CM-P00061769

Archives

Ref.TH.2602-CERN

A CRITIQUE OF ASTROPHYSICAL APPLICATIONS
OF HAGEDORN'S BOOTSTRAP

W. Nahm
CERN - Geneva

A B S T R A C T

Hagedorn's bootstrap should not be applied to hadronic matter at densities large against nuclear densities. The correct predictions of the thermodynamical model do not use any relation between the mass of the fireballs and their size, whereas the astrophysical applications depend on the unreasonable assumption that the size is independent of the mass. Moreover, the most spectacular prediction of the bootstrap, namely violent black hole explosions yielding 10^{15} g in the last millisecond, is completely unfounded, even if such an assumption is made.

Talk given at the Workshop on Hadronic
Matter at Extreme Energy Density
Erice
October 1978

Ref.TH.2602-CERN

8 January 1979

1. - INTRODUCTION

Astrophysicists discuss many different equations of state for hadronic matter at energy densities large against 1 GeV/fm^3 ¹⁾. However, for the calculation of black hole decay two models are the most popular : conventional field theory and Hagedorn's bootstrap. The models tend to be treated on the same footing. Expensive experiments have even be justified with the assumption that the bootstrap has a reasonable chance of being correct. Here I want to show that astrophysical applications of the bootstrap should not be believed, though Hagedorn's model is very successful in describing hadron collisions at accelerator energies ²⁾. In part this is a self-criticism.

In the asymptotically free non-Abelian gauge theories favoured today for the non-gravitational interactions, matter at high energy densities behaves like radiation ³⁾. This means in particular that the pressure is about one third of the energy density, and that the energy density increases like the fourth power of the temperature. For other renormalizable theories, it might well increase with a different power of the temperature, but there is no indication that a maximal temperature exists. In theories with an infinite number of fundamental fields one might have a maximal temperature or no homogeneous thermal equilibrium at all, but the divergences of such theories probably cannot be made sense of anyhow. Gravity also behaves differently, as the classical energy is not bounded from below if black holes are present. Gravity of course is weak and does not pose much of a problem below the Planck temperature of $10^{20} m_{\pi}$, but theories with strongly coupled tensor fields might well have an unexpected behaviour. Nevertheless, our present understanding of high energy physics indicates that below the Planck temperature the free gas behaviour is qualitatively a reasonable approximation.

Hagedorn's bootstrap yields very different results. There is a maximal temperature $T_0 \sim m_\pi$, such that one can reach arbitrary high energy densities at finite temperature, which would be very important for astrophysics. In the original version of the model the equilibrium pressure increases only logarithmically with the energy, in another version it even becomes constant. This is possible because the fluctuations of the energy density near T_0 become very big and the energy is concentrated in a few fireballs. This also has a further interesting consequence : the establishment of thermal equilibrium takes a long time. In particular, the expansion of the universe should easily disrupt the equilibrium, and the later decay of the fireballs down to low energy densities might create lots of entropy. This has been used by Carlitz, Frautschy and myself to explain the high value of the entropy per baryon ⁴⁾. I shall say something more about our model and its deficiencies in Section 3. What is more important, the prediction of a maximal temperature has much increased the hope of astronomers to observe the final explosions of black holes.

2. - BLACK HOLE EXPLOSIONS ACCORDING TO THE BOOTSTRAP

According to Hawking ⁵⁾, the temperature of a Schwarzschild black hole is inversely proportional to its radius

$$T = \frac{1}{4\pi R} \quad (1)$$

and for its mass one has

$$M = \frac{R}{2G} . \quad (2)$$

(we put $\hbar = c = k = 1$). This yields an entropy

$$S = \int \frac{dM}{T} = \frac{\pi R^2}{G} = 4\pi G M^2. \quad (3)$$

If T_0 really is the highest temperature, the black hole should explode after it has reached a radius $R \sim 0.1$ fm, i.e., a mass $M \sim 10^{14}$ g. To see what is meant by explosion consider the entropy of a hadron of high mass, which according to Hagedorn is

$$S \approx \frac{M}{T_0}. \quad (4)$$

When the radius of a black hole has decreased to $(2\pi T_0)^{-1}$, the entropy of a hadron of the same mass starts to get bigger than that of the black hole. According to the original version of Hagedorn's bootstrap, the volume should be comparable, too. Thus it becomes very probable that the black hole will change into a hadronic fireball.

