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RISING CROSS-SECTIONS AND THE TRIPLE POMERON

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ABSTRACT

An iterative scheme for the Pomeron is proposed. The first iteration generates diffractive large mass production and is found to explain satisfactorily the rise of σ_{tot} at ISR energies. This contribution is represented by a non-vanishing triple Pomeron coupling g . Asymptotically the iteration procedure generates a renormalization of the Pomeron trajectory that ends up at $1+g$. The Froissart bound is enforced through s -channel absorption. The behaviour of $d\sigma_{\text{el}}/dt$ can also be reproduced in the model.

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1. INTRODUCTION

We want to propose in this paper a perturbative view of the Pomeron which stems from two phenomenological features recently suggested by the ISR ¹⁾ and NAL ²⁾ data :

1. the diffractive production of large masses. The inclusive p spectrum shows at $x \sim 1$ a peak of the type expected from a triple Pomeron contribution, that increases for decreasing t and shows no indication to vanish at $t=0$.
2. the increase of pp total and elastic cross-sections ^{3),4)}. An analysis of the data on $(d\sigma_{el}/dt)(s,t)$ ^{4),5)} (performed in Section 2) shows that this increase is not related to a saturation of the Froissart bound, and is more easily described as a t channel phenomenon on which s channel unitarity (e.g., in the form of absorption) is superimposed. The effects of s channel unitarity corrections turn out to be crucial to understand the t dependence, but not the s dependence at $t=0$.

In our zeroth order perturbative approximation the Pomeron is generated by the sum of purely inelastic (i.e., non-diffractive) channels. Diffractive events (elastic or single diffractive of the type mentioned in point 1.) represent our first order correction and must be explicitly added. We will show in Section 2 that diffractive excitation of large masses gives an increasing contribution to σ_{tot} at ISR energy, and that the amount of the increase is of the same size as the one of σ_{tot} . This estimate relies on the assumption, suggested by the data, that there is no turn-over of the t distribution of the inclusive p spectrum for $x \simeq 1$, or it occurs at very small values of t . We will consistently pursue in this paper the first possibility, which is equivalent to the assumption that the triple Pomeron coupling does not vanish at $t=0$: $g_P(0,0,0) \neq 0$. If we start with a Pomeron with unit intercept, we therefore obtain a triple Pomeron contribution which rises indefinitely like $\ln s$ (or $\ln \ln s$ if $\alpha'(0) \neq 0$ ^{*)}). We will attribute to this mechanism the rise of σ_{tot} at ISR energies.

It is known, on the other hand, that if $g_P(0,0,0) \neq 0$, a factorizable Pomeron at $J=1$ is not consistent with unitarity. Double and higher diffractive events will become detectable at even higher energies and will contribute

*) As we will discuss better in Section 4, this increase may be temporary if $g_P(t,t,0)$ turns out to have a turn-over for small t ($|t| \leq 0.05$ or less) and vanishes at $t=0$. The relation between the non-vanishing of $g_P(0,0,0)$ and the properties of the fixed pole in the amplitude are discussed in the Appendix.

with higher powers of $\ln s$. This will inevitably lead to a renormalization of the Pomeron trajectory and to a violation of the Froissart bound. A model exhibiting these features is presented in Section 4.

Our attitude will then be as follows : we will use $g_P(0) \neq 0$ but introduce absorption in the elastic channel in order to restore the Froissart bound ⁶⁾. At asymptotic energies a model like this is expected to yield an inelastic cross-section which is a considerable fraction of the total, both rising like $\ln^2 s$ ^{*)}. However, even at ISR energies, the bulk of the total cross-section is built up by non-diffractive events with $g_P(0)$ fixing the magnitude of its increase. This suggests a perturbative expansion in $g_P(0) \ln s$, the validity of which is discussed in Section 4.

As we said, we need an absorption scheme to enforce the Froissart bound asymptotically. The simplest form of absorption already sufficient for that purpose is the elastic one; we shall therefore adopt it.

This form of absorption is known to cancel out and reverse the sign of the elastic contribution (AFS cuts) ⁷⁾ and produces the diffractive minima of $d\sigma_{el}/dt$. However, the elastic absorption is not sufficient to change the sign of the increasing triple Pomeron contribution, but only decreases somehow its size as discussed in Section 5.

On the basis of the s and t dependence of elastic pp data, we will attempt in Section 5 two possible parametrizations of the first order pole and first iteration (triple Pomeron contribution).

In the conclusive Section 6, we will summarize our results and compare our theoretical scheme with other approaches recently proposed in this field [cf., Refs. 10), 18)-20)].

*) If only elastic absorption is considered, $\sigma_{el} = \frac{1}{2}\sigma_{tot}$ asymptotically. In a more refined model in which also absorption by diffractive inelastic channels is introduced, one can, however, envisage the possibility that $\sigma_{el}/\sigma_{tot} \rightarrow c < \frac{1}{2}$, as $s \rightarrow \infty$.

2. TRIPLE POMERON AND RISING $\sigma_{\text{tot}}(s)$

We want to see in this section whether the assumption that the triple Pomeron region is responsible for most of the growth of σ_{in} is tenable. As a convenient framework to formulate the question, let us consider the baryon number conservation sum rule :

$$\sigma_{\text{in}}(s) = \int_0^{1-M^2/s} dx \int_0^\infty dp_1^2 \left(\frac{d\sigma^{(B)}}{dx dp_1^2} - \frac{d\sigma^{(\bar{B})}}{dx dp_1^2} \right) \quad (2.1)$$

For $x \gtrsim 0.95$ only the p contribution is important in (2.1). Calling $v_P(t)$ the p-p Pomeron vertex, $g_P(t_1, t_2, t)$ the triple Pomeron vertex, we have ^{*}

$$\begin{aligned} \frac{d\sigma^{(p)}}{dx dp_1^2} &\simeq \frac{1}{16\pi} \gamma_P^2(t) \gamma_P(0) (1-x)^{-2\alpha(t)+1} g_P(t, t, 0) = \\ &= G e^{Bt} (1-x)^{-2\alpha(t)+1}, \end{aligned} \quad (2.2)$$

with $\alpha(t) = 1 + 0.25t$, $B \simeq 4.5 \text{ GeV}^{-2}$ ^{**}) and $G \simeq 4 \text{ mb GeV}^{-2}$ ⁸⁾. Hence

$$\int_{.95}^{1-M^2/s} dx dp_1^2 \frac{d\sigma^{(p)}}{dx dp_1^2} \simeq \frac{G}{2\alpha'} \ln \frac{B + 2\alpha' \ln s/M^2}{B + 2\alpha' \ln 20} \quad (2.3)$$

This expression does not exhaust the diffractive contribution for two reasons. In the first place a tail of diffractively produced large masses is expected also for smaller values of x , where it is not possible to separate it from the background. The contribution to (2.1) of events of this kind with x smaller than some $\bar{x}$ is, however, constant with energy if scaling holds. Furthermore, also the proton spectrum near $x=-1$ presents obviously the same features. Therefore the diffractive cross-section has an extra factor of two. The effect of this symmetrical contribution is felt in the sum rule as

*) We have used the NAL data ²⁾ for $|t| < 0.5$ to estimate B , and the CERN-Holland-Lancaster-Manchester collaboration data at $|t| = 0.275$ to estimate the x dependence and the normalization ¹⁾.

