A universal description for the freezeout parameters in heavy-ion collisions

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Abstract

It is shown that the freezeout parameters estimated in the heavy-ion collisions all are well described by a constant value of the entropy density $s$ divided by $T^3$. The value of $s/T^3$ has been taken from the lattice QCD simulations at zero baryochemical potential $\mu_B$ and assumed to remain constant for finite $\mu_B$. This implies that the hadronic matter in rest frame of produced system can be determined by constant degrees of freedom. Furthermore, this condition has been used to predict the freezeout parameters at low temperatures.

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1 Introduction

At high temperature and/or density, it is conjectured that the confined hadronic matter likely dissolves into quark-gluon plasma (QGP) [1]. Reducing the temperature, the QGP will hadronize. At some temperature $T_{ch}$, the produced hadrons entirely freeze out. $T_{ch}$ is the temperature at the chemical freezeout. The freezeout parameters, $\mu_B$ and $T_{ch}$ can be determined in the thermal models by combining various ratios of integrated particle yields [2,3]. Such a way, we obtain a window in the $T_{ch} - \mu_B$ phase-diagram compatible to the experimental values. Both $\mu_B$ and $T$ are free parameters in these thermal fits. The search for common properties of the freezeout parameters in heavy-ion collisions has a long tradition [4,5]. Different models have been suggested, for example the average energy per average particle number $< E > / < n >= 1$ GeV [4], total baryon number density $n_b + n_{\bar{b}} = 0.12$ fm$^{-3}$ [5] and the interconnection amongst the constituents of

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the hadronic matter [6]. In this article, we assume that one best achieves this objective by assigning the entropy density \( s \) normalized to \( T^3 \) to a constant value [7]. The results can be summarized in the following way: the hadronic matter in rest frame of produced system can be determined by constant degrees of freedom \(^1\). For vanishing free energy, i.e. at the chemical freezeout, the *equilibrium* entropy gives the amount of energy which can’t be used to produce additional work. We can in this context define the entropy as the degree of sharing and spreading the energy inside the equilibrium system. Furthermore, we find that the strangeness degrees of freedom are essential at low collision energies, where the strangeness chemical potential \( \mu_S \) is as large as \( \mu_B \). According to the strangeness conservation in the heavy-ion collisions, we find that the higher is the collision energy, the smaller is \( \mu_S \) [9].

2 The model

In order to describe the freezeout phase-diagram, we basically have to be able to determine two characteristic regions. The first one is at \( \mu_B = 0 \) and finite \( T_{ch} \). The second one is characterized by \( T_{ch} = 0 \) and finite \( \mu_B \). Localizing the first region has been the subject of different experimental studies [10]. It has been found that \( T_{ch}(\mu_B = 0) \approx 174 \text{ MeV} \) [11]. The lattice estimation for the deconfinement temperature gives \( T_c(\mu_B = 0) = 173 \pm 8 \text{ MeV} \) [12]. This implies that the deconfinement and freezeout seem to be coincident at small \( \mu_B \). For the second region, we are left with effective models. As \( T \) is close to zero, one expects that the nucleons will dominate the resonance gas. We find that at \( T = 0 \) the baryo-chemical potential is corresponding to the normal nuclear density, \( n_0 \approx 0.17 \text{ fm}^{-3} \), i.e. \( \mu_B \approx 0.979 \text{ GeV} \). We have to mention here that this value can slightly be different according to the initial conditions [13]. As we shall see later, extrapolating our freezeout curve to the abscissa might result in a \( \mu_B \)-value very close to this estimation. But it is important to notice that the condition of constant \( s/T^3 \) breaks up at \( T = 0 \).

