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COULOMB EFFECTS IN OFF-SHELL $\pi\pi$ SCATTERING

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A B S T R A C T

Coulomb effects are shown to be strongly suppressed in the off-shell $\pi\pi$ scattering accessible from $\pi N \rightarrow \pi\pi N$ data, and hence Coulomb interference effects cannot be readily extrapolated to the pion exchange pole.

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1. - INTRODUCTION

Coulomb effects in elastic scattering are useful for determinations of the phase of the strong interaction amplitude from interference with the real one-photon exchange contribution at small momentum transfers. Hopes have been expressed ¹⁾ to exploit this Coulomb-nuclear interference in $\pi\pi$ scattering at low masses where the strong interaction amplitude is not too big ; in particular for $\pi^-\pi^+ \rightarrow \pi^-\pi^+$ below 500 MeV and for $\pi^+\pi^+ \rightarrow \pi^+\pi^+$ below 700 MeV. The on-shell Coulomb effects can readily be calculated. For $\pi^-\pi^+$ scattering, the result is

$$f_c = -\frac{e^2}{2\sqrt{s}} \left(\frac{s-u}{t} + \frac{t-u}{s} \right) \quad (1)$$

$$f_N = \frac{1}{2} \sum_{\ell, I} (2\ell+1) e^{i\delta_\ell^I} \sin \delta_\ell^I C_I P_\ell(\cos \Theta) \quad (2)$$

$$\frac{d\sigma}{d\Omega} = |f_c + f_N|^2 \quad (3)$$

where $C_I = 2/3, 1, 1/3$ for $I=0,1,2$, respectively. As an example, at $\sqrt{s}=500$ MeV, and using phase shifts ²⁾ $\delta_0^0 = 50^\circ$, $\delta_1^1 = 8.6^\circ$ and $\delta_0^2 = -7^\circ$, the result is shown on Fig. 1. Thus the Coulomb amplitude due to the first term of Eq. (1) is appreciable for small t and shows a constructive interference with the positive nuclear phase shifts.

The most useful source of data on $\pi\pi$ scattering comes from production experiments at high energies (such as $\pi^-p \rightarrow \pi^-\pi^+n$ at 17.2 GeV/c ³⁾, $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}$ at 7 GeV/c ⁴⁾, etc.) where one pion is virtual (exchanged) with a four-momentum $t_{be} < 0$ in the physical region of the production process. From a study of the amplitude structure, it is possible to extrapolate to the pion exchange pole and extract $\pi\pi$ cross-sections. The Coulomb effects in the $\pi N \rightarrow \pi\pi N$ (or $\pi N \rightarrow \pi\pi\Delta$) data will be very different, however, from those for on-shell $\pi\pi$ scattering. Thus the $1/t^2$ behaviour of $d\sigma/d\Omega$ is only valid for photon couplings to on-shell pions. When a photon transition between pions of different masses ($t_{be} < 0$ and μ^2 , respectively) is involved, the process will be more like Coulomb excitation with a $1/t$ behaviour of $d\sigma/d\Omega$. The next section gives a

general treatment of Coulomb excitation as appropriate to single pion production. Then follows a simple gauge invariant model for Coulomb effects in $\pi N \rightarrow \pi \pi N$ which illustrates the cancellation of Coulomb effects off-shell and bears out the conclusion that the physical region $\pi \pi$ distributions have much reduced Coulomb effects compared to on-shell $\pi \pi$ scattering.

2. - INELASTIC COULOMB EFFECTS

Consider the one-photon exchange contribution to the process $a + b \rightarrow c + f$ where $m_a = m_c = m_\pi = \mu$, $m_b = m$ and where f is "anything" of mass $\sqrt{s_f}$. Following the standard kinematics of inclusive electroproduction via W_1 and W_2 , one arrives at the expression (neglecting μ^2/m^2 judiciously) valid for small t :

$$\frac{d^2\sigma}{dt ds_f} = \frac{\alpha}{\pi} \frac{(-t_M - t)}{t^2} \left(\frac{s - s_f}{s - m^2} \right) \frac{\sigma_T(s_f)}{s_f - m^2} + O(t^0) \quad (4)$$

where $t_M = -\mu^2 (s_f - m^2)^2 / s^2$ and σ_T is the transverse photo-absorption cross-section on target b . For large s , t_M is small and the cross-section has effectively a $1/t$ peak at $t=0$ in contrast to the $1/t^2$ elastic Coulomb peak. This suppression of the $1/t^2$ peak to $1/t$ is valid for $f = \text{anything}$ and, since $\sigma_T(s_f)$ is a sum of moduli, the suppression will also be valid for any substate of f (e.g., πN) in any specific region of phase space (e.g., small t_{be}) provided that its relative contribution to $\sigma_T(s_f)$ is not negligible.

