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THE MISSING GAUGE CONDITIONS FOR
THE DUAL FERMION EMISSION VERTEX AND THEIR CONSEQUENCES

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A B S T R A C T

We find that the dual vertex for emitting a ground state fermion of mass m converts between generalized mesonic G gauges and generalized fermionic F gauges. These generalized gauges consist of infinite linear combinations of F 's and G 's which are, roughly speaking, square roots of Virasoro gauges and exhibit partial reflection and transmission behaviour at the vertex.

One of these Ward identities allows us to re-express the amplitude for fermion re-emission in the F_2 representation where the limit $m=0$ is well defined. If $m=0$ and $d=10$ certain meson channels are, we show, saturated by physical (ghost free) states.

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A grave difficulty of the dual resonance theory is that there has been no satisfactory dual theory of fermions (whether they are to be baryons or quarks ¹⁾). One possibility (originally due to Ramond ²⁾) describes the propagation of a single fermion emitting mesons ^{3),4)} (see Fig. 1a for a pictorial representation: the solid line is the fermion and the wavy lines the mesons).

However, when the scattering of two fermions is considered this theory runs into the difficulty that it is very difficult to ensure that the correct physical ghost free spectrum ⁵⁾ of mesons is exchanged between the two fermions (on the meson line indicated in Fig. 1b).

One way of enforcing the desired spectrum would be to insert the known meson physical state projection operator ⁶⁾ into the residues of the meson poles concerned. Since this projection operator is not Lorentz invariant it is not clear that the resultant amplitude is properly Lorentz covariant unless one can use some gauge conditions on the line concerned to restore the Lorentz covariance. Unfortunately it is precisely such gauge conditions which have eluded previous studies ^{7),8)} of the fermion emission vertex which appears in Fig. 1b adjacent to the indicated meson line.

In this note we shall find the missing gauge conditions. The fermion emission vertex converts into each other gauges which are infinite linear combinations of G_r 's and F_n 's in the mesonic and fermionic sectors respectively. These gauge conditions have been overlooked in the past because of their apparent complexity but, in fact, they have two features which make them rather simple and useful.

- i) The anticommutator of any pair of like gauges is a finite linear combination of the conventional Virasoro gauges already known.
- ii) All the gauge conditions exhibit a transmission and reflection behaviour - a finite linear combination of incident G_{-r} 's (F_{+n} 's) is partly reflected into an infinite linear combination of G_{+r} 's (F_{-n} 's) and partly transmitted into an infinite linear combination of F_{-n} 's (G_{+r} 's).

We shall consider the process in Fig. 1c whose amplitude was found in Ref. 7) by a duality argument based on what we call the " $\mathcal{F}_1$ " formalism. One of the above-mentioned gauge conditions can be used to convert the amplitude to an " $\mathcal{F}_2$ " formalism which is more convenient for the analysis of the spectrum of states and which reveals that the $\mathcal{F}_1$ formalism is singular in

the limit of zero ground state fermion mass m . Finally, we use another of our gauge conditions to show that if $m=0$ the transverse physical states which saturate purely mesonic amplitudes in ten space-time dimensions also saturate the residues of the meson pole indicated in Fig. 1c.

This is an important consistency check which is necessary for the successful calculation of the amplitude in Fig. 1b.

First define respectively arbitrary linear combinations of Ramond fermionic $(2), (3), (4)$ and Neveu Schwarz (NS) mesonic gauges (9) :

$$F_\phi = \frac{i}{\sqrt{2}} \oint_{\Gamma_F} \frac{dx}{2i\pi x} \phi(x) \Gamma(x) \cdot P(x) \quad (1)$$

$$G_\phi = \oint_{\Gamma_G} \frac{dx}{2i\pi x} \phi(x) H(x) \cdot P(x) \quad (2)$$