The partition function in the hadronic matter at finite \( T, \mu_B \) and \( \mu_S \) is given by the contributions of all hadron resonances up to 2 GeV treated as a free quantum gas [14,15,16,9,17]

\[
\ln Z(T, \mu_B, \mu_S) = V \frac{g}{2\pi^2} \int_0^\infty k^2 dk \ln \left[ 1 \pm \gamma \exp \frac{\mu_B + \mu_S - \varepsilon}{T} \right],
\]  

where \( \varepsilon = (k^2 + m^2)^{1/2} \) is the single-particle energy and \( \pm \) stand for bosons and fermions, respectively. \( g \) is the spin-isospin degeneracy factor. \( \mu_S \), the \(^\pi^2/4\)-scaling of \( s/T^3 \) gives the effective degrees of freedom of free gas [8]
strangeness chemical potential, has been calculated as a function of $T$ and $\mu_B$ under the condition that the average strange particle number $<n_s>$ equals to the average anti-strange particle number $<n_\bar{s}>$, the so-called strangeness conservation. $\gamma \equiv \gamma_q^n \gamma_s^m$ is the quark phase space occupancy parameter, with $n$ and $m$ being the number of light and strange quarks, respectively. In carrying out our calculations, we used $\gamma_q = \gamma_s = 1$, i.e. we assumed that the phase-space occupancy parameters of both light and strange quarks are in equilibrium [18,19] and the particle production is due to a chemically equilibrium process.

At $T = 0$, the hadron resonance gas model (HRGM) is no longer applicable. The laws of thermodynamics can’t hold in this limit. Corrections due to van der Waals repulsive interactions have not been taken into account in our calculations [9].

3 Results

![Graph](image)

**Fig. 1.** Lattice QCD results on the entropy density normalized to $T^3$ for $n_f = 2$ (full circles) and $n_f = 2+1$ (full squares) quark flavors at $\mu_B = 0$ [20,15] on the top of the results from the hadron resonance gas model (curves). The thick curves represent the results from hadron resonance gas model (HRGM) with rescaled masses. We find a well agreement with the lattice QCD simulations. The HRGM calculations with the physical resonance masses are given by the thin curves. The horizontal lines indicate to $s/T^3$-values for the physical resonance masses at the different quark flavors and critical temperature $T_c$.

We plot in Fig. 1 the lattice results on $s/T^3$ vs. $T/T_c$ at $\mu_B = 0$ for different
Fig. 2. The freezeout parameters according to constant $s/T^3$ on the top of the experimentally estimated (or thermally fitted) freezeout parameters (small full circles with errors). For non-strange hadron resonances, we use $s/T^3 = 5$ and for all hadron resonances we assign $s/T^3$ to 7. Both values have been taken from lattice QCD simulations at zero baryo-chemical potential (Fig. 1). We find that strangeness degrees of freedom are essential for reproducing the freezeout parameters at low incident energies. The condition of constant $s/T^3$ have been applied for $T > 5\text{ MeV}$. At smaller temperatures, the HRGM is no longer applicable.

quark flavors $n_f$ [20,15]. The quark masses used in the lattice calculations are heavier than their physical masses in vacuum. For a reliable comparison with the lattice QCD, the hadron resonance masses included in HRGM have to be re-scaled to values heavier than the physical ones [14,15]. As shown in Fig. 1, HRGM can very well reproduce the lattice results for the different quark flavors under this re-scaling condition. In the same figure, we draw the results for physical resonance masses as thin curves, i.e. the case if lattice QCD simulations were done for physical quark masses. The two horizontal lines point at the values of $s/T^3$ at the critical temperature $T_c$. As mentioned above, the critical $T_c$ and the freezeout temperature $T_{ch}$ are assumed to be the same at small $\mu_B$. We find that $s/T^3 = 5$ for $n_f = 2$ and $s/T^3 = 7$ for $n_f = 2 + 1$. These two values will be used in order to describe the freezeout parameters for different quark flavors. The normalization with respect to $T^3$ should not be connected with massless particles. Either the resonances in HRGM or the quarks on lattice are massive. We divide $s$ by $T^3$ in order to remove the $T$-dependence.

In HRGM, we calculate the temperature $T_{ch}$ at different $\mu_B$ according to constant $s/T^3$.