**) Since one can estimate $v_P(t) = v_P(0)e^{2t}$, $g_P(t, t, 0)$ turns out to be weakly dependent on t .

an energy dependence of the background. Therefore the change of

$$\int_0^{.95} dx dp_1^2 \left(\frac{d\sigma^{(B)}}{dx dp_1^2} - \frac{d\sigma^{(\bar{B})}}{dx dp_1^2} \right)$$

is identical in size to the energy dependence of (2.3).

In conclusion, the variation of the triple Pomeron contribution produces a change in $\sigma_{in}(s)$ going from an energy s_1 to an energy s_2 given by

$$\delta\sigma_{in}^{TP}(s_1, s_2) = \frac{G}{\alpha'} \ln \frac{B + 2\alpha' \ln s_2/M^{*2}}{B + 2\alpha' \ln s_1/M^{*2}} = \frac{2G}{B_{av.}} \ln \frac{s_2}{s_1}, \quad (2.4)$$

with $B_{av} = B + 2\alpha' \ln(s_{av}/M^{*2}) \simeq 6$ as estimated from the data ²⁾, we obtain $\delta\sigma_{in}(2800, 600) = 2.2$ mb.

We consider this result to be of the right size : the extra 1 mb still missing can be provided, for instance, by diffractive production of a large mass and a resonance (still belonging to our first perturbative order) and by higher orders (e.g., diffractive production of two large masses) ^{*}).

*) This approach has a phenomenological implication : the sum of events that do not show a rapidity gap larger than some Δy or that, having it, do not have a missing mass larger than, say 50 GeV^2 , should be nearly constant with energy. We will see, however, in Section 5.b that, in presence of strong absorption, terms which behave like $1/\ln s$ (i.e., cuts) can still give a small contribution to the rise of σ_{tot} .

3. PARTIAL WAVES AND FROISSART BOUND

In order to get some insight on the dynamics of the rise of σ_{tot} we find enlightening to discuss the amplitude $a(b,s)$ in impact parameter space. We define, as usual

$$a(b,s) = \frac{1}{16\pi} \int_{-\infty}^0 dt J_0(b\sqrt{t}) \text{Im} A(s,t), \quad (3.1)$$

where the normalization is such that $\text{Im} A(s,0) = \sigma_{\text{tot}}$ and the contribution to elastic unitarity in b space is simply $[a(b,s)]^2$, if we neglect the small $\text{Re} A(s,t)$. $\text{Im} A(s,t)$ is then given by $(16\pi(d\sigma/dt))^{1/2}$. Since the slope of $d\sigma_{\text{el}}/dt$ is about $2B \simeq 12$, the order of magnitude of $a(0,s)$ is

$$a(0,s) \simeq \frac{\sigma_{\text{tot}}}{16\pi B} \simeq \frac{1}{3}. \quad (3.2)$$

[Remember that, in our normalization, the unitarity bound is $a=1$, with $\sigma_{\text{el}} = \sigma_{\text{tot}}$, while for a black disk it is $a = \frac{1}{2}$ and $\sigma_{\text{el}} = \sigma_{\text{in}}$.] The experimental data on $d\sigma_{\text{el}}/dt$ increase steadily with s in the ISR range for $|t| < 0.15$, with a sharpening of the forward peak; the higher t data on the other hand show little or no energy dependence. A convenient parametrization of the increase $\delta A(t)$ from the lowest to the highest ISR energies is given by

$$\delta A(t) = A_1 e^{B_1 t}, \quad A_1 \simeq 4 \text{ mb}, \quad B_1 \simeq 12 \text{ Gev}^{-2}, \quad (3.3)$$

the Bessel transform of which is

$$\delta a(b) = \frac{A_1}{16\pi B_1} \exp\left(-\frac{b^2}{4B_1}\right). \quad (3.4)$$

With this parametrization $\delta\sigma_{\text{el}} \simeq 1 \text{ mb}$.

It is clear that - independently of the details of the parametrization chosen - two features emerge in impact parameter space :

- 1) $a(b,s)$ increases steadily with energy for all values of b ; $\delta a(b)$ has no particular structure at $b=0$ and in the chosen parametrization it has indeed a maximum $\delta a(0) \simeq 0.02$.

- 2) the width in b of $\delta a(b)$ is large compared to the width of a itself. In the parametrization (3.4) is about 48 GeV^{-2} .

We conclude from these remarks that, even if the small values of b are not very far from the unitarity bound for a black disk, the growth of $a(b,s)$ shows no sign of saturation. We would have expected, in such a case, a dip of $\delta a(b)$, at $b=0$, followed by a maximum at some $\bar{b} \simeq R_0 \ln s$: only when such a regime is reached will the dependence of σ_{tot} on the energy have the $\ln^2 s$ behaviour typical of the Froissart bound. We see, on the contrary, that at present energies the rise of σ_{tot} is mainly caused by the proton becoming blacker, rather than larger, and therefore a $\ln^2 s$ parametrization is unwarranted, and no meaning can be attributed to its coefficient. In our opinion, the growth of $a(b,s)$ at fixed b is determined by the t channel "potential"; hence we will try in this paper to understand the rise of σ_{tot} mainly in t channel terms.

As stated in the previous section, we think that it is possible to attribute the energy dependence of $\sigma_{\text{tot}}(s)$ [i.e., of $\text{Im}A(s,0)$], to the triple Pomeron contribution. In Section 5, we will try to understand the energy dependence of $\text{Im}A(s,t)$ also for $t \neq 0$ in these terms. In order to do so it is necessary to examine the form of the triple Pomeron contribution to the non-forward overlap function.

This can be easily done if one arbitrarily assumes an exponential factorizable form for the triple Pomeron vertex $g_P(t_1, t_2, t_3) = e^{a(t_1+t_2+t_3)} g_P$ the result is ($Y = \ln s$ from now on) :

$$a^{TP}(b,s) \simeq \frac{\gamma_P^3(0) g_P}{(16\pi)^2} \frac{\exp(-b^2/[6d+4\alpha'Y])}{(\frac{3}{2}d + \alpha'Y) \alpha'} \ln\left(1 + \frac{\alpha'Y}{d}\right). \quad (3.5)$$

Here we have parametrized the residue of the Pomeron-proton-proton vertex $v_P(t)$ as $v_P(0)e^{+ct}$, and $d=a+c$.

In the limit of small α' ($\alpha'Y \ll d$)

$$a^{TP}(b,s) \simeq \frac{2 \gamma_P^3(0) g_P(0)}{3 (16\pi)^2 d^2} \exp\left(-\frac{b^2}{6d}\right) \cdot Y, \quad (3.6)$$

and grows linearly in Y at fixed b ; thus even if the contribution of the triple Pomeron to σ_{tot} does not exceed the Froissart bound, the partial waves eventually violate the unitarity bound and need absorption.

