Spectral properties of nuclear matter

P. Bożek*
Institute of Nuclear Physics, PL-31-342 Cracow, Poland

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Abstract

We review self-consistent spectral methods for nuclear matter calculations. The in-medium $T$–matrix approach is conserving and thermodynamically consistent. It gives both the global and the single-particle properties the system. The $T$–matrix approximation allows to address the pairing phenomenon in cold nuclear matter. A generalization of nuclear matter calculations to the superfluid phase is discussed and numerical results are presented for this case. The linear response of a correlated system going beyond the Hartree-Fock+ Random-Phase-Approximation (RPA) scheme is studied. The polarization is obtained by solving a consistent Bethe-Salpeter (BS) equation for the coupling of dressed nucleons to an external field. We find that multipair contributions are important for the spin(isospin) response when the interaction is spin(isospin) dependent.

1 Introduction

Nuclear matter is an infinite system of strongly interacting fermions. Low energy nucleon-nucleon interactions in vacuum are well known from scattering experiments. The characteristic features of the nuclear force are the presence of a strongly repulsive core at small distances, attraction at moderate distances, and the appearance of a tensor component. The properties of nucleons in a strongly interacting medium are modified. This is taken into account by the dressing of nucleon propagators by a self-consistently calculated self-energy. Nonzero imaginary part of the self-energy implies a finite lifetime of

*Electronic address: piotr.bozek@ifj.edu.pl
quasiparticles and gives them nontrivial spectral properties; nucleons propagate off the energy shell. The description of nuclear matter in the language of such dressed nucleons with a broad spectral function has been the subject of intensive studies in the last years.

2 In medium $T$ matrix

The description of properties and excitations of a many-body system can be performed in the language of finite-temperature Green's functions. The Green's functions’ approach is most easily formulated in the imaginary time formalism [1], suitable for formal presentations and perturbative calculations, but the real-time formalism [2] is better adapted for numerical calculations. In medium propagators (Green’s functions) are dressed by a self-energy term. The physical approximation enters in the choice of a suitable expression for the self-energy used in the calculation. Analytical properties of the Green’s functions are guaranteed by the dispersion relation between the imaginary and real parts of the self-energy and by the use of the Dyson equation

$$G^{-1} = G_0^{-1} - \Sigma,$$  \hspace{1cm} (1)

where $G$ and $G_0$ represent the dressed and vacuum propagators respectively and $\Sigma$ is the self-energy.

The choice of a specific form of the self-energy should be adapted to the physical system under consideration, but a general procedure to derive nonperturbative approximations in many-body systems has been proposed [3]. The self-energy is written as a functional derivative of a two-particle irreducible generating functional $\Sigma = \frac{\delta \Phi}{\delta G}$.

Due to the presence of a short range repulsive core, ladder diagrams in the free nucleon-nucleon interaction $V$ must be resummed. The resulting correlated part of the two-particle Green’s function is called the in-medium $T-$matrix [4] (Fig. 1)

$$T = V + VGGT.$$  \hspace{1cm} (2)

Although the ladder diagrams for the $T$ matrix have the same form as for the Brueckner $G-$matrix the resulting expressions are different. The $T-$matrix equation is defined for in-medium Green’s functions. At zero temperature the two-nucleon propagator in Fig. 1 represents the propagation a particle-particle pair (excitations above the Fermi energy) or a hole-hole pair (excitations below the Fermi energy), whereas in the Brueckner scheme the blocking operator forces the two nucleons in the ladder to be of the particle-particle type. The second, even more important, difference comes at the level of the self-energy,
Figure 1: Definition of the $T-$matrix as a sum ladder diagrams in the interaction.

\[
\begin{array}{c}
\text{Figure 2: The self-energy in the } T\text{-matrix approximation which in the } \Phi\text{-derivable } T\text{-matrix approximation must be taken self-consistently in the form (Fig 2)} \\
\Sigma = T \text{Tr}TG .
\end{array}
\]

The self-consistency means that the nucleon propagators at all stages of the approximation are dressed by this self-energy. It requires an iterative procedure for the solution of the coupled equations (2), (3), and (1). The $T$ matrix obtained in a given iteration is used to generated the self-energy and the dressed propagators. The dressed propagators are then used to calculate the in-medium $T$ matrix in the next iteration.