In Ref. 4) it was argued that the decay time of a fireball of this mass should lie between 10^{-4} sec and 10^5 y. Astrophysicists usually use the lower limit or even shorter decay times, yielding very powerful explosions. I insisted that the decay time is 10^5 y, if one really sticks to the model ⁶⁾. However, this calculation neglected gravity, and Hawking and Page correctly pointed out that this is inadmissible, as for fireballs heavier than Planck's mass of 10^{-4} g the gravitational energy cannot be neglected against the kinetic energy ⁷⁾, though of course below 10^{14} g the gravity contribution to the fireball mass is small. They argued that gravity should increase the decay rate, as in thermodynamical equilibrium the collisions between the fireballs would become more numerous, and therefore also the decays. Unfortunately this argument is completely wrong, as it is based on a given equilibrium density of fireballs, independent of the gravitational interaction. But if one starts from an equilibrium with $G=0$ and then turns on gravity, most of the collisions would yield fireballs of higher mass, until a new equilibrium is reached, with a much lower density of fireballs and drastically reduced rates of collisions and decays.

Indeed, Hawking and Page's argument starts at a wrong point. Intuitively, fireball fragments should have greater difficulty to get apart, if they attract each other gravitationally.

Taking gravity into account, one also expects a smooth transition between hadrons and black holes. For masses less than 10^{14} g, black holes should spontaneously change into hadrons, and for masses larger than 10^{15} g, hadrons will be inside their Schwarzschild radius. For hadron masses between 10^{-5} g and 10^{14} g the spectrum should interpolate between Eqs. (3) and (4). Thus let us repeat the bootstrap calculations taking into account gravity.

Let $\sigma(E)$ be the density of states per energy, i.e.,

$$\sigma(E) = \frac{V_0}{(2\pi)^3} \int \rho(m) \delta(E - \sqrt{p^2 + m^2}) d^3p dm, \quad (5)$$

where $\rho(m)$ is the hadronic mass spectrum. Neglecting unessential complications the bootstrap equation should look like

$$\sigma(E) \approx \sum_{n=2}^{\infty} \frac{1}{n!} \int \delta(E - \sum_{i=1}^n E_i + \sum_{i,j} \frac{G E_i E_j}{r_0}) \prod_{i=1}^n \sigma(E_i) dE_i, \quad (6)$$

where r_0 is of the order of 1 fm. With

$$f(E) = \ln \sigma(E) \quad (7)$$

this yields

$$f(E) \approx \max \left\{ \sum_i f(E_i) \middle| \sum_i E_i + \sum_{i,j} \frac{G E_i E_j}{r_0} = E \right\}. \quad (8)$$

Below 10^{14} g the gravitational contribution can be treated as a perturbation :

$$T_0 f(E) = E + G h(E) + O(G^2), \quad (9)$$

yielding

$$h(\sum_i E_i) - \sum_{i,j} \frac{E_i E_j}{T_0} \geq \sum_i h(E_i), \quad (10)$$

that is

$$h(E) = \frac{E^2}{2r_0}. \quad (11)$$

Thus

$$S(E) = \ln \sigma(E) \approx \frac{E}{T_0} + \frac{GE^2}{2r_0 T_0} \quad \text{for } E \ll 10^{15} g, \quad (12)$$

which interpolates between Eqs. (3) and (4).

To evaluate the decay probability into fireballs with energies E_i we have to take into account both the phase space and the final state interaction. In particular, the kinetic energy of the fireballs should be sufficient to get them at least a distance of $2r_0$ apart.

Now the preceding equations show immediately that for all E_i the value of $\Sigma f(E_i)$ is approximately the same, such that phase space does not favour particular decay configurations, as far as exponential factors are concerned. On the other hand, a decay product of energy E_1 needs at least a kinetic energy of $GE_1(E-E_1)/(2r_0)$ to get away, such that the decay probability is suppressed by a factor $\exp(-GE_1(E-E_1)/(2r_0 T_0))$. Thus typical decay products have at most an energy

$$E' \lesssim \frac{2r_0 T_0}{GE} . \quad (13)$$

Because of limitation of space a fireball cannot decay into more than, say, 12 parts at once. The time per decay is certainly larger than r_0 , as the velocities are small. Thus

$$\frac{dE}{dt} \ll \frac{24T_0}{GE} . \quad (14)$$

This is rather slow, and in the last second much less than 10^9 g will be freed. In the conventional model the explosion energy will be higher, as already the electromagnetic radiation yields a contribution