$H(x)$ is the usual NS field without γ_5 . It, γ_5 and the fields $\Gamma(x)$ and $P(x)$ are as defined in Ref. 7). The problem is to find the class of functions $\phi(x)$ which yield Ward identities. The contours of integration Γ_F and Γ_G will be specified later when we know ϕ , but must encircle the origin and lie on a single sheet of the integrand. Of course

$$F_{x^n} = F_n \quad n \text{ an integer}$$

$$G_{x^r} = G_r \quad r \text{ a half integer}$$

If L_ϕ denotes the obvious generalization of the Virasoro generators [so $L_{x^n} = L_n$ in the notation of Refs. 7) and 8)] we find

$$\begin{aligned} \{ F_{\phi_1}, F_{\phi_2} \} &= 2 L_{\phi_1 \phi_2} + 2 c_F(\phi_1, \phi_2) \\ \{ G_{\phi_1}, G_{\phi_2} \} &= 2 L_{\phi_1 \phi_2} + 2 c_G(\phi_1, \phi_2) \end{aligned} \quad (3)$$

where the c numbers differ only with respect to the integration contours which correspond to those in F_ϕ and G_ϕ respectively:

$$c_\alpha(\phi_1, \phi_2) = \frac{D}{4} \oint_{\Gamma_\alpha} \frac{dx}{2i\pi x} \phi_1(x) \left[\left(x \frac{d}{dx} \right)^2 - \frac{1}{4} \right] \phi_2(x) \quad (4)$$

$\alpha = F, G$

D is the space-time dimension (ultimately taken to be 10).

We shall now see what class of functions ϕ should be considered by the following simple argument: from (3) we see

$$[G_\phi]^2 = L_{\phi^2} + c_\phi(\phi, \phi)$$

If G_ϕ is to be a gauge operator working to the left then L_{ϕ^2} must be one automatically: but we already know all the gauges of this, the Virasoro form - for example the simplest is

$$L_0 - L_{-1} = L\left(1 - \frac{1}{x}\right)$$

It then follows that

$$G_{\sqrt{1-\frac{1}{x}}} = \sqrt{L_0 - L_{-1}}$$

is a good candidate for a gauge. In fact this will be correct and the choice

$$\phi(x) = f(x) \sqrt{1 - \frac{1}{x}} \quad (5)$$

will give the most general possible gauge where $f(x)$ is a function analytic in a domain including the points 0 and 1 with the exception of possible poles at 0. Remembering that $H(x)$ has a square root branch point at $x=0$ and $\Gamma(x)$ does not we see that the contours Γ_F and Γ_G illustrated in Fig. 2 do precisely satisfy the previously mentioned requirements.

Suppose $V_0(1)$ denotes the untwisted vertex for the emission of a ground state fermion of momentum k and mass m

$$V_0(1) = e^{-L_1^d} \tilde{W}_0(1) : e^{-ik \cdot Q(1)} :$$

in the notation of Ref. 7), so that it converts between the fermionic sector on the left and the mesonic sector on the right. Then the most general Ward identity satisfied by $V_0(1)$ is

$$F_\phi V_0(1) + V_0(1) \left[\gamma_5 G_\phi - i \frac{k \cdot \gamma}{\sqrt{2}} f(1) \right] = 0 \quad (6)$$

This follows from Eq. (5.2) of the article by Corrigan and Goddard⁸⁾ when one takes into account that they considered a twisted vertex rather than the untwisted one.

Iterating Eq. (6) we have

$$\left[\{ F_{\phi_1}, F_{\phi_2} \} + m^2 f_1(1) f_2(1) \right] V_0(1) = V_0(1) \{ G_{\phi_1}, G_{\phi_2} \} \quad (7)$$

Because of (3) this is a Virasoro gauge identity. More precisely using (3) it gives

$$\left[L_{\phi_1 \phi_2}, V_0(1) \right] = \left[\frac{D}{16} - \frac{m^2}{2} \right] f_1(1) f_2(1) V_0(1) \quad (8)$$

$L_{\phi_1 \phi_2}$ is to be interpreted in the appropriate way depending on whether it is to the left or right of $V_0(1)$. We have used the fact that $C_G - C_F$ is the integral of (4) around the point 1 and can be evaluated to give $D/16 f_1(1) f_2(1)$ since the integrand has a simple pole there. Since $\phi_1 \phi_2 = ((1-x)/x) f_1(x) f_2(x)$ we see that (8) is the most general possible Virasoro gauge condition. The quantity in the square bracket on the right-hand side of (8) is the expectation value of L_0 for the emitted ground state meson and was taken to be 1 in the previous work so that (8) generalizes those results.