The numerical calculations are very demanding due to the fact that the nucleons are dressed by nontrivial spectral functions coming from the imaginary part of the self-energy (Fig 3); additional integrations over the energy of off-shell nucleons appear. First results on the self-consistent in-medium $T$ matrix appeared only in the recent years \cite{5, 6}. In \cite{5, 7} the spectral function of the dressed nucleon was parameterized by three Gaussians and in \cite{8, 9} using a three poles approximation for the in-medium propagator. The first solution using full spectral functions for the dressed propagators in nuclear matter was obtained numerically in ref. \cite{6} at finite temperature, for a simple nuclear interaction. This approach, using numerical algorithms significantly speeding the calculations, was then generalized to realistic interactions and zero temperature \cite{10, 11}. The solution of
the $T$–matrix scheme at finite temperature was also obtained in [12].

At low temperatures two difficulties show up. The first one is related to the transition to superfluidity in cold nuclear matter. This effect can be taken into account quite naturally within suitably generalized $T$–matrix approaches. This is the subject of the next section. We note however that most of the existing $T$–matrix calculations are done for the normal phase of the nuclear matter also at zero temperature. The second difficulty with low temperature nuclear matter is technical and is related to the appearance of a well defined quasi-particle peak in the spectral function of the in-medium nucleon. Simple discretization algorithms for the energy integration break down in that case. The spectral function must be separated into background and quasi-particle contributions for momenta close to $p_F$. At the time of this writing, there is only one numerical implementation of this procedure for the nuclear matter $T$ matrix (see [10] for details).

First calculations [5, 6, 7] have shown that the self-consistency for the self-energies of nucleons is very important. The effective scattering is reduced in the self-consistent calculation when compared to an approximation neglecting the imaginary part of the self-energy [13]. Also the value of the critical temperature for the superfluid phase transition goes down when using fully dressed propagators in the Thouless criterion for superfluidity.

Further developments have addressed the binding energy in the $T$–matrix approximation [14, 15, 9, 16]. The role of the correlations, high momentum states, and low energy tails in the spectral function on binding has been discussed [15]. Formally, one should
Figure 4: The binding energy and the Fermi energy (upper panel) and two (formally equivalent) expressions for the pressure (lower panel) as function of density in the $T-$matrix and the Brueckner-Hartree-Fock calculations.

not expect better results for the binding energy in the $T-$matrix approximation than in the standard $G-$matrix approach with higher order terms in the hole line-expansion [17]. However, the authors of ref. [16] claim that the $T-$matrix calculation, which does not take into account ring diagrams contribution, is better adapted to describe nuclear matter in finite nuclei. An important observation is made in ref. [14], where the role of the thermodynamical consistency of the $T-$matrix approximation is discussed. As expected from the $\Phi-$derivability of the scheme, the $T-$matrix calculation gives consistent results for the global quantities in the system and its single particle properties (Fig. 4). In particular the $T-$matrix results fulfill the Hugenholz-Van Hove theorem and the Luttinger relation for the value of the Fermi momentum. In practice, such a consistency of the single particle energy can be obtained within the Brueckner theory only approximately by taking rearrangement terms to a given order.

The single-particle properties obtained in the self-consistent iteration of the $T-$matrix scheme represent a reliable estimate of the optical potential and of the single-particle width in medium [10]. Nuclear matter in the non-superfluid phase behaves as a Fermi liquid, with the standard phase-space scaling of the scattering width with the distance to the
Fermi energy and with the temperature \[10\]. The spectral function in finite nuclei has been estimated taking into account the local density and the neutron-proton asymmetry \[18\] and compared to experimental data. Thermodynamic properties of the asymmetric \[19\] and pure neutron nuclear matter \[20\] have been discussed. We note that the \(T\)–matrix calculation can be performed very easily at finite temperature \[6, 10, 12\] (it is even simpler than at zero temperature).