$$\frac{dE}{dt} \sim 4\pi R^2 \sigma T^{-4} = (15 \cdot 2^8 \pi R^2)^{-1} , \quad (15)$$

where $\sigma = \pi^2/60$ is the Stefan-Boltzmann constant. Thus due to the electromagnetic radiation alone, the black hole will evaporate 10^9 g in its last second, which is more than for the bootstrap, but still not very high. A few fields more yield a factor of 10, but that's it. Thus for reasonable densities of black holes, one has not much of a chance to see their "death cries". Astronomers have tried hard to see the γ 's from the explosions, but in vain. Still the searches have some use as they put limits on the black hole producing fluctuations in the early universe ⁸⁾. But in my opinion one should decide about such searches without taking the bootstrap into account, and this for two reasons. Firstly and contrary to the present belief of astrophysicists, the bootstrap would yield less violent explosions than field theory, as I have tried to show. Secondly, I see no experimental support whatsoever for the bootstrap type

behaviour of hadronic matter at high energy densities, in spite of the successes of the bootstrap at accelerator energies.

Before discussing this point, let me say something about the application of the bootstrap to the cosmological entropy production, which does not fare much better than the application to black hole explosions.

3. - THE BOOTSTRAP AND COSMOLOGICAL ENTROPY PRODUCTION

Using the bootstrap, Carlitz, Frautschi and myself tried to derive the density distribution in the universe and the large entropy per baryon ⁴⁾. Simplifying the arguments, we reasoned like this. The causally connected pocket of our universe has about the correct mass for a black hole. From Einstein's equations we know that this feature is approximately time invariant. Thus let us go back to 10^{-23} sec after the big bang, where bootstrap hadrons just should come into being. The mass of a black hole of this size is $10^{15} \text{ g} \sim 10^{39} m_{\pi}$, and this will also be the approximate mass of our hadrons. Within somewhat less than $10^{39} \times 10^{-23}$ sec they should decay into 10^{39} particles, among them a sizeable fraction of photons. Then thermal equilibrium should be restored. Before this restoration the numbers of photons and baryons are approximately equal, but afterwards the number of photons will have increased by a large factor, which is to be compared with the observed number of about 10^8 or 10^9 photons per baryon.

Let us evaluate this factor. According to Einstein's equations, the volume increases like the square of the time. Thus before restoration of thermal equilibrium the number density of photons is 10^{-39} fm^{-3} , whereas the energy density is about $10^{-39} m_{\pi} \text{ fm}^{-3}$ (both figures will be larger if the fireball decay time is shorter). Now the equilibrium number density of photons increases like the third power of the temperature, which

is proportional to the fourth root of the energy density. Thus during the restauration of thermodynamical equilibrium the photon number has to increase by a factor of somewhat less than $(10^{39})^{1/4}$, which is certainly in the right ballpark.

However, this scenario is in contradiction with many other observations. Almost no primordial helium is produced, as the radiation era is too short. We tried to transform this vice into a virtue, noting that the gravitational growth of density fluctuations is unimpeded, if there is no radiation to damp it. However, we greatly overestimated the growth rate by an unjustified application of linear perturbation theory, as has been pointed out by Peebles⁹⁾. As a consequence, the observed large inhomogeneities, which extend over 10^{-2} of the radius of the causally connected pocket of our universe, cannot have arisen from a homogeneous initial state. After all this may be a good thing, because initial inhomogeneities tell us a bit more about what happened before the big bang.

As far as the photon number per baryon is concerned, there is now hope for a better theory using initial conditions with baryon number zero, and C, P and T violations arising naturally in unified gauge theories^{10),11)}. But even without the problems concerning helium production, etc., we should have been reticent to apply the bootstrap to cosmology. This may sound strange, because Hagedorn's model was the first one to give reasonable and quite often accurate prediction for hadronic cross-sections, and is still widely and successfully used by experimenters. But I shall try to dissect the model into two pieces with no deep connection between each other. One half alone yields the experimental success, the other half yields the controversial astrophysical predictions, and is responsible for the most important wrong predictions at accelerator energies.

My presentation of the model will be as elementary as possible. Moreover, I shall describe it not as it really came into being, but in the dry, idealized, and inaccurate way such things are usually described in textbooks.

