EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN ISR-CO-OP/80-32

A PROCEDURE FOR OBTAINING TRANSVERSE WALL IMPEDANCE AND WORKING LINE
FROM TRANSFER FUNCTION MEASUREMENTS

by

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Presented at XIth International Conference on High Energy Accelerators
CERN, Geneva, July 7 - 11, 1980

Geneva, Switzerland
July 1980

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ABSTRACT

In order to measure the variation of betatron tune versus momentum, the "working line", the effect of the wall impedance must be eliminated. The method is based on a comparison between the longitudinal particle distribution calculated from the transfer function of a slow and a fast wave, and the measured longitudinal distribution. The difference between the two is due to the transverse wall impedance. A matching routine yields the correct longitudinal and betatron distributions, the working line and the stability diagram. However, the data have to be treated prior to this operation to eliminate a number of parasitic effects in order to ensure a sufficient precision. The results of the method are presented.

1. INTRODUCTION

In order to guarantee good physics conditions it is important to know the variation of the betatron tune with momentum, i.e. the "working line" for ISR stacks. A method has been developed to extract this information from the transverse beam transfer function measurements which can be made on stacks without causing an observable perturbation to the physicists. These measurements are made with a dual channel fast Fourier transformer spectrum analyser (FFT)1).

2. THEORY1),7)

When a beam is subjected to a transverse excitation, then the particle motion can be suitably described by the following equation

$$\ddot{x} + Q^2 \Omega^2 x = g \quad (1)$$

where x is displacement, $\Omega/2\pi$ revolution frequency, Q betatron wavenumber or tune and g excitation. Assume $g = G e^{j\omega t}$ and the particle displacements are expected to be of the same nature, i.e. $x = X e^{j\omega t}$. The solution becomes

$$\frac{X}{G} = \frac{1}{(Q\Omega - \omega)(Q\Omega + \omega)} = \frac{1}{[(n-Q)\Omega - \omega_e][(n+Q)\Omega - \omega_e]} \quad (2)$$

whereby $\omega = n\Omega - \omega_e$. Only the response of a whole beam can be observed, not that of a single particle, hence we put

$$ID = \frac{\bar{X}}{G} = \frac{1}{N} \int \frac{F d(Q\Omega)}{[(n-Q)\Omega - \omega_e][(n+Q)\Omega - \omega_e]} \quad (3)$$

where $F(Q\Omega)$ is the particle betatron distribution function, N the number of particles. This simplifies to

$$\pm ID = \pm \frac{1}{2Q_0 \Omega_0 N} \int \frac{F(\omega)}{\omega - \omega_e} d\omega \quad (4)$$

remembering that a large response is only obtained when the excitation lies within a fast (+) or a slow (-) wave. The integral in (4) is called the transverse dispersion integral. It splits into a real principal value (PV) and an imaginary value ($-j\pi F$) whereby the latter sign is determined by the requirement of damped solutions. This solution is modified by the environment of the beam which is described by the transverse impedance.

This impedance is defined as follows 2)

$$Z_T = j \int \frac{(E + B \times V) ds}{I \bar{X}} \quad (5)$$

with, E and B fields set-up by the beam, V velocity, ds variable describing the beam path, I beam current. The force acting on the beam can be written as

$$F = \frac{e I \bar{X} Z_T}{j 2 \pi R} \quad (6)$$

with e unit charge, $2 \pi R$ machine circumference. To take the Z_T into account one must add the driving term $F/\gamma m_0$ to the r.h.s. of (1), in other words replace

$$G \text{ by } G_0 + \frac{Z \bar{X}}{j}, \text{ where } Z = \frac{e I Z_T}{2 \pi R \gamma m_0}. \quad (7)$$

So the solution becomes

$$\frac{\bar{X}}{G_0} = +/ - \frac{ID}{1 +/ - jIDZ} \text{ or } \frac{G_0}{\bar{X}} = +/ - \frac{1}{ID} + jZ. \quad (8)$$

This is represented in the following elementary block diagram of a simple feedback control system.

$$\text{The inverse } \frac{G_0}{\bar{X}} = +/ - \frac{1}{ID} \quad (9)$$

corresponds to the stability diagram. We call the real term of the beam response (PV) the reactive response and the imaginary one ($-j\pi F$) the resistive response to be consistent with Z.