3 Pairing

![Figure 5: The spectral function in the superfluid phase, including the the diagonal self-energy (solid line) and both the diagonal and anomalous self-energies (dashed line).](image)

A general property of fermions interacting through an attractive potential is the transition to a superfluid state at low temperatures \[21\]. This phenomenon is usually not taken into account in nuclear matter studies, because the value of the superfluid gap, as extracted from the properties of finite nuclei, is expected to be small. On the other hand naive estimates of the neutron-proton pairing gap yield a value of several MeVs \[22\]. This value, obtained from a mean-field gap equation, is modified by many-body effects.

A fundamental approach in the study of superfluid nuclear matter should aim at the description of the cold nuclear matter system dealing with a strong repulsive core in the interaction and at the same time with the formation of the superfluid state. A generalization of nuclear matter calculations must include the usual ladder diagram resummation.
in the normal phase. At the same time it should describe the formation of the superfluid long-range order in the two-particle correlations. Such correlations appear as a singularity of the $T$ matrix for $T = T_c$ at the energy of twice the fermionic chemical potential. This corresponds to the formation of a two-fermion bound state. At temperatures below $T_c$ such fermionic (Cooper) pairs have a binding energy allowing for the condensation and for the formation of a superfluid order parameter.

![Figure 6: Inverse of the $T$–matrix in the pairing (deuteron) channel in the superfluid phase. The usual $T$ matrix (dashed line) and the generalized $T$ matrix with off-diagonal components (solid line) are shown [23]. The generalized $T$ matrix has a singularity at the Fermi energy for zero total momentum of the pair.](image)

In ref. [24] a first attempt to generalize nuclear matter calculation to the superfluid phase has been discussed. At small temperatures a superfluid state forms leading to nonzero expectations of the off-diagonal (anomalous) Green’s functions and self-energies [25]. The off-diagonal self-energy is obtained from a gap equation but with dressed propagators. The $T$–matrix in the superfluid must be constructed in a way to conserve the singularity at twice the chemical potential also for $T < T_c$. Such a scheme was considered in [24], giving the first calculation of the superfluid nuclear matter problem, including both the ladder resummation of the nuclear interaction and the superfluid properties obtained with a gap equation modified by many-body effects. The nucleon propagator in the superfluid is dressed by a diagonal self-energy $\Sigma$ coming from a modified $T$–matrix approximation and an off-diagonal anomalous self-energy $\Delta$; it has two poles on both
sides the Fermi energy (Fig. 5). The most interesting result of this first, and up to now unique, study is the observation of a strong reduction of the superfluid gap in the correlated nuclear matter as compared to results from a mean-field gap equation [24]. This effect was analyzed in details [26] and can be explained by an effective modification of the density of states at the Fermi energy. The reduction of the superfluid energy gap is especially important for neutron-proton pairing close to the gap closure in symmetric nuclear matter [27].

Possible generalizations of the $T$–matrix approximation to the superfluid phase have been discussed by Haussmann [28]. The simplest scheme, which is $\Phi$–derivable and includes the required ingredients, i.e. the ladder diagrams resummation in the diagonal self-energy and the gap equation for the off-diagonal self-energy, requires the introduction of an additional $T$–matrix describing two-particle correlations for anomalous propagators. The generalized $T$–matrix has a singularity at twice the Fermi momentum for $T \leq T_c$ (Fig. 6) and the imaginary part of the self-energy has a gap around the Fermi energy, corresponding to an energy gap for possible excitations in a scattering [23]. This last, important property of the approximation does not hold for the simple scheme discussed in [24]. Explicit calculations show that corrections to the gap equation coming from ladder diagrams in the off-diagonal self-energy are small [23]. These small corrections make the superfluid gap energy dependent.