4. - THE THERMODYNAMIC MODEL AD USUM DELPHINI

The success of the model depends on the correct description of the longitudinal and transverse dynamics of hadron collisions. The longitudinal dynamics has simply been fitted in a way yielding limiting fragmentation. The transverse dynamics depends on the exponential mass spectrum of the resonances. These are the good features.

On the contrary, the astrophysical applications depend on the assumption of a universal volume for all hadronic resonances and fireballs, which is (a) unreasonable, (b) unnecessary for the empirical success of the model, and (c) not in agreement with the observed hadron spectrum. On the contrary, strings and bag models share the good features of the bootstrap, but avoid the bad ones. Hopefully these models can be derived from QCD.

Let me consider these points separately. Early attempts to predict cross-sections for hadron collisions in the 30 GeV range on the basis of phase space alone failed completely. In particular the obvious peaking in forward and backward directions could not be reproduced, but had to be explained from the internal structure of the hadrons. Phenomenologically, the effect was described by intermediate states with a number of fireballs moving along the collision axis. For the moment we regard fireballs just as highly excited pieces of hadronic matter with no important externally imposed constraints, such that for their decays a distribution according to phase space is reasonable.

For heavy fireballs the recoil can be neglected, and the decay products have a thermal spectrum, with temperature

$$T = \left(\frac{d}{dM} \ln \rho(M) \right)^{-1}, \quad (16)$$

where $\rho(M)$ is the density of states per mass of the fireballs.

Now let us consider the one-particle spectrum coming from a collision with total energy $2E$. Apart from quantum number effects, which are unimportant for our purpose, we have to take a thermal spectrum in the rest frame of each fireball and to transform it by a Lorentz transformation into the laboratory frame. The fireballs have to have small transverse momenta, as otherwise we could not describe the peaking along the collision axis. For simplicity let us assume that their transverse momenta are zero, but later we shall show that more reasonable values do not change the results significantly.

Let us denote the thermal spectrum at temperature T with $\theta(\vec{p}, T)$, where $\vec{p}$ is the momentum of the emitted particle. Then with $x = P/E$, where P is the fireball momentum,

$$\frac{d\sigma}{d\vec{p}} = \int f(x, M, E) L_x \theta(\vec{p}, T(M)) dx dM. \quad (17)$$

Here $f(x, M, E)$ is up to a normalization the mean density of fireballs with given M and x . With L_x we denote the Lorentz transformation to the rest frame of the fireball. We still have to determine one function $T(M)$ of one variable and one function $f(x, M, E)$ of three variables.

I shall explain below how Hagedorn found that

$$\rho(M) \sim \exp(M/T_0) \quad (18)$$

for sufficiently high masses, such that according to Eq. (16)

$$T(M) \approx T_0. \quad (19)$$

Note by the way that a kinetic energy of order T_0 restricts the momenta to values of order $(MT_0)^{\frac{1}{2}}$, such that Eq. (5) yields

$$\rho(M) \sim M^{-\frac{3}{2}} \sigma(M). \quad (20)$$

Equation (19) helps a lot, because we can integrate over M , obtaining

$$\frac{d\sigma}{d\vec{p}} = \int F(x, E) L_x \theta(\vec{p}, T_0) dx, \quad (21)$$

where only one function of two variables remains to be determined, namely

$$F(x, E) = \int f(x, M, E) dM. \quad (22)$$

Now Hagedorn compared with the experiments and found that $F(x, E)$ is practically independent of E , if the collision energy is high enough. Later this phenomenon was rediscovered by Benecke, Chou, Yang and Yen and called limiting fragmentation.

Well, I have been cheating a little bit, because Hagedorn did not use

$$x = \frac{v\lambda}{\gamma_0}, \quad (23)$$

but

$$\lambda = \text{sign}(v) \frac{\gamma_0 - 1}{\gamma_0}, \quad (24)$$

where v is the fireball velocity, γ the corresponding Lorentz factor

$$\gamma = (1 - v^2)^{-1/2} \quad (25)$$

and γ_0 the Lorentz factor of the incoming particles. For large γ this makes no difference, as the variables x and λ are approximately equal. For small velocities, λ is of course a poor choice, as it depends non-analytically on v . However, the central region is anyhow better described in terms of the rapidity, which for vanishing transverse momenta is just

$$y = \frac{1}{2} \ln \frac{1+v}{1-v} \quad (26)$$

Using this variable, one obtains experimentally a flat distribution near $y=0$. This means local invariance of the hadronic matter with respect to longitudinal Lorentz transformations. Therefore such a distribution stays flat, when the fireballs decay.

