\mu\nu} \square^2 V^{\mu\nu} - \frac{i}{2} \square^2 \bar{\lambda} \gamma^\mu \partial_\mu \square^2 \lambda + \frac{1}{2a^2} (\square^2 D)^2. \end{aligned} \quad (5.13)$$

In the vector meson sector the eigenmass spectrum now consists of

$$\mu^2_{\lambda=1} = 0, \quad \mu^2_{\lambda=2,3} = \mu^2_{\pm} = \pm i|a|. \quad (5.14)$$

†) One has

$$\begin{aligned} \phi_i(x) = & \sum_{\vec{p}, \lambda} \left(\frac{1}{2\omega_\lambda V} \right)^{1/2} [u_i^\lambda(p) a^\lambda(p) e^{ip^\lambda x} \\ & + \left(\frac{1}{2\omega_\lambda^* V} \right)^{1/2} [u_i^{\lambda*}(p) a^{\lambda*}(p) e^{-ip^{\lambda*} x}] \}, \end{aligned}$$

where the column vectors $u_i(p)$ satisfy the condition

$$[K_{ij} p^2 + M_{ij}] u_j^\lambda(p) = 0.$$

Thus once again one has a set of complex conjugate poles, but now with a massless gauge particle (a photon) remaining. This constitutes an interesting example, since gauge supersymmetry possesses specific residual gauge invariances in addition to global supersymmetry after spontaneous breaking.

One may recall that in a theory which exhibits the Lee-Wick phenomenon, one may define a unitary S-matrix by allowing the in and out states to involve only the normal states¹⁵⁾. However, there is a loss of strict causality for a theory of this type¹⁶⁾. For the Lee-Wick electrodynamics, the mass parameter of the heavy boson appearing in the theory is of the order of ~ 10 GeV and acausal effects are expected, in intervals $< 10^{-23}$ s. For gauge supersymmetry the size of the mass parameter is much larger, i.e. $\sim 10^{18}$ GeV, so that acausal effects would manifest themselves only in exceedingly small intervals, i.e. $< 10^{-40}$ s!

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