4 Linear response with dressed vertices

Processes occurring in the dense nuclear matter are modified by medium effects [29]. This happens for neutrino rates in neutron stars and for particle or photon emission in hot nuclear matter. In particular, the effect of the off-shell propagation of nucleons in the medium is important for the subthreshold particle production in heavy ion collisions. The role of correlations for the neutrino emission could be especially important for processes in hot stars.

If the interaction with an external perturbation is small the dynamics of the system in the external field can be described in terms of linear response functions. The response functions incorporate all the correlations due to the self-interaction of particles in the system. For normal Fermi liquids the response function to long-wavelength perturbations can be calculated within the Fermi liquid theory [30]. However, if one is interested in nuclear systems at higher temperatures or for perturbations with large momentum a different approach is required. The Fermi liquid theory does not allow for multipair excitations of the system in the external potential. Such multipair excitations are important for higher energies and momenta of the external perturbation or in the presence of tensor
interactions \cite{31}.

We take the interactions between the nucleons as a sum of a mean-field interaction that is based on the Gogny parameterization \cite{32} and a residual interaction (scalar or isospin dependent). The isospin dependent residual interaction is obtained from the scalar one multiplying by the factor $\frac{1}{2}(1 + \tau_1 \tau_2)$. The self-energy is taken in the second direct Born approximation for the residual interaction (Fig. 7). The residual interaction induces a finite width to nucleon excitations in the medium. Such a dressing of nucleons is expected in any approach going beyond the simple mean-field, e.g. the $T$–matrix approximation discussed previously.

When the description of the correlated system goes beyond the mean-field approximation the difficulty involved in a consistent calculation of the response function is severely increased \cite{33, 34}. A naive calculation of the polarization bubble using dressed propagators

$$\Pi = Tr \Gamma_0 G_{ph} \Gamma_0 ,$$

where $G_{ph}$ is the particle-hole propagator with dressed nucleons and $\Gamma_0$ is the free vertex for the coupling of the nucleon to an external field, is a very bad estimate for the response function \cite{35}. In particular, it severely violates the $\omega$-sum rule \cite{36}.

A general recipe for calculating the in-medium coupling of the external potential to dressed nucleons is known \cite{37}. The in-medium vertex describing the coupling of an external perturbation to nucleons is given by the solution of the BS equation (Fig. 8)

$$\Gamma = \Gamma_0 + Tr K G_{ph} \Gamma ,$$

where $K$ denotes the particle-hole irreducible kernel. The kernel $K$ of the BS equation should be taken consistently with the chosen expression for the self-energy, it is given by the functional derivative of the self-energy with respect to the dressed Green’s function $K = \delta \Sigma / \delta G$ \cite{37}. Using the dressed vertex obtained as a solution of the BS equation the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\caption{Diagrams for the self-energy. The first two diagrams are the Hartree-Fock contribution for the Gogny interaction. The last diagram is the contribution of the residual interaction in the second order.}
\end{figure}
Figure 8: The Bethe-Salpeter equation for the dressed vertex. The particle-hole irreducible kernel $K$ is denoted by the box and the fat and the small dots denote respectively the dressed and the bare vertices for the coupling of the external field to the nucleon.

\[ \Pi = \ldots \bigtriangleup \ldots \]

Figure 9: The polarization function expressed using the dressed vertex for the coupling of the external field to the dressed nucleon.

Response function in the correlated medium can be obtained from the diagram in Fig. 9

\[ \Pi_{(ST)} = Tr \Gamma^0_{(ST)} G_{ph} \Gamma_{(ST)} , \]

(6)

$\Gamma_{(ST)}$ is the in-medium vertex for the coupling in a given spin-isospin $(ST)$ channel. The exact form of the BS equation in the real-time formalism (for different types of vertices $\Gamma_{(ST)}$) can be found in [35, 38]. The solution of the BS integral equation is obtained by iteration. It is a serious numerical task involving the calculation of two-loop diagrams with dressed propagators and broken rotational symmetry due to the presence of an external field.