In terms of λ such a flat distribution in y yields

$$F(\lambda, E) = (\lambda^2 + 2\lambda/(\gamma_0 - 1))^{-1/2} \quad (27)$$

At 30 GeV this function should be reasonable up to $\lambda = 1/3$, one unit of rapidity away from the maximal rapidity. Indeed between $\lambda = 0.02$ and $\lambda = 0.3$ it agrees well with Hagedorn's $F(\lambda)$, of course up to a normalization. However, Hagedorn kept his $F(\lambda)$ finite at $\lambda = 0$, which introduces at $y=0$ a spurious hole into the y distribution. Moreover, below $\lambda = 0.1$ an energy dependence should be taken into account.

These unrealistic features of the longitudinal dynamics can easily be remedied by working with y instead of λ ¹²⁾⁻¹⁴⁾,

which indeed is done at ISR energies. The unrealistic assumption of zero transverse momentum of the fireballs also causes no problems. From the decay recoils the fireballs will anyhow acquire some transverse momentum which approaches a thermal distribution. Thus one may as well assume a thermal distribution from the start, which is by far enough to fulfil the uncertainty relation. Even then the corresponding transverse velocities will be so small that one can forget about them.

In such a model it should not make much of a difference at which generation of fireball decay one starts. In particular the individual contributions of high mass resonances are washed out, though it is useful to take the very last generation of resonances explicitly into account. The first generation of fireballs are not even well-defined entities. Regarding the hadronic matter created by energetic collisions one just considers regions which are small enough such that no longitudinal peaking of the relative velocities is marked. But the hadronic matter need not split into such parts before particles are emitted.

Such a behaviour of the longitudinal dynamics had to be assumed in Hagedorn's model, whereas nowadays limiting fragmentation and a flat y distribution can be derived in bag and string models ¹⁵⁾⁻¹⁷⁾. The other basic ingredient for the success of the model is the exponentially increasing mass spectrum of the fireballs. On the contrary, nothing need be assumed about their size, except that it should be not too large, such that the states can be produced in typical collisions. This is not an important restriction, as the mean observed size of fireballs ("clusters") is anyhow only 1-2 GeV, independent of the collision energy.

One might even argue that at these low masses the exponential increase is just an accident, and that it would be sufficient to use just the resonances of the naïve quark model. However, the fireball excitation function $f(x, M, E)$ might well have a tail reaching to much higher masses. In particular one should note that the thermodynamical model allowed a successful calculation of the production rates of anti-deuterons and anti-He³ (18). In QCD one may also expect the production of larger fireballs in collisions where multigluon exchange yields flux tubes with a colour flux in a higher SU(3) multiplet.

To gain a better understanding of the significance of a fixed fireball volume in Hagedorn's bootstrap, let us look at his derivation of the exponential mass spectrum, and let us compare it with the collective excitations of strings and bags, which also yield an exponential spectrum. Hagedorn made one basic assumption which was hotly contested in the sixties, but seems to be universally accepted today : there is no qualitative difference between resonances and fireballs.

Thus we may build up the fireballs inversely to their decay by letting the decay products resonate with each other. Both Hagedorn's model and the bag and string models assume that the interaction is rather soft : for a given location in phase space of the constituents the resonance is essentially uniquely determined and the binding energy is not large. This means in particular that $\rho(M)$ cannot increase faster than exponentially, because otherwise

$$\rho(M+M') \gg \rho(M)\rho(M') \quad (28)$$

for high masses.

That hadronic interactions are sort, in other words that there are no strong forces involved, is up to a point well born out by the experiments, as large transverse momenta are rare. Note also that for a spectrum increasing faster than exponentially the time for every decay step of a fireball would increase with its mass, as the phase space for the decay products would be smaller than that of the fireball itself. Pion emission would dominate, thus the decay time would behave like

$$t \sim \frac{\rho(M)}{\rho(M-m_\pi)}, \quad (29)$$

whereas evidently no hang-up in fireball decay is observed. On the contrary, for black holes just such a phenomenon is to be expected, as according to Eq. (3) $\log \rho(M)$ is proportional to M^2 .