If $\alpha' \neq 0$, asymptotically the s channel partial waves vanish like $\ln Y/Y$, i.e., slightly slower than in a regular Regge pole case, and have a maximum (at $b=0$) for $Y \simeq d/\alpha'$. With the value of $G = v_P^3(0)g_P(0)/16\pi \simeq 10 \text{ GeV}^{-4}$ estimated in Section 2, this maximum can lead to violation of the black disk bound $a = \frac{1}{2}$, if $d \leq 2$ *).

*) Of course this violation of the unitarity bound is prevented by the introduction of elastic absorption : in particular in the self-consistent approach of Ref. 6), the asymptotic limit of the $x \simeq 1$ inclusive spectrum

$$\frac{1}{\sigma_{\text{in}}} \int \frac{d\sigma}{dx dp_{\perp}^2} dp_{\perp}^2$$

is changed into the integrable form $(1-x)^{-1} \ln^{-3}(1-x)$ ⁹⁾. We have checked that at ISR energy the effects of absorption on the x dependence of (2.2) are not important [it amounts to a corrective factor of the form $1+(c/\ln(1-x))$] and therefore we are justified in using a triple Pomeron form in comparing with the data. The effects of absorption on the normalization and the t dependence are likely to be more important; the former are included in the analysis of Section 5, the latter require unfortunately a detailed assumption on $g_P(t_1, t_2, t_3)$ for an estimate to be given.

4. PERTURBATIVE MODEL OF THE POMERON RENORMALIZATION

The phenomenological indications of Section 2 suggest the assumption that $g_P(0,0,0) = (16\pi/(\sigma_{\text{tot}})^{\frac{3}{2}})G \simeq 0.5 \text{ GeV}^{-1}$, which in turn explains satisfactorily the rise of σ_{tot} . As already pointed out, t channel iteration leads in this case to renormalization of the trajectory and violation of the Froissart bound, with need for absorption. We shall now study a multiperipheral-like overlap function M , generated by single and multiple diffractive events, and we shall include absorption later on (Section 5).

Our zeroth approximation M_0 is just the multiperipheral overlap function coming from the iteration of low energy clusters, i.e., the solution of an integral equation with a low energy kernel (e.g., meson exchange). The contribution of diffractive events (through M_0) is obtained by including M_0 in the kernel. The complete inelastic overlap function M is therefore given by a sum of terms which are illustrated in Fig. 1. The integral equation which represents the sum of the series is given in the J plane by :

$$\begin{aligned} M(J,t) &= D(J,t) [G_0(t) + g g_0(t) D_1(J,t)]^2, \\ D(J,t) &= D_0(J,t) + g^2(t) D_1(J,t) D(J,t), \end{aligned} \quad (4.1)$$

where a factorizable triple Pomeron vertex has been assumed, and $D_0(J,t)$ is the J plane propagator of M_0 , while $D_1(J,t)$ is the propagator of the M_0 - M_0 cut, with an effective coupling g_0 to the proton.

The solution of the preceding equation is

$$M(J,t) = [D_0^{-1}(J,t) - g^2 D_1(J,t)]^{-1} [G_0 + g_0 g D_1(J,t)]^2. \quad (4.2)$$

As explained before, we assume that non-diffractive events contribute a nearly constant cross-section. We set therefore $\alpha_0 = 1 - \epsilon$. We shall see later on that for our value of g , the actual, very small value of $|\epsilon|$, is irrelevant; in particular we do not need $\epsilon \neq 0$. Corrections to the inelastic Pomeron come from single or multiple diffractive exchanges, which are characterized by the coupling constant g . The smallness of g_P forces g to be small with respect to g_0/M and G_0/M .

For the time being let us neglect shrinkage, so we start with M_0 represented by a fixed pole at $J = 1 - \epsilon$. Then, at $t = 0$, one has

$$D_0(J,0) = (J-1+\epsilon)^{-1} , \quad D_1(J,0) = (J-1+2\epsilon)^{-1} . \quad (4.3)$$

This gives the perturbative expansion

$$M(J,0) \simeq \frac{G_0^2}{J-1+\epsilon} + \frac{2g_0gG_0}{(J-1+\epsilon)(J-1+2\epsilon)} + \dots \quad (4.4)$$

which, neglecting absorption, gives the estimates

$$G_0^2 \simeq \sigma_{in} , \quad g_0^2 \simeq \sigma_{el} \quad (4.5)$$

The latter piece in (4.4) is the triple Pomeron contribution to the overlap function, which, in our case, has the energy dependence

$$\begin{aligned} M_1(s,0) &= 2g_0gG_0 \frac{1}{\epsilon} (s^{-\epsilon} - s^{-2\epsilon}) \simeq \\ &\simeq 2g_0gG_0 \ln s \simeq \frac{2G}{B_{av}} \ln s , \quad (\epsilon \ln s \ll 1) , \end{aligned} \quad (4.6)$$

where G/B_{av} is given by (2.3). This fixes the magnitude of g . By (4.5) we get

$$g \simeq G / (B_{av} \sqrt{\sigma_{el} \sigma_{in}}) \quad (4.7)$$

The exact form of $M(J,t)$ is, of course, given by Eq. (4.2) and with the specifications (4.3) takes the form

$$\begin{aligned} M(J,0) &= [(J-1+\epsilon)(J-1+2\epsilon) - g^2]^{-1} . \\ &\cdot \left[G_0^2 (J-1+2\epsilon) + 2g_0gG_0 + \frac{(g_0g)^2}{J-1+2\epsilon} \right] . \end{aligned} \quad (4.8)$$

The output poles are given by the roots of the denominator as follows

$$J-1 = -\frac{3}{2}\epsilon \pm \left(\frac{1}{4}\epsilon^2 + g^2 \right)^{1/2} . \quad (4.9)$$

Two different regimes are of interest, according to whether $g \ll \frac{1}{2}\epsilon$ or $g \gg \frac{1}{2}\epsilon$. In the first case, the displacement of the poles is very small, of order g^2/ϵ , and one essentially recovers the perturbative expansion (4.4)¹⁰⁾. However, the estimate (4.7) shows that g is at least $\simeq 0.05$, in order to be consistent with the estimated value of G [Eq. (2.2)], and is about $\simeq 1/10$, after absorption has been put in (Section 5). The near constancy of σ_{tot} followed by the rise at ISR energies shows that the alternative $g \gg \frac{1}{2}\epsilon$ is more appealing. In such a case, the displacement of the poles is of order g , and the leading trajectory violates the Froissart bound^{*)}.

In this situation, the precise value of ϵ plays little role, and we shall put $\epsilon = 0$. Then one has

$$M(J, 0) = \frac{1}{2} \frac{(G_0 + g_0)^2}{J-1-g} + \frac{1}{2} \frac{(G_0 - g_0)^2}{J-1+g} - \frac{g_0^2}{J-1} \quad (4.10)$$

which gives an s dependence

$$\tilde{M}(s, 0) = (G_0^2 + g_0^2) \cosh gY + 2G_0g_0 \sinh gY - g_0^2 \quad (4.11)$$

$(Y \equiv \ln s)$.