A linearly independent set of Ward identities (5), (6) is found by considering the individual term in the Laurent expansion of $f(x)$ i.e.,

$$\phi_n(x) = x^n \sqrt{1 - \frac{1}{x}} = x^n \frac{\sqrt{x-1}}{\sqrt{x}} \quad n=0 \pm 1 \pm 2 \dots \quad (9)$$

Let us return to the case $\phi_0(x) = \sqrt{1-1/x} = \sqrt{(x-1)/x}$. Then

$$G_{\phi_0} = i \left\{ G_{-\gamma_2} - \sum_{n=0}^{\infty} (-1)^n \binom{\gamma_2}{n+1} G_{n+\gamma_2} \right\} \quad (10)$$

$$F_{\phi_0} = \sum_{n=0}^{\infty} (-1)^n \binom{\gamma_2}{n} F_{-n} \quad (11)$$

since on Γ_G $|x| < 1$ while on Γ_F $|x| > 1$ and we must expand $\phi_0(x)$ respectively in powers x^n and x^{-n} . Our interpretation of (6), (10) and (11) is that a $G_{-\frac{1}{2}}$ incident from the right is partly reflected as an infinite sum of $G_{n+\frac{1}{2}}$'s $n \geq 0$ and partly transmitted as an infinite sum of F_{-n} $n \geq 0$ which could eventually annihilate on tree attached to the right and left respectively.

It is this reflection and transmission behaviour which has prevented these gauges from being recognized in the past. Recently, the understanding of how spurious states formed by a physical tree can decouple from physical Pomerons ¹⁰⁾ taught us that such behaviour is perfectly acceptable and indeed useful (as we shall see again later on with respect to decouplings from fermion anti-fermion states).

All the other gauge conditions have a similar interpretation. If $n \leq 0$ we have a finite number of G gauges incident from the right and if $n > 0$ finite number of F gauges incident from the left. In each case there is an infinite number of reflected and transmitted gauges.

For example for $n > 0$, (6) and (9) lead to

$$\begin{aligned} & \sum_{r=0}^{n-1} (-1)^r \binom{\frac{1}{2}}{r} F_{n-r} V_0(1) \\ &= - \sum_{r=n}^{\infty} (-1)^r \binom{\frac{1}{2}}{r} F_{n-r} V_0(1) - V_0(1) i \gamma_5 \sum_{r=0}^{\infty} (-1)^r \binom{\frac{1}{2}}{r} G_{n+r-\frac{1}{2}} \\ &+ V_0(1) i \frac{k \cdot \gamma}{\sqrt{2}} \end{aligned} \quad (12)$$

The left-hand side consists of the n incident F 's, the right-hand side of reflected F 's and transmitted G 's respectively.

The vertex $V_0(1)$ whose gauge properties we have just discussed was originally derived ⁷⁾ by dualizing amplitudes written down in what we shall call the $\mathcal{F}_1$ picture ⁹⁾. That is the amplitude for the process in the Fig. 1b was shown to be proportional to

$$\langle R1 | V_0(1) u(k) | NS1 \rangle \quad (13)$$

where $|NS1\rangle$ and $|R1\rangle$ denote respectively $\mathcal{F}_1$ Neveu-Schwarz and