The results for the response functions in different spin-isospin channels [38] show that the calculation using dressed propagators and dressed vertices obtained as solutions of the BS equation is close to the RPA approximation. It is expected due to cancellations of self-energy and vertex corrections. Such cancellations occur for the exact solution of the system as well as within consistent approximation derived from a generating functional $\Phi$. For the scalar residual interaction the $\omega$-sum rule takes the simple form

\[ - \int \frac{\omega d\omega}{2\pi} \text{Im}\Pi^r_{(ST)}(q, \omega) = \frac{q^2}{2m} , \]

(7)

in all the spin isospin channels $(ST)$. The above sum-rule severely constraints acceptable forms of the response functions in the case of a scalar residual interaction and leads to
very small multipair contributions in most of the kinematical regions. Some differences can occur only when collective modes are present, as discussed latter.

The situation is very different for a more general form of the residual interaction. The kernel of the BS equation depends very much on the isospin channel, e.g. in the channel \( ST = 11 \) there are no vertex corrections from the residual interaction at all. The propagators are dressed by a nontrivial self-energy due to the residual interaction but the vertex corrections are small or even absent. Therefore, the cancellation between self-energy and vertex corrections, observed for a scalar interaction, can no longer be maintained. This has its implications for the \( \omega \)–sum rule which gets modified for responses with \( T = 1 \) \cite{38}. In the case of the isospin dependent residual interaction, in-medium dressed nucleons couple in the same way as free nucleons to isovector potentials. In Fig. 10 the response functions obtained with the isospin dependent interaction are compared to the response functions from the Fermi liquid theory. For \( T = 0 \) channels the correlated response func-

![Figure 10: The imaginary part of the polarization in the RPA approximation (dashed-dotted line), from the self-consistent calculation with dressed nucleons and vertices (solid line) and the naive one-loop polarization with dressed nucleons (dashed line). All results are for an isospin dependent interaction \( \frac{1}{2}(1 + \tau_1 \tau_2)V \), \( q = 210 \text{MeV} \), and \( T = 15 \text{MeV} \).](image-url)
tion is similar to the one particle-one hole response function. On the other hand, the isovector response is closer to the naive one-loop result, without vertex corrections.

![Figure 11](image.png)

Figure 11: The imaginary part of the polarization when a collective mode is present; density response (upper panel) and isospin response (lower panel). The results are obtained in the RPA approximation (dashed-dotted line), and in the full calculation with dressed vertices for an isospin dependent residual interaction (solid line) and a scalar one (dashed line). For the density response the result is independent of the type of the residual interaction.

To study the role of multipair configurations on the collective modes we increase the value of the Landau parameters. Within the Fermi liquid theory a collective excitation at zero temperature is a discrete peak in the imaginary part of the response function. The state corresponding to a collective excitation cannot couple to incoherent one particle-one hole excitations. At finite temperature such a coupling is possible, it can be calculated and the finite temperature width of the collective state is usually small. The collective state can acquire a finite width (also at zero temperature) due to the coupling to multipair configurations [30]. The description of the damping of collective states from such processes goes beyond the Fermi liquid theory.
The result of the calculation with self-energy and vertex corrections is compared to the RPA result in Fig. 11. The collective mode in the density response has a large width in the full calculation. This corresponds to the coupling of the sound mode to multipair excitations giving rise to a finite decay width, even at zero temperature (the small width of the RPA collective mode is due to the finite temperature \( T = 15\text{MeV} \)). For the isovector response shown in the lower panel of Fig. 11 the Fermi liquid theory predicts the presence of a well defined collective state (dashed-dotted line). For a scalar residual interaction the response function in the correlated system shows a collective state at similar energy (dashed line). It has a larger width due to the contribution of multipair configurations, analogously as in the density response. On the other hand, an isospin dependent residual interaction leads to an isovector response with a large imaginary part at high energies (Fig. 10). For the chosen interaction we observe the extreme scenario of the disappearance of the collective mode when the effects of the residual interaction are taken into account (solid line in the lower panel of Fig. 11). Generally we expect a whole range of behavior depending on the energy of the collective state and on the strength of isospin dependent terms in the residual interaction but always the collective state is broader than in the Fermi liquid theory, due to the coupling to multipair configurations. The same phenomena are expected also for the spin wave collective state in the presence of spin dependent residual interactions.