It is interesting to note that perturbation theory in the J plane is not applicable in the limit $\epsilon \rightarrow 0$, due to the $\sqrt{g^2}$ singularity of (4.9), and to the singular residues of (4.8). However, one still has an acceptable perturbative expansion in $g \ln s$, due to the contributions of non-leading poles. In fact, from (4.11), we get

*) Should the triple Pomeron vertex $g_P(t, t, 0) \equiv g_P(t)$ vanish at $t=0$, $(-\infty \int dt g_P^2(t)/16\pi)^{\frac{1}{2}}$ gives the trajectory renormalization if $\alpha' = 0$, and the output trajectory is likely to overcome $J=1$. (It has to, if D_1 has a $J=1$ singularity, cf., end of Section.) If $\alpha' \neq 0$, however, then $g_P(0)=0$ allows a self-consistent Pomeron¹⁰⁾ at $\alpha_P(0)=1$, provided $\epsilon = \int_{-\infty}^0 dt g_P^2(t)/|t| 32\pi\alpha'$. The triple Pomeron contribution to M will have the form $A-B/\ln s$ ($A, B > 0$). We therefore have (after elastic absorption) a contribution to σ_{tot} that increases to a constant. If $g_P(t)=0$ for $|t| < t_0$, the magnitude of this increase will be reduced (with respect to the estimate given in the text) by an energy dependent factor $r = \exp(-B_{\text{av}} t_0)$. Putting $B_{\text{av}} \simeq 6 \text{ GeV}^{-2}$ and t_0 very small, say $t_0 \simeq 0.04$ ²⁾ results in a rather weak effect, because $r \simeq 3/4$. [The case $g_P(0)=0$ is better discussed in the Appendix.]

$$\tilde{M}(s,0) \simeq G_0^2 + 2g_0 G_0 g Y + \frac{1}{2} (G_0^2 + g_0^2) g^2 Y^2 + \dots \quad (4.12)$$

to be compared with (4.4) and (4.6). As a consequence, the triple Pomeron contribution, which is not by itself of "renormalization" type, still gives an estimate of the renormalization of the trajectory which is of order g , instead of g^2/ϵ . This will be very important at energies for which the regime $M \sim s^{\frac{1}{2}}$ takes over.

Let us now discuss the physical content of the expansion (4.12). M_0 being the solution of a multiperipheral equation with meson exchange obviously has no threshold. In other words, its scale is the usual $s_0 \sim 1 \text{ GeV}^2$. Therefore the iterative scheme described before has no other scale than s_0 . This implies that those inelastic events containing an exchanged M_0 responsible for a small subenergy contribute to M_1 , i.e., to the correction terms to M_0 . Inelastic Pomeron exchanges contribute therefore also to events which cannot be classified as diffractive at a glance, because they do not contain any large rapidity gap. In other words, the events that generate M_0 can be separated only after subtraction of a low energy tail coming from diffractive events, and extrapolated in accordance with duality prescriptions.

If one accepts this interpretation, the expansion parameter is $g \ln s/s_0$, and with our estimate it is already rather large, of the order of 0.5 at ISR energies; hence doubly diffractive events could be already sizeable, even if not identifiable as such ^{*)}.

An attractive alternative, which has been recently considered by Abarbanel, Chew and co-workers ¹¹⁾, is to define M_0 as the overlap of all events which do not contain rapidity gaps larger than a prescribed amount, $\ln \tilde{s}$. For instance, in a multiperipheral language with elementary π exchange, M_0 would be the solution of an integral equation, the kernel of which is the complete π - π interaction, including the low energy Pomeron, but cut off at $\tilde{s}$.

*) M_0 defined in this way could decrease asymptotically like $s^{-\eta}$, with η very small. In this case the need for rather large corrections is obvious, since they have to make up for the decreasing M_0 .

In this case the integral equation must give rise to corrections arising from events with a large gap. Formally one has again Eq. (4.1), but D_1 is now generated by a P-P cut with a $\Theta(s-\tilde{s})$, everything below $\tilde{s}$ being already contained in M_0 , and in our approach P is obtained by absorbing M. P will no longer be a pole, however D_1 will still be a cut (at $J=1$) equivalent - for the purpose of the integral equation for M - to the usual pole-pole cut. The factorization property of the multi-Pomeron exchange amplitude needed in order to write (4.1) is, however, no longer guaranteed if P is not a pole. It can be recovered in a π exchange multi-peripheral model where the non-factorizable P represents the high energy behaviour of the $\pi\pi$ kernel.

In this approach the high energy behaviour of M_0 depends on $\tilde{s}$, and we know that with $\ln(\tilde{s}/s_0) \simeq 2+3$, M_0 can be identified with the overlap functions of nearly all the events observed below ISR energies, and is therefore associated to a constant cross-section. Furthermore, there is a natural threshold $\bar{s}$ that depends on $\tilde{s}$, below which the iterative corrections are negligible due to $t_{\min}$ effects (the correction strictly vanishes for $s < \tilde{s}$). The effective expansion parameter is then $g \ln s/\bar{s}$, which can be rather small, even at ISR energies. A measure of the 2nd order corrections is given in this approach by the rise of the cross-section for events with two gaps larger than $\ln \tilde{s}$.

Let us now briefly study the t dependence of the output poles in M. If M_0 is still a fixed pole, Eqs. (4.8) and (4.9) are unchanged, with G_0 , g_0 and g all t dependent ^{*}). In particular, if we parametrize $g(t) = g e^{bt}$, the slope of the output pole in M is

$$J'_{out}(0) = b g. \quad (4.13)$$

If, on the other hand, M_0 is a moving pole, Eq. (4.8) has to be modified by putting

$$\begin{aligned} D_0^{-1}(J,t) &\simeq J-1+\varepsilon-\alpha'_0 t, \\ D_1(J,t) &\simeq \frac{g^2(t)}{\alpha'_0 \Delta} \ln \left[1 + \frac{\alpha'_0 \Delta}{J-1+2\varepsilon-\frac{1}{2}\alpha'_0 t} \right] \end{aligned} \quad (4.14)$$

^{*}) While $g=g(0)$ is given in terms of $g_p(t,t,0)$, and can be therefore estimated from the diffractive production data, $g(t)$ depends on $g_p(t_1, t_2, t)$ and therefore its t dependence cannot be deduced from the data alone.

where α'_0 is the slope of M_0 , and Δ is the width of the t distribution. For the position of the leading output singularity, and for a small slope, the effect of the logarithmic branch point is much the same as that of a pole, so that one still has approximately

$$J_{out-1} \simeq -\frac{3}{2}\epsilon + \frac{3}{4}\alpha'_0 + \left[\frac{1}{4} \left(\epsilon + \frac{\alpha'_0}{2}t \right)^2 + g^2(t) \right]^{1/2}. \quad (4.15)$$

when ϵ is small compared with g , the output $t=0$ slope is

$$J'_{out} \simeq \frac{3}{4}\alpha'_0 + b g. \quad (4.16)$$

Note that formulae (4.13) and (4.16) give different output slopes, unless b is assumed to be rather big in one case and small in the other. This corresponds to the two physical pictures which are discussed in Section 5.