In general one might expect that soft interactions yield

$$\log \rho(M_1+M_2) \ll \log (\rho(M_1)\rho(M_2)), \quad (30)$$

as states of mass M_1+M_2 usually can be obtained in very many ways by putting states of masses M_1 and M_2 together. For example in a pion gas one may distribute the pions in many different ways into two packages. But if the inequality (30) is true, $\rho(M)$ increases slower than exponentially. Only if the fireballs somehow remember the way they have been put together, one may have an approximate equality in (30) and correspondingly an exponential spectrum.

So far the agreement between the conventional theories and Hagedorn's bootstrap, but now comes the big difference. If one considers collective excitations in string or bag models, the size of a fireball increases proportionally to its mass. Thus

if one puts two fireballs together, they still occupy different volumes and the parts still can be identified to some extent. Instead of the inequality (30) one therefore obtains an approximate equality. In other words, the entropy is an extensive quantity, just as for ordinary matter :

$$S(\lambda E, \lambda V) = \lambda S(E, V). \quad (31)$$

Therefore the exponential increase of $\rho(M)$ is a rather trivial geometrical effect, which does not say much about the equation of state for high densities. Consider for example ordinary water in a state dominated by droplets. Volume and mass are proportional, and their density of states per mass increases exponentially. The entropy of the total system can be obtained in the usual way from the entropy of the droplet. But now increase the density until the droplets fuse, and still further : one obviously has to switch to a completely different description of the system.

Of course one may use slightly different versions of this geometrical effect, having either a constant energy per volume, a constant surface tension, or a constant tension along a string. The difference matters only if one calculates the power of M in front of the exponent in $\rho(M)$. Configurations of open strings are the easiest to count. Adding one unit length to the string, the number of configurations increases by some constant factor

$$\sigma(E + \Delta E) = c(\Delta E) \sigma(E), \quad (32)$$

such that the density of states with energy E is a pure exponential

$$\sigma(E) \approx C \exp(E/T_0). \quad (33)$$

For open surfaces one can add unit pieces of surface anywhere along the boundary, which yields some multiple counting. If $l(E)$ is the length of the boundary, one finds

$$\sigma(E+\Delta E) l(E+\Delta E) = c(\Delta E) \sigma(E) l(E) \quad (34)$$

For smooth surfaces, $l(E)$ should increase like $E^{\frac{1}{2}}$, such that

$$\sigma(E) \sim E^{-\frac{1}{2}} \exp(E/T_0). \quad (35)$$

Similarly for volumes

$$\sigma(E) \sim E^{-\frac{2}{3}} \exp(E/T_0). \quad (36)$$

$\rho(M)$ is always given by Eq. (20).

A geometrical effect of just this kind is responsible for the exponential increase of the mass spectrum in the old Pomeranchuk model of fireball decay, as he also considered fireballs with a volume proportional to their energy^{19),20)}. However, one again obtains a different power of M in front of the exponential, as he admitted arbitrary distributions of the energy within the volume, but fixed the geometry of the volume itself.

In contrast, Hagedorn does something completely different. In the original version of his model, he put all the degrees of freedom into one volume of fixed size, with a radius given

by the pion mass. In this model one cannot simply explain why the hadrons remember how they have been put together, one has to postulate it. In other words, as, e.g., a $\rho(\pi\pi)$ resonance must be different from the corresponding $(\rho\pi)\pi$ resonance, even if the momenta involved are the same, one must postulate that the formation of resonances is not associative.

Describing strings in this language, one has associativity but not commutativity :

Putting the pieces together in the order ACB yields obviously a string different from ABC, but $(AB)C$ and $A(BC)$ are the same.

In my opinion, the assumption of a constant volume of all resonances cannot be maintained. On the one hand one cannot identify a dynamic mechanism which confines them to one fermi cube, as large forces are incompatible with the assumptions of the model. But the assumption of a constant volume is also impossible to reconcile with the observed linear Regge trajectories.

Consider the expectation value of the orbital angular momentum

$$\vec{\ell} = \sum_n \vec{r}^n \times \vec{p}^n \quad (37)$$

where $\vec{p}^n$ is the momentum on the n^{th} constituent. Obviously,

$$\langle l \rangle \leq MR, \quad (38)$$

where R is the radius of the system. Thus for a constant volume the maximal angular momentum of hadrons of mass M should be

$$l_{\max} \sim M, \quad (39)$$

if the constituents are enclosed in a potential well of infinite height. For a confining potential $V(r) \sim \exp(r/r_0)$ the maximal angular momentum is somewhat larger, but only by a factor $\log(Mr_0)$, as not much phase space outside $r=r_0$ is available. On the contrary for strings and bags

$$R_{\max} \sim M \quad (40)$$

and

$$l_{\max} \sim M^2, \quad (41)$$

as it is observed. Thus for hadrons on the leading trajectories one sees directly that their energy density does not vary strongly with M .

