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A NEW VALUE OF THE ANOMALOUS MAGNETIC MOMENT
OF THE ELECTRON

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A B S T R A C T

The contribution to the sixth order anomaly $a_e^{(6)}$ from light-by-light scattering subgraphs is recomputed. The result is:

$$a_e^{\gamma-\gamma} = (\alpha^3/\pi^3)(0.366 \pm 0.010)$$

This result agrees with a previous calculation done at SLAC. The accuracy is improved by a factor of 4. With the currently accepted values for many of the other diagrams, the sixth order anomaly is:

$$a_e^{(6)} = (1.16 \pm 0.07) (\alpha/\pi)^3,$$

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The contribution to the sixth-order anomaly of the muon $a_\mu^{(6)}$ from the six light-by-light scattering subgraphs has recently been recomputed by one of us ¹⁾ with an improved accuracy compared to the existing result ²⁾.

It was logical to do the same work for the electron, (see figure), replacing the ratio $R_1 = (m_e/m_\mu)^2$ (muon case) by $R_1 = 1.0$ (electron case).

But, with $R_1 = 1.0$, doing the integration naïvely and using the RIWIAD ³⁾ integration subroutine, it was found that the successive iterations do not give a convergent result, the error (90% confidence level) being of the same order of magnitude as the value of the integral. A new version of RIWIAD ⁴⁾ gives, in such a case, a message suggesting to perform either a mapping or a change of variables in order to smoothen the function to be integrated.

Therefore, we tried to perform a few changes of variables (two in our case), but the result, using nevertheless 1.000.000 subvolumes, was only slightly modified. Examples of a few iterations are given:

0.2738 ± 0.1456	(5th iteration)
0.2369 ± 0.2893 (!)	(8th iteration)
0.2050 ± 0.1613	(9th iteration)
0.2335 ± 0.1867	(10th iteration)

A preset of the interval's structure of five iterations with 50.000 hypercubes was used. We finally made the following change of variables

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= (1 - x_1)x_2 \\ z_7 &= (1 - x_1) \dots (1 - x_6)x_7 \end{aligned}$$

and switched z_2 and z_8 exactly as in Ref. 2) which gave a fair convergence.

The new version of RIWIAD also accounts for the systematic error observed in the previous version ⁵⁾. Three major runs (the second one with the SLAC algebra ⁶⁾) were then performed:

- 1) 8 iterations with 500.000 subvolumes and a preset of interval's structure of 11, 149, 4, 4, 2, 4, 3
value found: 0.379 ± 0.021
final $\chi^2 = 7.98$;
- 2) 9 iterations with 1.000.000 subvolumes and a preset of interval's structure of 7, 60, 3, 4, 2, 2, 2
value found: 0.367 ± 0.022
final $\chi^2 = 7.51$;
- 3) 12 iterations with 1.750.000 subvolumes and a preset of interval's structure of 11, 1103, 2, 3, 2, 2, 2
value found: 0.3610 ± 0.0129
final $\chi^2 = 20.75$.

We have taken the weighted average of these three runs. The resulting value

$$a_e^{\delta-\delta} = \frac{\alpha^3}{\pi^3} (0.366 \pm 0.010)$$

agrees very well with the previous calculation ²⁾ which gave 0.36 ± 0.04 . The accuracy is improved by a factor of 4.

Combined with the currently accepted values for the contributions from many of the other sixth order graphs ^{7),8)} we get for the total sixth order contribution to the anomalous magnetic moment of the electron :

$$a_e^{(6)} = (\alpha/\pi)^3 (1.16 \pm 0.07)$$

This, together with the lower orders, gives the following value for the electron anomaly up to sixth order :

$$a_e^{\text{th}} = (1\ 159\ 651.2 \pm 2.5) \times 10^{-9}$$

[The value of α used is the presently accepted one :

$$\alpha^{-1} = 137.036\ 08\ (26).]$$

R E F E R E N C E S

- 1) A. Peterman, CERN preprint TH.1566 (September 1972).
- 2) J. Aldins, S.J. Brodsky, A. Dufner and T. Kinoshita, Phys.Rev. D1, 2378 (1970).
- 3) B. Lautrup, Proceedings of the 2nd Colloquium on Advanced Computer Methods in Theoretical Physics, Marseille (1971). (This numerical integration method is due originally to G. Sheppey and is often called LSD or SPCINT.)
- 4) J. Calmet and A. Peterman, to be published.
- 5) This explains the different results reported in J. Calmet and A. Peterman, CERN preprint TH.1584.
- 6) We thank very much Dr. S.J. Brodsky for providing us with a listing of the original calculation.
- 7) J. Mignaco and E. Remiddi, Nuovo Cimento 60A, 519 (1969) ;
S. Brodsky and T. Kinoshita, Phys.Rev. D3, 356 (1971) ;
J. Calmet and M. Perrottet, Phys.Rev. D3, 3101 (1971) ;
M.J. Levine and J. Wright, University of Illinois Preprint (July 1973).
- 8) T. Kinoshita and P. Cvitanovic, Phys.Rev.Letters 29, 1534 (1972).
We have not used their values because an improved result is to be published.

Two of the six diagrams contributing to a_e via scattering of light-by-light through electrons.