3. - A GAUGE INVARIANT MODEL FOR COULOMB EFFECTS

It is instructive to illustrate the cancellation of Eq. (4) at $t_{ac} = 0$ for $\pi^- p \rightarrow \pi^- \pi^+ n$ by a simple model. The diagram of Fig. 2a is implicitly assumed in a Chew-Low extrapolation but is not gauge invariant. The simplest set of gauge invariant diagrams to consider are those of Figs. 2a and 2b. The kinematic region of interest in $\pi\pi$ studies is at high energy s_{ab} , low subenergy s_{cd} and small momentum transfer t_{be} . At $p_{lab} = 17.2$ GeV/c, for example, when $s_{cd} = 0.25$ GeV² the kinematically allowed range of s_{de} is from 3.7 GeV² to 30 GeV². Thus a treatment of Coulomb effects in $\pi N \rightarrow \pi\pi N$ requires effectively a model for electroproduction (at $q^2 = t_{ac} \sim 0$) of single pions on nucleons at intermediate and high energies s_{de} and small momentum transfer t_{be} . The simple elementary π exchange (Fig. 2a) plus nucleon Born term (Fig. 2b) should serve as a rough guide for small values of t_{be} and moderate energies s_{de} . Form factors are set equal to 1, since the kinematic region where t_{ac} and t_{be} are both small is to be studied. Only that part of the nucleon Born term is retained, which is in the same nucleon spin amplitude as the π exchange, and so may interfere with it. Evaluating diagrams 2a and 2b gives, up to an over-all normalization,

$$\begin{aligned}
 F_C &= A + B_1 + B_2 \\
 A &= \frac{s_{cd} - u_{ad}}{t_{ac}} \cdot \frac{\sqrt{-t_{be}}}{\mu^2 - t_{be}} \\
 B_1 &= \frac{s_{ab} - u_{bc}}{t_{ac}} \cdot \frac{\sqrt{-t_{be}}}{m^2 - s_{de}} \\
 B_2 &= \frac{\sqrt{E_e + m} \sqrt{2m}}{t_{ac} (m^2 - s_{de})} \left\{ (E_f - m)(P_a^z + P_c^z) - (E_a + E_c)P_f^z \right\}
 \end{aligned} \tag{5}$$

for the spin amplitude proportional to $\chi_b \sigma_z \chi_e$, where E and p are the laboratory (b at rest) energy and three-momenta and the z direction is along p_e .

For $t_{ac} \sim 0$, there is a cancellation ^{*)} between A and B_1 which reproduces the suppression shown in Eq. (4). Relative to A and B_1 , B_2 is only important ^{**)} for $|t_{be}| < 0.01$ and it contributes a $1/t_{ac}^2$ term to $d^2\sigma/dt_{ac} ds_f$. On integration over t_{be} , however, this reduces to a $1/t_{ac}$ behaviour as the previous section would require. This simple model thus illustrates the cancellations in Coulomb effects coming from gauge invariance considerations.

To estimate Coulomb-nuclear interference effects in $\pi N \rightarrow \pi \pi N$, one needs a simple model for the strong interaction amplitudes. Between $t_{be} = -0.01$ and -0.1 , the dominant amplitude is well approximated ⁵⁾ by elementary pion exchange. For illustration, a strong interaction amplitude with elementary pion exchange, the form factor set equal to 1, and on-shell $\pi\pi$ phase shifts is used. In order to compare with on-shell $\pi\pi$ scattering, an off-shell $\pi\pi$ differential cross-section is defined from $\pi N \rightarrow \pi \pi N$ observables (angles θ, ϕ in the Gottfried-Jackson frame)

$$\frac{d\bar{\sigma}}{d\Omega}(s_{cd}, t_{be}) = \int_0^{2\pi} \frac{d\phi}{2\pi} \cdot \frac{(t_{be} - \mu^2)^2}{-t_{be}} \cdot \frac{1}{K} \cdot \frac{d^4\sigma}{dt_{be} ds_{cd} d\Omega}$$

(6)

$$K = \left(\frac{g^2}{4\pi}\right) \frac{q_{cd} \sqrt{s_{cd}}}{4\pi m^2 P_{LAB}^2}$$

