CM-P00060401

Archives

Ref.TH.1650-CERN

ASPECTS OF THE FUNDAMENTAL THEORY
OF PROTON-PROTON SCATTERING

A. Martin
CERN - Geneva

A B S T R A C T

After recalling the existence of a high energy bound on proton-proton total cross-sections, we discuss the various phenomena which occur when these cross-sections rise and especially when they have the qualitative behaviour of the bound: rising elastic cross-sections, shrinking diffraction peak, validity of the Pommeranchuk theorem for total and elastic cross-sections, existence of a positive real part of the forward amplitude at high energies.

Invited talk at the Meeting of the Royal Society
on Proton-Proton Scattering at High Energies

8 March 1973

1. THE BOUND ON TOTAL CROSS-SECTIONS

There is, strictly speaking, no fundamental theory of proton-proton scattering at high energies. There are models, but it is not my purpose to speak about them and therefore I shall restrict myself to the very general bounds that one can derive from the general requirements of microcausality, conservation of probability and existence of a particle with minimum, non-zero, mass. That means that strictly speaking all of what I shall say applies to the fictitious world of strong interactions, without photons and neutrinos.

The existence of a minimum mass is responsible for the short range of the forces, half the pion Compton wavelength, corresponding to the exchange of two pions between the protons (one-pion exchange contributes only to the real part of the scattering amplitude). Now it would be too naïve to conclude from this that a proton is necessarily seen by another proton as an object of fixed size because the forces have not a sharp cut-off and the exponential tail of the interaction might become more and more effective as the relative energy of the two protons increases. This is the basis for the bound which says that the total cross-section cannot increase faster than the square of the logarithm of the energy

$$\sigma_{\text{total}}(E) < \frac{\pi}{m_{\pi}^2} (\log E)^2 \quad (1)$$

m_{π} = pion mass, E lab. energy.

The bound was first established by Froissart (1961) on the basis of Mandelstam analyticity and polynomial boundedness. However, it was established later by the author (Martin 1966) that these assumptions are not necessary and that one could use as a starting point microcausality, conservation of probability, and positivity. The bound (1) has therefore now a kind of inescapable character. Let us notice that although the scale in the logarithm is unknown in (1), one can make scale independent statements on moments of total cross-sections, as was done by Common (1970) and Yndurain (1970).

Up to quite recently, the bound seemed to be rather academic since the proton-proton total cross-section seems to remain flat, as a function of energy, approximately equal to 39 millibarns. However, recent experiments (Amaldi et al. 1973, Amendolia et al. 1973), which are being described by other speakers at this meeting, indicate that total cross-sections are rising at high energies. It is in fact possible to fit the cross-sections with an expression $A+B(\log E)^2$, for instance, as Amaldi proposed it, $38.5 + 0.5 \left[\log \left(\frac{s}{137} \right) \right]^2$ millibarns, where s is the square of the centre-of-mass energy in $(\text{GeV})^2$. Of course, other fits are possible, the data do not exclude an increase with a single power of the logarithm or even a saturation of the total cross-section at a large finite value in the infinite energy limit. However, we think that there is at present a sufficient motivation to speculate under the assumption that the bound (1) is qualitatively saturated, i.e., that the cross-section does increase like $(\log E)^2$ with a numerical coefficient in front which is not maximal (the maximum allowed being about 60 millibarns).

2. BEHAVIOUR OF ELASTIC CROSS-SECTIONS WHEN TOTAL CROSS-SECTIONS GROW

If total cross-sections behave like $(\log E)^2$, this implies that we have exhausted the maximum allowed number of partial waves, which is (Łukaszuk and Martin 1966)

$$L_{\max} = \frac{k}{2m\pi} \log E \quad (2)$$

Now, using the unitarity condition on partial waves, it is easy to prove that

$$(\sigma_{\text{total}})^2 < c \left(\frac{L_{\max}}{k} \right)^2 \sigma_{el} \quad (3)$$

which shows that if $\sigma_{\text{total}} \sim (\log E)^2$ also $\sigma_{el} \sim (\log E)^2$. In practice it is indeed observed that the elastic cross-section is also rising at ISR energies. Now, if the total cross-section does increase like $(\log E)^2$, this means, by the optical theorem, that the forward amplitude increases like $E(\log E)^2$. Then if we want to conserve probability the width of the diffraction peak must be sufficiently narrow, otherwise

the elastic cross-section would become bigger than the total cross-section at a sufficiently high energy (Leader 1963, Martin 1963).