3. PRINCIPLE OF WORKING LINE MEASUREMENT

In order to establish the betatron tune variation with momentum in a stack it is necessary to identify both in a fast and a slow wave, a response which is generated by the same sample of particles. From the frequency difference one can calculate q, the non integral part of the tune Q, and from the average frequency the momentum or average radial position $\langle r \rangle$:

$$Q = \frac{\Delta \omega}{2\Omega}$$

$$\frac{\Omega_0}{2\pi} = \frac{\sqrt{\gamma^2 - 1}}{\gamma} \frac{c}{2\pi R}$$

$$\frac{d\Omega}{d\langle r \rangle} = \Omega_0 \frac{1}{R} \left(\left(\frac{\gamma_T}{\gamma} \right)^2 - 1 \right) \quad (10)$$

(c velocity of light, $\gamma = E/E_0$, E_0 proton rest mass, γ_T : γ value at transition).

Consider the case where the influence of the environment on the response of an excited beam is negligible, in other words $Z = 0$. The resistive response gives the particle betatron distribution function for a fast and a slow wave. The two total integrals over each wave must be equal since they concern the same particles. Partial integrals which start from the low momentum side and which are equal, cover the same particles, i.e. the other boundary identifies the same particles in the stack. Previous formulae can be applied to the second boundary to make a complete working line measurement. This method was originally proposed to be used with the natural Schottky spectra³⁾ and is illustrated in Fig. 1.

Fig. 1 Principle of working line measurement

When the influence of the beam environment cannot be neglected, as is the case in the ISR, ways have to be found to reconstruct the particle distribution to which the explained method can be applied. For this the transverse impedance should be determined for the measuring frequency and then Z can be calculated from (7).

4. MEASUREMENT OF TRANSVERSE IMPEDANCE (2) FROM BTF

4.1 Resistive part of Z

Neglecting some scaling factors and signs (4) can be rewritten as

$$ID = PV - j\pi F \quad (11)$$

where $PV = \int_{\omega_1}^{\omega_2} \frac{F(\omega)d\omega}{\omega - \omega_e}$ ω_1, ω_2 being the edges of the distribution. For ω_e close to ω_1 , PV will be positive, while for ω_e close to ω_2 , PV will be negative. This means that ID describes an angle of π radians in the polar diagram since $F(\omega_1) = F(\omega_2) = 0$ at the edges. This is equally the case for $1/ID$. The inverse response however will describe an angle different from π and this is caused by the resistive part of Z .

4.2 Reactive part of Z

The reactive part of Z shifts the origin of the inverse diagram along the ordinate. Suppose that this shift is known. Then the integration method explained in the previous paragraph can be used and for every momentum (radial position) in the stack, one can calculate Q(q). From the betatron distribution and the correlated momenta, one can calculate a corresponding longitudinal distribution. However this distribution can also be directly measured in the ISR and can be compared with the longitudinal distribution computed from the beam transfer function. Different origins in the inverse diagram can be tried out and the resultant longitudinal distribution compared with the measured one. The origin which yields the best match is a measure for the reactive part of the impedance. It has been pointed out⁴⁾ that the measurement of the longitudinal distribution may be influenced as well by the corresponding longitudinal impedance. Experiments in the ISR have shown that the margin of longitudinal stability is extremely large⁵⁾ and hence the influence on the density measurement may be neglected.

5. SOFTWARE IMPLEMENTATION AND RESULTS

The set up of the instrumentation as well as the data acquisition, data analysis and displays are performed by the ISR control computer.

Since the ISR are usually tuned close to an integer, the fast and slow waves are only 30 KHz apart and this is close enough that the reactive response (PV) of the beam at one band overlaps the other band (see Fig. 2a). This parasitic response is corrected by means of the value of the signal outside the stack as follows:

$$PV = A \ln \left(\frac{\omega_1 - \omega}{\omega_2 - \omega} \right) \quad (12)$$

for $\omega < \omega_1 < \omega_2$ or $\omega > \omega_1 > \omega_2$ and A is the constant density of the virtual betatron distribution. Phase errors due to residual cable delays are compensated by computations as well.

The search for the value of the reactive part of Z_T uses an iterative algorithm in agreement with 4.2.

The results can be split in two groups, i.e. tune measurements and transverse impedance measurements. Figure 2 gives an example of a working line measurement on a stack. A good correspondance has been found with the measurements performed with RF knock-out⁶⁾.

Measurements of Z_T at 10 MHz on a number of stacks have been done as well. For the horizontal plane we notice much smaller values of the reactive part than in the vertical plane, i.e. less than 1 M Ω /m compared with around 6 M Ω /m in the vertical plane. This is explained by the fact that the chamber cross section is elliptic with the longer axis in the horizontal plane. The resistive part of Z_T has been found to be a few tenths M Ω /m.

8. ACKNOWLEDGEMENTS

Many people contributed to the implementation of the fast Fourier transformer spectrum analyzer⁸⁾. Our special thanks go to those who contributed directly to this particular project, i.e. L. Durieu, D. Kemp and J-C. Juillard.

Fig. 2 a : signals from FFT; b : calculated betatron distributions and stability diagram; c : working line measurement.

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