Gorenstein, Miransky, Shelest and Zinoviev²⁰⁾, and Hagedorn, Montvay and Rafelski at this workshop introduced versions of the bootstrap in which both non-associativity and a volume proportional to the fireball mass contribute to the exponential rise of the mass spectrum. Such models are threatened by Occam's razor, but there are more serious objections, too. The volume effect alone yields one non-degenerate leading

Regge trajectory. If one now adds non-associativity, the multiplicity along this trajectory should increase rapidly, in contradiction to the observations. One can also argue, of course, that deep inelastic scattering, large transverse momenta, ψ spectroscopy, etc., more and more reveal a rather conventional internal dynamic of the hadrons, which is difficult to reconcile with non-associativity at distances of the order of one fermi.

5. - CONCLUSIONS

When people tried to apply the bootstrap to hadronic matter at large energy densities the results have been wrong (as for black hole decay) or inconsistent with the observations (as for entropy production in the early universe). There is no point in looking for improved applications, as an analysis of the foundations of the bootstrap shows that its fireballs should be objects with an energy density of order 1 GeV/fm^3 . We have to accept that our laboratory experiments yield no direct observations of the states of high energy density relevant for cosmology. Of course, states of high energy density are initially created by colliding Lorentz contracted protons, but they are far from thermodynamic equilibrium.

Nevertheless, the bootstrap might yield interesting phenomenological models for phase transitions near energy densities of 1 GeV/fm^3 . Below this energy density one should have a gas of hadronic resonances and bound states. Near the transition point they should cluster to larger and larger fireballs, and finally fuse. Above the transition point one has to describe the state in terms of the hadronic constituents. This direction has been explored by Cabibbo and Parisi²¹⁾, and by Hagedorn, Montvay, Rafelsky and Satz at this workshop. If the bootstrap will have useful astrophysical applications, then in this density regime.

REFERENCES

- 1) V. Canuto - Ann.Rev.Astron.Astrophys. 12 (1974) 167 ;
13 (1975) 335.
- 2) H. Grote, J. Ranft and R. Hagedorn - Atlas of Particle
Production Spectra, CERN (1970).
- 3) M.B. Kislinger and P.D. Morley - Phys.Rev. D13 (1976) 2765.
- 4) R. Carlitz, S. Frautschi and W. Nahm - Astronomy and
Astrophysics 26 (1973) 171.
- 5) S.W. Hawking, Commun.Math.Phys. 43 (1975) 199.
- 6) W. Nahm - Nuclear Phys. B68 (1974) 111.
- 7) S.W. Hawking and D.N. Page - Astrphys.J. 206 (1976) 1.
- 8) B.J. Carr - Astrophys.J. 201 (1975) 1.
- 9) P.J.E. Peebles - "The effect of a lumpy matter distribu-
tion on growth of irregularities in the expanding universe"
(October 1973), unpublished.
- 10) M. Yoshimura - Phys.Rev.Letters 41 (1978) 281.
- 11) J. Ellis, M.K. Gaillard and D.V. Nanopoulos - CERN Preprint
TH. 2596 (1978).
- 12) C.E. DeTar - Phys.Rev. D3 (1971) 128.
- 13) L. Van Hove - Physics Reports 1 (1971) 347.
- 14) J. Ranft - Phys.Letters 41B (1972) 613.
- 15) F.E. Low - Phys.Rev. D12 (1975) 163.
- 16) J. Kogut and L. Susskind - Phys.Rev. D9 (1974) 3501.
- 17) X. Artru and G. Mennessier - Nuclear Phys. B70 (1974) 93.
- 18) R. Hagedorn - Nuclear Phys. B24 (1970) 93.
- 19) I.Ya. Pomeranchuk - Dokl.Akad.Nauk SSSR 78 (1951) 889.
- 20) M.I. Gorestein, V.A. Miransky, V.P. Shelest and
G.M. Zinoviev - Phys.Letters 45B (1973) 475.
- 21) N. Cabibbo and G. Parisi - Phys.Letters 59B (1975) 67.