*) This can be seen as follows. In the physical region $t_{ac} < -\mu^2(\mu^2 - t_{be})^2/s_{cd}^2$ from considering t_{min} for the subprocess $\pi\pi \rightarrow \pi\pi$ and $t_{ac} < -\mu^2 \cdot (s_{de} - m^2)^2/s_{ab}^2$ from considering the over-all process $\pi N \rightarrow \pi + \text{anything}$. The minimum value of $|t_{ac}|$ for fixed s , s_{cd} and t_{be} is thus given by choosing s_{de} such that the two limits are equal. Thus $(s_{de} - m^2)/s_{ab} = (\mu^2 - t_{be})/s_{cd}$ which yields the required cancellation between A and B_1 .

**) In a model with scalar baryons, only A and B_1 are present (with no $\sqrt{-t_{be}}$ factors) and the cancellation between them gives a $1/t_{ac}$ behaviour to the differential cross-section for all values of t_{be} .

For the simple strong interaction amplitude discussed above, $d\overline{\sigma}/d\Omega$ is the same as $d\sigma/d\Omega$ for on-shell $\pi\pi$ scattering. This is not true for the Coulomb and Coulomb-nuclear interference effects, however, as shown in Fig. 1. Retaining only the diagram of Fig. 2a in the Coulomb calculation would have yielded the same Coulomb interference effects off-shell as on-shell with the definition of Eq. (6). The constraint of gauge invariance, which necessitates a cancellation at $t_{ac} \sim 0$ for inelastic Coulomb transition, or equivalently the introduction of a diagram such as that of Fig. 2b, results in the strong suppression of Coulomb effects in virtual $\pi\pi$ scattering illustrated in Fig. 1.

Since the Coulomb effects are so suppressed, a more refined calculation is not presented. The ingredients would be : (i) a parametrization of the strong interaction production amplitudes obtained from amplitude analyses ⁵⁾ ; (ii) a model ⁶⁾ for shallow ($q^2 \sim 0$) electroproduction of single π mesons at intermediate and high energies. The nuclear and Coulomb contributions to the four nucleon spin amplitudes for each value of the variables s_{ab} , s_{cd} , t_{ac} , θ and t_{be} could then be calculated - and hence any observable interference effect. The general feature of a $1/t_{ac}$ behaviour of the Coulomb contribution to the cross-section, that has been illustrated here, would, of course, remain.

In conclusion, Coulomb effects in $\pi N \rightarrow \pi\pi N$ or $\pi N \rightarrow \pi\pi\Delta$ data will be strongly suppressed compared to the effects in on-shell scattering, and attempts to identify Coulomb interference effects in scattering angular distributions obtained by extrapolating $\pi N \rightarrow \pi\pi N$ or $\pi N \rightarrow \pi\pi\Delta$ data to the pion pole must be regarded with suspicion.

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FIGURE CAPTIONS

Figure 1 The curves marked on-shell are the $\pi\pi$ elastic scattering differential cross-sections against $\cos\theta$ or momentum transfer t_{ac} at 500 MeV centre-of-mass energy. The inset shows the full angular distribution. The strong interaction amplitude f_N is taken from the phase shifts quoted in the text and the Coulomb amplitude f_c is calculated according to Eq. (1). In a simple model for $\pi^-p \rightarrow \pi^- \pi^+ n$ at 17.2 GeV/c, the strong interaction amplitudes come from combining elementary π exchange and on-shell $\pi\pi$ scattering (reasonable for $0.01 < -t_{be} < 0.1$) while the Coulomb amplitudes come from a term combining elementary π exchange and on-shell $\pi\pi$ Coulomb effects (Fig. 2a) and a second term (Fig. 2b) to preserve gauge invariance. Combining these nuclear and Coulomb amplitudes and defining an off-shell $\pi\pi$ differential cross-section as a function of t_{be} as in Eq. (6) of the text results in the curve shown marked off-shell. The curve is almost identical for t_{be} values from -0.01 to -0.1 except for the kinematic limits on t_{ac} represented by the cross-hatched lines for different values of t_{be} . The Coulomb interference effects are greatly reduced off-shell and cannot be extrapolated to the pion pole in the same way as the nuclear contribution.

Figure 2 Two diagrams contributing to the Coulomb interference effects in $\pi^-p \rightarrow \pi^- \pi^+ n$ at small t_{ac} .

FIG.1

(a)

(b)

FIG. 2