$\mu_B$ is corresponding to the collision energy. On the other hand,
\( \mu_S \), the strangeness chemical potential, has been calculated in dependence on \( \mu_B \) and \( T \). The resulting \( T \) and \( \mu_B \) are plotted in Fig. 2 on the top of the freezeout parameters (\( T_{ch} \) and \( \mu_B \)) which, as mentioned above, have been estimated by thermal fits of various ratios of the particle yields produced in different heavy-ion collisions. The dotted curve represents our results for \( n_f = 2 \). In this case, only the non-strange hadron resonances are included in the partition function, Eq. (1). The entropy is given by \( \partial T \ln Z(T, \mu_B, \mu_S)/\partial T \). The condition applied in this case is \( s/T^3 = 5 \). The solid curve gives \( 2 + 1 \) results (two light quarks plus one heavy strange quark). Here all resonances are included in the partition function and the condition reads \( s/T^3 = 7 \). We find that both conditions can satisfactory describe the freezeout parameters at high collision energy. We notice that the \( n_f = 2 \) curve doesn’t go through the SIS data points. Therefore, we can conclude that the non-strange degrees of freedom alone might be non-sufficient at the SIS energy.

We have to mention here that both the entropy density \( s \) and the corresponding temperature \( T_{ch} \) decrease with increasing \( \mu_B \). The entropy density is much faster than \( T \), so that the ratio \( s/T^3 \) becomes greater than 7 at very large \( \mu_B \). In this limit, thermal entropy density \( s \) is expected to vanish \(^2\), since it becomes proportional to \( T \) (third law of thermodynamics). The quantum entropy [21,22,23,24,25,26] is entirely disregarded in these calculations.

4 Conclusion and outlook

We used HRGM in order to map out the freezeout curve according to constant \( s/T^3 \). Taking its value from the lattice QCD simulations at \( \mu_B = 0 \) and assuming that it remains constant in the entire \( \mu_B \)-axis, we obtained the results shown in Fig. 2. We find that the freezeout parameters \( T_{ch} \) and \( \mu_B \) are very well described under this condition. We also find that the strangeness degrees of freedom seem to be essential at low collision energies, where the strangeness chemical potential \( \mu_S \) is as large as \( \mu_B \). At high collision energies, \( \mu_B \) decreases and correspondingly \( \mu_S \). We conclude that the given ratio \( s/T^3 \) characterizes very well the final states observed in all heavy-ion collisions. The hadronic abundances observed in the final state of heavy-ion collisions are settled when \( s/T^3 \) drops to 7, i.e. the degrees of freedom drop to \( 7 \pi^2/4 \). Meanwhile the changing in the particle number with the changing in the collision energy is given by the baryo-chemical potential \( \mu_B \), the energy that produces no additional work, i.e. the stage of vanishing free energy, gives the entropy at the chemical equilibrium. At the chemical freezeout, the equilibrium entropy represents the amount of energy that can’t be used to produce additional work.

\(^2\) At \( T = 0 \) the thermal entropy equals to zero as the system gets degenerate.
In this context, the entropy is defined as the degree of sharing and spreading the energy inside the system that is in chemical equilibrium.

Regarding to the sharp peak of the $K^+ / \pi^+$ ratio at SPS energy [27], we still face the problem that the thermal models (HRGM), on the one hand side, describe very well the freezeout parameters at each collision energy individually, i.e. they are assumed to be able to reproduce all particle ratios. On the other hand, we find that such a sharp peak can’t be reproduced by the thermal models [28]. Indeed, we find that the $K^+ / \pi^+$ ratio at the SIS, AGS and low SPS energies can be reproduced by the thermal models. But a large overestimation is to be observe at top SPS and all RHIC energies. If we assume that the uncertainty in the experimentally estimated (or fitted by the thermal models) $K^+ / \pi^+$ ratio is small, then one might think that this experimental observation might indicate to some critical phenomena, which we don’t include in the thermal models.

One way to avoid this dilemma might be assuming two separate conditions for the freezeout parameters. One of them is to be applied at small $\mu_B$, where we can’t distinguish between the freezeout and the deconfinement. The other one should be able to describe the freezeout parameters at large $\mu_B$. Both conditions should be able to reproduce the particle ratios at all collision energies. The region, i.e. $\mu_B$-values, where the two conditions intersect hopefully will be corresponding to the collision energy, at which the sharp peak of the $K^+ / \pi^+$ ratio has been observed.

In [19], we have worked out an additional model. We have allowed the phase space occupancy parameters $\gamma_q$ and $\gamma_s$ to take values other than that of equilibrium.

References