5. ENERGY DEPENDENCE OF THE DIFFRACTION PEAK

In this section we want to see what can we learn from a study of the shape and energy dependence of $d\sigma_{el}/dt$. In order to compute this quantity we need some assumption on the t dependence of M_0 (generated by the overlap function of the purely inelastic events), and of the triple Pomeron contribution, the higher perturbative terms being in our scheme still negligible at ISR energies (cf., Section 4).

The rough qualitative features of the data on $d\sigma_{el}/dt$ can be summarized by distinguishing regions of small t 's ($|t| \lesssim 0.15$), of intermediate t 's ($1 > |t| > 0.15$) and of large t 's. In the intermediate and large t regions, the data seem energy independent in normalization and slope, in the first region an increase of slope, and consequently of normalization at $t=0$ is observed.

On the basis of these data we want to discuss two extreme points of view (cf., also Section 4) :

- A) the "inelastic" Pomeron M_0 is responsible for the intermediate and large t behaviour and is not shrinking. In this case the shrinkage of the small t region must be attributed to the triple Pomeron contribution, that has therefore to be very sharp in t ;
- B) the inelastic Pomeron shrinks, and thus is responsible for the features of the small t region, whereas the increasing contribution of a broad triple Pomeron masks the shrinkage of M_0 outside the small t region.

The other theoretical ingredient that we should specify at this stage is the role and form of the elastic channel absorption that we assume.

A) Non-shrinking M_0 , sharp M_1

In this picture any form of absorption will play little role for small t , because a sharp M_1 populates the large b (impact parameter) region, where absorption is small. However, this separation into two components of very different steepness is tenable up to $|t| \simeq 1 \text{ GeV}^2$ only if absorption (e.g., in the form of cancelling cuts) is not very strong. Otherwise strong cuts obtained by absorbing M_1 through A_0 would cause excessive s dependence for $t > 0.2$. For consistency, we shall assume in this case an absorption of the eikonal type, which turns out to be rather weak ^{*)}. To first order in

^{*)} A comparison of formula (5.1) with (5.7) shows that the latter contains an additional series of negative terms, of which the first is $(-2a_0^2)$.

the perturbation M_1 , but to all orders in M_0 , one has in this case

$$a_1(b) = m_1(b) [1 - 2 a_0(b)] , \quad (5.1)$$

where

$$\tilde{M}(s, t) = M_0(t) + M_1(t) \ln(s/\bar{s}) + \dots \quad (5.2)$$

is the inelastic overlap function before absorption, to which the t channel analysis of Section 4 applies. $A_0(t)$ is the amplitude obtained by absorbing M_0 , i.e., in the eikonal model

$$a_0(b) = \frac{1}{2} (1 - e^{-2 m_0(b)}) , \quad (5.3)$$

and contains those diffractive features which seem to be energy independent, like the dip at $|t| \simeq 1.5$ ^{*)}.

The meaning of Eq. (5.1) is transparent. To the triple Pomeron contribution M_1 we will add a - supposedly small - negative correction which is an absorptive M_1 - A_0 cut. If we forget for the moment this correction, the parametrization we are seeking is analogous to the (3.3) given previously. We shall therefore put

$$A_0 = \sigma_0 e^{B_0 t} , \quad M_1 = \sigma_1 e^{B_1 t} . \quad (5.4)$$

A reasonable fit to the data in the ISR range, using the weak absorption expressions (5.1) and (5.2) and the parametrization (5.4). is obtained for ^{**) (5.5)}

$$\begin{aligned} \sigma_0 &\simeq 33.5 \text{ mb} , & \sigma_1 &\simeq 3.3 \text{ mb} , \\ B_0 &\simeq 5 \text{ GeV}^{-2} , & B_1 &\simeq 10 \text{ GeV}^{-2} , & \bar{s} &\simeq 60 \text{ GeV}^2 . \end{aligned} \quad (5.5)$$

*) It has been pointed out to us that a very reasonable fit to A_0 is obtained by an eikonalization of an M_0 given by the square of the proton form factor ¹²⁾. In what follows, we will be concerned only with values of $|t| \lesssim 1 \text{ GeV}^2$: for this purpose $a_0(b)$ is well represented by a Gaussian.

**) The role of the scale factor $\bar{s}$ has been discussed at length in Section 4. The actual value used here is strongly dependent on how steep a triple Pomeron one is willing to accept.

Indeed, we obtain

$$\begin{aligned}
 A(s,t) &= \sigma_0 e^{B_0 t} + \sigma_1 \ln \frac{s}{s_0} \left[e^{B_1 t} - \frac{2 \bar{a}(0) \bar{B}_0}{\bar{B}_0 + B_1} \exp\left(-\frac{B_1 \bar{B}_0}{B_1 + \bar{B}_0} t\right) \right] \approx \\
 &\approx 33.5 \left[e^{5t} + 0.1 \ln \frac{s}{s_0} \left(e^{10t} - 0.25 e^{3.75t} \right) \right], \quad (5.6)
 \end{aligned}$$

where an average value of the slope, $\bar{B}_0 \simeq 6$ has been used in the actual calculation of (5.1).

The negative absorptive term (which cancels only 1/4 of the M_1 contribution to the total cross-section) sharpens the correction term for small t 's. Altogether, we find a sort of break in the slope (due to the compensation of the triple Pomeron and absorptive correction, which in our parametrization occurs at $t \sim 0.2$). For smaller t we find a considerable shrinkage while for intermediate t the shrinkage is small. We give in the Table the slopes as computed with the parametrization (5.4).

Let us finally remark that σ_1 has been normalized to the total increase (4 mb) of σ_{tot} in the ISR range. Therefore, we include in the phenomenological formula (5.6) those corrective terms (like diffractive production of a resonance and a large mass and of two large masses) which, in our scheme, give the extra 1 mb rise we need (cf., Section 2). The triple Pomeron strength g of Section 4 is instead normalized to a 2.2 mb rise in the ISR range, with an extra 4/3 factor to take into account absorption. From Eqs. (4.6) and (5.6) we get

$$g \approx \frac{\sigma_1}{2 \sqrt{s_0} \sigma_{\text{in}}} \frac{2.2}{3} \approx 0.08 \quad (5.6)$$

(This gives for our expansion parameter : $g \ln s/\bar{s} \simeq 0.25$ in this case.)

B) Shrinking M_0 , broad M_1

The previous picture is attractive because the behaviour of $A(s,t)$ is directly interpreted in terms of our two mechanisms, but presents some difficulties if one starts from a multiperipheral prejudice on the generation of M_0 from inelastic events. In fact, in this framework it is hard to understand why M_0 does not shrink. Furthermore, it is a general feature of

multiperipheralism that the higher is the multiplicity, the steeper is the overlap function. Since the diffractive events which build up M_1 have a multiplicity that is expected to grow indefinitely, but is certainly smaller than average, it is unnatural (in a MPM) to attribute to these events the very steep corrective term needed in the previous section *).