5 Summary

We present a new method of calculations for nuclear matter. The method discussed in this work is using in-medium Green’s functions’ formalism and is based on the summation of diagrams with a retarded propagation of a pair of fermions. The sum of such ladder diagrams is the in-medium scattering matrix (the \( T \)-matrix). Self-consistency in this approach means that the fermion propagators in the \( T \)-matrix diagrams are dressed in a nontrivial self-energy. The self-energy itself is obtained from the sum of ladder diagrams. This self-consistency requires the use of full spectral functions depending on the momentum and energy of the particle. The propagation off the mass shell, i.e. the full dependence of the nucleon propagator on the energy, is a serious difficulty in numerical applications. The progress achieved in the last years allows to perform extensive calculation of the nuclear matter properties in the \( T \)-matrix approach at zero and finite temperatures.

Attractive nuclear interactions lead to the formation of a superfluid phase at low temperatures. Cooper pairs of fermions are formed, analogously as for the electron superconductivity in metals. The second important achievement here reported is the generalization
of the ladder diagram summation to the superfluid phase. We discuss equations which allow for a simultaneous and consistent treatment of the short range nuclear interactions, the bound state formation (Cooper pairs) in the $T$-matrix and the superfluidity in nuclear matter. Model calculations have allowed us to estimate the influence of the superfluidity on standard many-body effects in nuclear matter. At the same time, the influence of many-body corrections on the superfluid gap can be assessed. These corrections, reducing the value of the gap, can be described by an effective superfluid gap equation, similar to the mean-field Bardeen-Cooper-Schrieffer equation with an effective interaction and with a renormalized value for the order parameter.

The self-consistent $T$-matrix approximation is a thermodynamically consistent approximation, a so called conserving approximation. Single-particle properties obtained within this approximation are consistent with global quantities describing the system, such as the binding energy or the pressure. Unlike other methods, it allows to obtain directly single-particle properties: optical potentials, single-particle widths, and spectral functions. These observables can be compared to experimental values.

The calculation of processes occurring in the dense medium, e.g. particle absorption, emission, neutrino cross-sections, requires the knowledge of different kinds of linear responses of the system to external probes. In a correlated system, described using dressed propagators, the calculation of the linear response is a very difficult task. The problem lies in the need for a consistent dressing of the vertices corresponding to the coupling of dressed nucleons to an external field. The in-medium vertices are obtained from a solution of the BS equation, where the kernel of the equation is derived from the same generating functional $\Phi$ as the self-energy. This procedure guarantees the fulfillment of conservation laws and sum-rules. It must be contrasted with incomplete procedures with only self-energy effects without vertex corrections. The linear response with self-energy and vertex corrections beyond the Hartree-Fock+RPA approximation includes the effects of multipair excitations. For fermions interacting with a scalar potential such effects are small, except for the damping of collective modes, where the multipair configurations are important. The situation is very different for spin or isospin dependent interactions. In that case the spin or isospin response is very different from the RPA result; multipair effects are essential.

Applications discussed in this talk are restricted to nuclear systems. We note that the same methods can be applied to other many-body fermionic systems, high $T_c$ superconductors, fermions near the Feshbach resonance, the electron gas.

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References


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