This means

$$\Delta T = C/(\log E)^2 \quad (4)$$

ΔT being the width of the diffraction peak measured in momentum transfer squared. In fact, in this limiting situation, it is not only the width of the diffraction peak which is constrained; it is its shape. In mathematical terms, what has been established (Auberson, Kinoshita, Martin 1971) is that inside the diffraction peak we have a "scaling" property : the ratio of the amplitude to the forward amplitude is asymptotically a function of a single variable $\tau = t(\log E)^2$, and this function is an entire function of order $\frac{1}{2}$. In physical terms, it means that we cannot fit the differential cross-section with the exponential of a polynomial in t with coefficients depending on s (which is indeed what is observed).

To conclude this section, I would like to put in a word of warning. First, it could very well be that the cross-section increases like $\log E$ instead of $(\log E)^2$. Then the predictions concerning the elastic scattering are somewhat changed : one can accept a constant elastic cross-section with a width $\Delta T = (\log E)^{-2}$ or an elastic cross-section behaving like $\log E$ with a width $\Delta T = (\log E)^{-1}$. Second, even if σ_{total} does increase like $(\log E)^2$, we can see that we have not yet reached the asymptotic regime because, at ISR energies, the diffraction peak does not seem to shrink more than at lower energies (to say the least !), so that we are still very far from the behaviour given by Eq. (4). This, of course, can go on for some time because of the smallness of the elastic cross-section, as compared to the total one, at presently accessible energies.

3. CONSEQUENCES OF ANALYTICITY IN ENERGY : THE POMERANCHUK THEOREM, THE REAL PARTS

The results of Section 2 follow from the bound on L_{max} from analyticity of the amplitude with respect to momentum transfer and from conservation of probability. The analyticity in energy is not

used explicitly, but only implicitly, in the obtention of $L_{\max}$. On the contrary, in this section we want to study direct consequences of the analyticity of the scattering amplitude with respect to energy, which is often summarized in fixed momentum transfer dispersion relations.

First, let us start with the old question of the Pomeranchuk theorem. One version of the Pomeranchuk theorem is that the ratio of the total particle-particle cross-section and total particle-anti-particle cross-section approaches unity at infinite energy. The "theorem", however, is valid under certain conditions which are not necessarily fulfilled. The principle of the proof is to compute, by dispersion relations, the real part of the forward amplitude under the assumption that the ratio of the cross-sections does not approach unity and to prove that the real part thus obtained is unacceptable. Unfortunately, if the cross-sections approach finite, different, constants at $E = \infty$, the real part is found to behave like $E \log E$ and it is impossible to find any objection of a fundamental nature against such a behaviour. One can only say that this goes against physical intuition, which makes us think that high energy scattering is dominantly diffractive.

If, on the contrary, the cross-sections go to infinity, it has been shown by Eden (1966) and Kinoshita (1966) that a contradiction with unitarity arises if the ratio of the cross-sections does not approach unity. In the limiting case where $\sigma_{pp} \sim (\log E)^2$ one finds

$$\frac{|\sigma_{pp} - \sigma_{\bar{p}p}|}{|\sigma_{pp} + \sigma_{\bar{p}p}|} < C(\log E)^{-1} \quad (5)$$

So, paradoxically, the Pomeranchuk "theorem" is a theorem when cross-sections rise but not when they are flat.

In passing, I would like to mention another Pomeranchuk-like theorem (Cornille and Martin 1972). It has been known for some time that due to the positivity of the absorptive part of the forward amplitude, the only possible high energy limit of $\frac{d\sigma}{dt}(pp)/\frac{d\sigma}{dt}(\bar{p}p)$ in the forward direction is unity. This has been shown to hold in the whole

diffraction peak. Hence the ratio of the width of the diffraction peaks in particle-particle and particle-antiparticle scattering approaches unity at infinite energy. Hence, if persistent shrinking is observed in pp collisions, persistent shrinking should also be seen in $\bar{p}p$ collisions at sufficiently high energies.

We now turn our attention to the question of the real part of the scattering amplitudes. If $\sigma_{pp} \sim (\log E)^2$ this means that the imaginary part of the even signature amplitude (which is the average between the proton-proton and proton-antiproton amplitudes) behaves like $C E (\log E)^2$. According to Khuri and Kinoshita (1965), this means that the real part of the even signature amplitude behaves like $C \pi E \log E$. In general, if the imaginary part of the even signature amplitude behaves like $E (\log E)^\beta$, the ratio $\rho = \text{Re } F^+ / \text{Im } F^+$ is equivalent to $\frac{\beta}{2} \pi (\log E)^{-1}$ (notice the positive sign of ρ).