A simple, even if somewhat arbitrary estimate of the expected t dependence of the triple Pomeron overlap function can be obtained assuming for $g_P(t_1, t_2, t_3)$ the factorizable exponential parametrization used in Section 3, i.e., $g \simeq e^{a(t_1+t_2+t_3)}$. Then the data on the t dependence of the proton inclusive spectrum give a small value for a ($a \lesssim 0.5$) and the calculation of Section 3 gives a width of the triple Pomeron overlap of the order of the elastic overlap function, i.e. twice as broad as the diffraction peak.

We want therefore to examine now the possibility of a broad triple Pomeron contribution. The small t sharp shrinking peak must therefore be attributed mainly to M_0 . Since in this picture the triple Pomeron contribution is concentrated in a small range of b around $b=0$, absorption effects are important and non-linear, and different assumptions on the precise nature of the elastic absorption can give rise to a sizeable change. Together with the eikonal scheme, we will therefore consider the stronger absorption scheme suggested by Gottfried and Jackson :

$$a(b, s) = a^2(b, s) + (1 - 2a(b, s)) m(b, s) . \quad (5.7)$$

Since in this approach it is hard to have an intuitive picture of the interplay of our two terms, we will try to learn directly from the data what $m(b, s)$ looks like.

For simplicity, we assume for $A(s, t)$ a parametrization with a break at $t=0.1$

$$A(s, t) = \sigma_T(s) e^{B(s)t} \Theta(\bar{t} - |t|) + \sigma'_T e^{Bt} \Theta(|t| - \bar{t}) . \quad (5.8)$$

*) Rather than a multiperipheral scheme, these features suggest a geometrical interpretation, in which high multiplicity comes from central (i.e., small b) collisions, and diffractive events are naturally associated to glancing (i.e., large b) collisions.

In particular for the two extreme ISR energies $s_1 = 600 \text{ GeV}^2$ and $s_2 = 2800 \text{ GeV}^2$ the parameters σ and B are assumed to be : $B(s_1) = 11.8$; $B(s_2) = 13.1$; $B = 10.5$; $\sigma_{\text{tot}}(s_1) = 39 \text{ mb}$; $\sigma_{\text{tot}}(s_2) = 43 \text{ mb}$, and σ'_{tot} is fixed by the requirement of continuity at $t=0.1$, and shows very little energy dependence.

Inserting this parametrization in (5.1) and (5.7), we obtain the values of $m(b, s_1)$ and of $\delta m(b) = m(b, s_2) - m(b, s_1)$ shown in Fig. 2. In both cases δm seems to deviate strongly from an exponential in b^2 , and shows a steep peak followed by a very broad tail; this two-component structure is particularly clear if we use strong absorption. The broad large b^2 contribution is generated in this scheme by the shrinkage of M_0 . If M_0 is a Regge pole of intercept 1, the contribution to δm coming from the energy dependence of M_0 is negative at small b and integrates to zero. Hence δm_1 has to cancel the negative δm_0 at small b , in addition to producing the observed peak.

A rough, but adequate, parametrization of δm can thus be given in both cases (weak or strong absorption) in terms of a moving Regge pole with $\alpha = 1 + 0.35t$. Its contribution is assumed Gaussian and yields $m_0(b, s_1) = 0.36 e^{-b^2/24}$ and $m_0(b, s_2) = 0.33 e^{-b^2/26}$.

The triple Pomeron contribution required to add to δm_0 to reproduce δm has, in both cases, a width of the expected order of magnitude $\simeq e^{-b^2/11}$, whereas its normalization at $b=0$ is 0.16 and 0.096, respectively.

This second picture has the attractive feature of allowing a broad (in t) triple Pomeron contribution, but we have to acknowledge the fact that it is far less transparent than the former one. Absorptive effects, especially with strong absorption, are non-linear and it is impossible to separate in a simple way A into different contributions. In particular, the shrinkage at $t=0$ depends very much on absorption, since, e.g., $\langle b^2 \rangle$ does not increase from s_1 to s_2 in m .

Also the actual mechanism of the rise of σ_{tot} is, in principle, not simple in this picture, since it results from the interplay of three effects :

- 1) the disappearance of the negative cut, which results in an increase of the number of non-diffractive events;
- 2) the growth of M_1 , related to diffractive events;
- 3) the growth of M_1 increases the absorption of M_0 , and thus reduces the number of non-diffractive events.

The effect of point 2) causes the increase of σ_{tot} related to the increase of diffractive events that forms the basis of our approach. The contribution of M_1 is reduced by absorption both at the level of the rate of diffractive events and, more severely, thanks to the non-linearity of absorption, at the level of increase of A . Hence the number of diffractive events provides a slight overestimate of the increase of σ_{tot} , the difference being given by point 3). A numerical estimate shows that this effect is quite negligible in our situation. Not so negligible is the effect 1), that gives a positive contribution of about $3/4$ mb to σ_{in} , this number being related to an increase of non-diffractive events. This would be a welcome addition to the 2.2 mb estimated in Section 2 from the diffractive contribution.

6. DISCUSSION

Altogether, we have proposed an iterative scheme for the Pomeron, in which diffractive inelastic events (already sizeable at ISR energies) give the order of magnitude of the expansion parameter, and will ultimately lead to saturation of the Froissart bound.

In this framework, we have obtained the following results :

- a) at ISR energies, the triple Pomeron contribution gives most of the rise of σ_{tot} , and - in a more model dependent way - explains the shape and energy dependence of the diffraction peak.
- b) our expansion parameter turns out to be $g \ln s/\bar{s}$, (cf., Section 4) where the magnitude of g is determined by the size of diffractive production, and turns out to be of the order of $1/10$. Hence a perturbative approach is tenable at ISR energies, and the first order correction is the triple Pomeron contribution.
- c) after summation of the perturbation series, our Pomeron pole has intercept $(1+g)$. Absorption implements the Froissart bound and therefore, at very high energy, we expect σ_{tot} going like $\ln^2 s$, with a coefficient controlled by g :

$$\sigma_{\text{tot}} / \ln^2 s \xrightarrow{s \rightarrow \infty} 8\pi \alpha' g \simeq 0.24 \text{ mb.}$$

The smallness of this parameter, and the very slow approach to asymptopia is due to the fact that the radius of saturation of the unitarity bound ¹³⁾ is determined by $(\alpha'g)$. We remark again that this $\ln^2 s$ behaviour appears in our approach only at extremely high energies and is completely unrelated with the increase observed at ISR.

Let us now compare our approach with others in the field. We share with other authors a larger than one Pomeron which, through eikonalization ¹⁴⁾ or absorption ⁶⁾ gives an ultimate $\ln^2 s$ increase of the cross-section.

The consideration of the Pomeron as an iterative object is also a property of the dual models. In this approach, the Pomeron is generated by the production of two distinct super-resonances (Fig. 3a). The production of three super-resonances (Fig. 3b) gives rise to a double pole at the dual Pomeron position, which must therefore contribute to the Pomeron trajectory renormalization. Let us notice that the dual loop theory gives a scheme in which renormalization and unitarization are treated consistently; its analysis ¹⁵⁾ can throw better light on the interconnection of s and t channel iterations.

The iterative process gives an interpretation of the "discrepancy" found ¹⁶⁾ in the inclusive sum rules when the triple Pomeron does not vanish at $t=0$. Indeed, it is well known that the sum rule is equivalent to a unitarity condition ¹⁷⁾. The fact that an input pole at $\alpha(0)=1$ gives rise to a logarithmic behaviour in the output just reflects our iteration procedure. Our ultimate solution will be given by the interplay of this iteration and the subsequent absorption, and, as discussed before, will lead to a Pomeron singularity which is consistent with the sum rule, but is not a factorizable pole.

The idea that diffractive events generate contributions which renormalize the Pomeron has been advanced by many authors ^{10),18)-20)}. The common feature of these approaches is that they ask for a self-consistent Pomeron in a simple t channel picture (pole + cut). This forces them to have a triple Pomeron coupling vanishing at $t=0$ if $\alpha_P(0)=1$, and a non-rising σ_{tot} at real asymptotic energies. Our aim being to have a model for rising cross-sections, we cannot have a self-consistent Pomeron pole. Our iteration procedure starting from a factorizable pole (t channel picture, still adequate up to ISR energies) will end up with a non-factorizable singularity (s channel picture, at real asymptotic energies) which will not have a simple J plane structure.

Gribov's Reggeon calculus ¹⁸⁾ stems from an analysis of families of Feynman diagrams. It predicts negative cut sign, so that total cross-sections reach a constant limit from below, and elastic cross-sections decrease monotonically to zero. The S matrix approach that we follow and the diagrammatic one are not necessarily incompatible. In the case of the elastic cuts, for instance, it is known ⁷⁾ that elastic absorption effects on inelastic channels reverse the sign of the elastic cut into a negative one in agreement with Feynman diagram calculations. The compensation mechanism provided by elastic absorption works also for other contributions (as the triple Pomeron of this paper) but does not operate a complete cancellation, leaving a net positive result. Nevertheless, one could invent more elaborate absorption schemes which could provide complete cancellations, thus leading to Gribov's results. We remark, however, that this cannot occur for inelastic diffractive contributions which increase more than the purely inelastic one, as in the case $g_P(0) \neq 0$ which we have explored in this paper.

The approach of Ter-Martirosyan ¹⁰⁾ seems to us slightly different from Gribov's and closer to what we do, in the sense that diffractive events explicitly add to inelastic ones, and renormalize the Pomeron at $J=1$. Again, $g_P(0)=0$ and the constant asymptotic σ_{tot} is reached from below.

In the approach of Chew and collaborators ¹⁹⁾, all contributions are positive, cuts as well as the triple Pomeron. Having no negative damping contributions, they start with a Pomeron below $J=1$ and ensure that renormalization corrections are sufficiently small, so as not to overcome the bound one. Therefore, in particular the triple Pomeron vertex has to vanish for $\alpha_P(0) \rightarrow 1$. Its contribution to the absorptive part takes then the form $(A-B/\ln s)^{10)}$, $A, B > 0$, and may give a local increase of the total cross-section, depending on compensations with the positive elastic cut.

Summing up, all these different approaches are theoretically appealing, if total cross-sections are asymptotically constant, and $g_P(0)=0$. Our attempt - in a more modest framework - has been to correlate the possible failure of these two properties. We are confident that better experimental data will soon indicate more clearly which direction must be followed.

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TABLE : Slopes of $d\sigma_{el}/dt$ calculated from the parametrization (5.4).

ISR momentum	$\frac{\sigma_1}{\sigma_0} \ln \frac{s}{s_0}$	t = 0	t = 0.05	t = 0.4
11.8	0.21	11.9	11.7	10.7
15.4	0.27	12.4	12.0	10.8
22.6	0.35	12.9	12.5	10.9
26.6	0.37	13.1	12.6	11.0

A P P E N D I X

CUT VS CONTRIBUTIONS OF DIFFRACTIVE EVENTS

It is well known ^{18),21),22)} that there is a relation between the vanishing of $g_P(0)$ and the absence of a multiplicative fixed pole in the Pomeron particle amplitude. We want to discuss here this relation in a perturbative treatment of diffractive events, and identify the pole and cut contributions if $g_P(0) = 0$.

The whole overlap function M' defined as the contribution from events having one large rapidity gap, is given by (cf., Fig. 1) :

$$M' = \frac{1}{16\pi} \int \frac{ds_1}{s_1} \int \frac{ds_2}{s_2} \int dt \sigma_P(s_1, t) \sigma_P(s_2, t) \left(\frac{s}{s_1 s_2} \right)^{2\alpha_P(t)-2} \quad (A.1)$$

Let us take for the particle Pomeron absorptive amplitude the simplified parametrization

$$\sigma_P(s, t) = \gamma_P^2(t) \delta(s-1) + \gamma_P(0) g_P(t) s^{\alpha_P(0)-1} \Theta(s-1) \quad (A.2)$$

where we have assumed for simplicity a single scale $s_0 \simeq 1 \text{ GeV}^2$ and we have omitted non-leading terms and other low energy contributions. This simple form contains already a fixed and a moving pole. Indeed one has

$$\int_0^s ds_1 \frac{s_1^{-2\alpha_P(t)+1}}{s_1} \sigma_P(s_1, t) \underset{s \rightarrow \infty}{\simeq} N(t) + \frac{\gamma_P(0) g_P(t) s^{\alpha_P(0)-2\alpha_P(t)+1}}{\alpha_P(0) - 2\alpha_P(t) + 1} \quad (A.3)$$

where

$$N(t) = \gamma_P^2(t) - \frac{\gamma_P(0) g_P(t)}{\alpha_P(0) - 2\alpha_P(t) + 1} \quad (A.4)$$

is, by definition, the fixed pole residue. Equations (A.3) and (A.4) show the known fact that the moving pole has a spurious singularity at $\alpha_P(0) = 2\alpha_P(t)-1$, that can be cancelled either by an ad hoc zero of $g_P(t)$, or by an opposite singularity in the fixed pole, which is therefore multiplicative.

Inserting Eq. (A.2) into (A.1), we get

$$M' = M'_{el} + M'_1 + M'_2 \quad (A.5)$$

where ($Y \equiv \ln s$)

$$M'_{el}(s) = \frac{1}{16\pi} \int dt \gamma_P^4(t) s^{2\alpha_P(t)-2}$$

$$M'_1(s) = \frac{2\gamma_P(0)}{16\pi} \int dt \gamma_P^2(t) g_P(t) \frac{s^{\alpha_P(0)-1} - s^{2\alpha_P(t)-2}}{\alpha_P(0) - 2\alpha_P(t) + 1} \quad (A.6)$$

$$M'_2(s) = \frac{\gamma_P^2(0)}{16\pi} \int dt \frac{g_P^2(t)}{\alpha_P(0) - 2\alpha_P(t) + 1} \left[Y s^{\alpha_P(0)-1} - \frac{s^{\alpha_P(0)-1} - s^{2\alpha_P(t)-2}}{\alpha_P(0) - 2\alpha_P(t) + 1} \right]$$

It is clear from these formulae that the cut contribution $s^{2\alpha_P(t)-2}$ occurs with a discontinuity $N^2(t)$.