If one assumes in addition that the odd signature amplitude is negligible at high energies, for instance that it behaves like E^α with $\alpha < 1$, one sees that these results apply to the proton-proton amplitude itself. So, with this additional assumption, we predict that :

i) if the pp cross-section behaves like $(\log E)^2$

$$\rho = \frac{\text{Re } F}{\text{Im } F} \simeq \frac{\pi}{\log E};$$

ii) if the pp cross-section behaves like $\log E$

$$\rho \simeq \frac{\pi}{2 \log E}.$$

These are of course asymptotic statements which could be regarded as consequences of dispersion relations. In fact, what is known experimentally is that, in a large energy interval, ρ is negative, and that the most recent measurement at the ISR, by Amaldi et al. (1973), gives a positive value consistent with zero within experimental errors. It is therefore very desirable to try to predict when the ρ changes sign under the assumption that cross-sections tend to infinity. Then one has to know the cross-sections not only asymptotically, but also at relatively low energy.

This is what has been done by Bourrely and Fischer (1973) who have used as starting point analytic amplitudes depending on sufficiently many parameters, the assumption $\sigma_{\bar{p}p} - \sigma_{pp} \sim E^{\alpha-1}$ being taken into account. Then the parameters have been adjusted to make the imaginary part of the amplitude fit the total cross-sections. One parameter is adjusted to low energy measurements of the real part. The prediction is that ρ indeed crosses over and is already +6 to +10 per cent at 2000 GeV equivalent laboratory energy, depending on the choice $\sigma \sim \log E$ or $\sigma \sim (\log E)^2$. It continues growing beyond 10^5 GeV, finally reaches a maximum and, later, follows the asymptotic regime. It appears in the work of Bourrely and Fischer that the results are not sensitive to the details of the parametrization, but that they depend on what is assumed for the behaviour of the cross-sections beyond the experimentally accessible domain of energies.

4. CONCLUDING REMARKS

To conclude, I would like to point out the interest of a measurement of the real part of the pp amplitude near the highest available energy (2000 GeV, equivalent) since this would differentiate between possible behaviours of total cross-sections at still higher energies. I would also like to say that any new information about proton-anti-proton amplitudes, either total cross-sections, widths of diffraction peaks, or real parts would be welcome even at energies as "low" as those of the NAL accelerator or the SPS at CERN. If it would turn out, for instance, that the $\bar{p}p$ cross-section continues to decrease well below the pp cross-section, we would be led to a dramatic revision of our ideas about particle interactions via local fields and microscopic causality.

Finally, let me indicate to the interested reader that the most recent and complete review on the subject is the article of Roy (1972) in Physics Reports.

R E F E R E N C E S

- Amaldi U., Biancastelli R., Bosio C., Matthiae G., Allaby J.V.,
Bartel W., Block M.M., Cocconi G., Diddens A.N., Dobinson R.W.,
Litt J., and Wetherell A.M., 1973, Phys. Letters 43B, 231-236 and
submitted to Physics Letters.
- Amendolia S.R., Bellettini G., Braccini P.L., Bradaschia C.,
Castaldi R., Cavasinni V., Cerri C., Del Prete T., Foa L., Giromini P.,
Laurelli P., Menzione A., Ristori L., Sanguinetti G., Valdata M.,
Finnochiario G., Grannis P., Green D., Mustard R., and Thun R., 1973,
submitted to Physics Letters.
- Auberson G., Kinoshita T., and Martin A., 1971, Phys.Rev. D3, 3185-3194.
- Bourrely C., and Fischer J., 1973, private communication.
- Common A.K., 1970, Nuovo Cimento 69A, 115-132.
- Cornille H., and Martin A., 1972, Phys. Letters 40B, 671-674.
- Eden R.J., 1966, Phys. Rev. Letters 16, 39-41.
- Froissart M., 1961, Phys. Rev. 123, 1053-1057.
- Khuri N.N. and Kinoshita T., 1965, Phys. Rev. 137B, 720-729.
- Kinoshita T., 1966, Perspectives in Modern Physics, R.E. Marshak
editor (New York, Interscience) 211-223.
- Leader E., 1963, Phys. Letters 5, 75-76.
- Lukaszuk L., and Martin A., Nuovo Cimento 52A, 122-145.
- Martin A., 1963, Phys. Rev. 129, 1432-1436.
- Martin A., 1966, Nuovo Cimento 42A, 930-954.
- Roy S.M., 1972, Physics Reports 5C, 125-196.
- Yndurain F.J., 1970, Phys. Letters 31B, 368-370.