If $\alpha_P(0)=1$ and $g_P(0) \neq 0$, the cut becomes singular and cancels the singularity of the pole term in M'_1 and M'_2 . Indeed, the whole contribution is finite, and if $\alpha' \neq 0$ is asymptotically given by

$$M'_{el}(s) \simeq \gamma_P^4(0) / 32\pi \alpha' Y$$

$$M'_1(s) \simeq \gamma_P^3(0) g_P(0) \ln Y / 16\pi \alpha' \quad (A.7)$$

$$M'_2(s) \simeq \gamma_P^2(0) g_P^2(0) Y \ln Y / 32\pi \alpha'$$

Note that in this case one cannot separate the pole and cut contributions in the asymptotic behaviour. In the limit $\alpha' \rightarrow 0$ and if $\alpha_P(0)=1-\epsilon$ one could recover the results given in the text (Sections 2 and 4), provided the perturbation theory in the J plane is applicable, i.e., $g \ll \epsilon$ (cf., Section 4).

If $\alpha_P(0)=1$, but $g_P(t)$ has at least a linear zero at $t=0$, the fixed pole is not singular, and

$$N(0) = \gamma_P^2(0) - \Gamma \gamma_P(0) ; \quad \Gamma = \lim_{t \rightarrow 0} \frac{g_P(t)}{2\alpha' |t|} \quad (\text{A.8})$$

In this case one has the asymptotic behaviour

$$M'_2(s) \simeq \gamma_P^4(0) / 32 \pi \alpha' Y + \dots$$

$$M'_1(s) \simeq \frac{\gamma_P(0)}{8\pi} \int \frac{dt}{2\alpha' |t|} \frac{\gamma_P^2(t) g_P(t)}{2\alpha' |t|} - \frac{2\gamma_P(0)\Gamma}{32\pi\alpha'Y} + \dots (\text{A.9})$$

$$M'_2(s) \simeq \Delta \gamma_P^2(0) Y - \frac{\gamma_P^2(0)}{16\pi} \int dt \frac{g_P^2(t)}{(2\alpha' t)^2} + \frac{\gamma_P^2(0)\Gamma^2}{32\pi\alpha'Y}$$

By collecting the $1/Y$ terms, we see that the cut contribution is

$$M'_{\text{cut}} \simeq \frac{N(0)^2}{32 \pi \alpha' Y} \quad (\text{A.10})$$

The constant terms can be interpreted as residue renormalizations, while the term in Y comes from a double pole in the J plane, and therefore represents a first order trajectory renormalization of the amount Δ (cf., footnote p.10).

The fact that Δ gives a renormalization can be proved by a t channel iteration procedure. If we start with a Pomeron below one (our "inelastic" M_0) still having $g_P(0)=0$, we can impose the renormalized Pomeron to be at $\alpha_P(0)=1$ ¹⁰⁾ and identify it as the one responsible for diffractive events. The solution of the integral equation represented in Fig. 1 is then given in the J plane by

$$M(J,0) = \frac{\left(G_0 + \frac{1}{16\pi} \int dt \frac{\gamma_P^2(t) g_P(t)}{J-1-2\alpha't} \right)^2}{J-1+\varepsilon - \frac{1}{16\pi} \int \frac{dt}{J-1-2\alpha't} \frac{g_P^2(t)}{J-1-2\alpha't}} \quad (\text{A.11})$$

The condition that the output pole has $\alpha_P(0)=1$ implies that

$$\xi = \Delta = \frac{1}{16\pi} \int \frac{dt}{2\alpha' |t|} g_P^2(t) \quad (A.12)$$

which gives

$$M(J,0) = \frac{\left(G_0 + \frac{1}{16\pi} \int \frac{dt}{J-1-2\alpha' t} g_P^2(t) \right)^2}{(J-1) \left(1 + \frac{1}{16\pi} \int \frac{dt}{2\alpha' |t| (J-1-2\alpha' t)} g_P^2(t) \right)} \quad (A.13)$$

From Eq. (A.13), by adding the elastic overlap function, one can easily recover the cut contribution given in Eq. (A.10), as well as the residue renormalization, in a power expansion in $g_P(t)$.

We have discussed so far only the t channel iteration procedure, and therefore obtained a positive cut contribution identical to the one of Abarbanel, Chew and co-workers¹¹⁾. The introduction of elastic absorption will change the sign of the elastic contribution $\gamma_P^2(0)/Y$ ⁷⁾, but will only modify in normalization, not in sign, the triple Pomeron contribution $(-2\gamma_P^3(0)\Gamma)/Y$, which therefore still increases with energy, (cf., footnote p.10). We notice that in Gribov's approach, the cut contribution is the same as in Eq. (A.10), but with a minus sign. In other words, both the elastic contribution $\gamma_P^2(0)$ and the triple Pomeron contribution $(-2\gamma_P^3(0)\Gamma)$ to the fixed pole residue are reversed in sign.

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FIGURE CAPTIONS

Figure 1 : Iterative structure of diffractive exchanges in the inelastic overlap function M (before absorption).

Figure 2 : Computed values of $a(b,s_1)$, $\delta a(b,s_1,s_2)$ from the parametrization (5.8) and of $m(b,s_1)$, $\delta m(b,s_1,s_2)$ in the eikonal (dashed line) and Gottfried-Jackson (dashed-dotted line) absorption schemes.

Figure 3 : Dual diagrams which give rise to
a) the Pomeron singularity and
b) its first iteration.

$$\begin{aligned}
 & \text{Diagram (M)} = \text{Diagram } G_0 \text{ } D_0 \text{ } G_0 \text{ (M}_0\text{)} + 2 \text{ Diagram } g \text{ (M}_1\text{)} + \\
 & + \text{Diagram } \text{---} + \text{Diagram } D_1 \text{ } \text{---} + \dots
 \end{aligned}$$

The diagram shows a series of Feynman diagrams representing a perturbative expansion. The first row shows a circle with four external lines, labeled (M), equal to a sum of three terms. The first term is a diagram with two vertices labeled G_0 connected by a wavy line labeled D_0 , with four external lines, labeled (M₀). The second term is a coefficient 2 multiplied by a diagram with two vertices labeled g connected by a wavy line, with four external lines, labeled (M₁). The second row continues the series with a plus sign followed by a diagram with two vertices connected by a wavy line, each with two external lines. This is followed by a plus sign and a diagram with two vertices connected by a wavy line, each with two external lines, labeled D_1 . The series ends with a plus sign and an ellipsis.

FIG.1

FIG. 2

FIG.3