Ramond trees to be defined below [actually (13) was only derived when $D/16 - m^2/2 = 1$ but the analysis can be repeated in the case of general m]. We shall also define trees $|R2\rangle$ and $|NS2\rangle$ in the $\mathcal{F}_2$ picture. For purely mesonic processes the nomenclature is standard ⁹⁾, but in the Ramond case (which was actually first written down in the $\mathcal{F}_2$ formalism ^{2),3)}) it is new and will be justified by Eq. (24) which converts between the two pictures and will be proved below. The definitions are

$$|NS1\rangle = \frac{1}{L_0-1} V_{NS} \frac{1}{L_0-1} \dots V_{NS} G_{-1/2} |0\rangle \quad (14)$$

$$|NS2\rangle = \frac{1}{L_0-1/2} V_{NS} \frac{1}{L_0-1/2} \dots V_{NS} |0\rangle \quad (15)$$

$$|R1\rangle = \frac{1}{\bar{L}_0 + \frac{m^2}{2}} V_1 \frac{1}{\bar{L}_0 + \frac{m^2}{2}} \dots V_1 |0\rangle u \quad (16)$$

$$|R2\rangle = \frac{1}{F_0 - \frac{im}{\sqrt{2}}} V_2 \frac{1}{F_0 - \frac{im}{\sqrt{2}}} \dots V_2 |0\rangle u \quad (17)$$

where

$$V_1 = \{ F_0, V_2 \} \quad (18)$$

and for pion emission e.g., $V_2 = \Gamma_5 : e^{-ik \cdot Q(1)} :$ and $\bar{L}_0$ is a redefined fermion L_0 such that

$$\bar{L}_0 = F_0^2 = L_0 - \frac{D}{16} \quad (19)$$

Notice that in the NS case it is the propagators and states which differ between the two pictures while in the Ramond case it is the propagators and vertices. In the NS case we have the gauge conditions ⁹⁾

$$\left. \begin{aligned} G_r |NS2\rangle &\approx 0 \\ G_{-1/2} |NS2\rangle &\approx |NS1\rangle \end{aligned} \right\} \quad r \geq \frac{1}{2} \quad (20)$$

where $\approx$ means equal up to terms which will vanish on the mass shell or in matrix elements by the cancelled propagator arguments. The corresponding statements for Ramond trees are

$$\left. \begin{aligned} F_N |R1\rangle &\approx 0 \\ (F_0 - \frac{im}{\sqrt{2}}) |R1\rangle &\approx 0 \\ |R1\rangle &= |R2\rangle \end{aligned} \right\} \quad N \geq 1 \quad (21)$$

The last equation follows on substituting

$$V_1 = (F_0 + \frac{im}{\sqrt{2}}) V_2 + V_2 (F_0 - \frac{im}{\sqrt{2}})$$

[see Eq. (18)] and using cancelled propagator arguments. We shall now show that the gauge condition corresponding to ϕ_0 will allow us to convert the amplitude (13) for the process in Fig. 1c to the $\mathcal{F}_2$ picture. Taking $\phi = \phi_0$ in Eq. (6) and multiplying on the right by $\delta_5 u(k)$ we have, using the Dirac equation

$$(\gamma \cdot k - m) u(k) = 0 \quad (22)$$

$$(F_{\phi_0} + \frac{im}{\sqrt{2}}) V_0(1) \delta_5 u(k) = -V_0(1) u(k) G_{\phi_0} \quad (23)$$

Taking the matrix element

$$\langle R1 | \dots | NS2 \rangle$$

and using the facts that according to (10) and (20)

$$G_{\phi_0} |NS2\rangle \approx i |NS1\rangle$$

with, according to (11) and (21)

$$\langle R1 | (F_{\phi_0} + \frac{im}{\sqrt{2}}) \approx i\sqrt{2} m \langle R2 |$$

we see that

$$\langle R2 | V_0(1) \delta_5 u(k) | NS2 \rangle = \frac{-1}{\sqrt{2} m} \langle R1 | V_0(1) u(k) | NS1 \rangle \quad (24)$$

The right side is proportional to the amplitude (13) for the process in Fig. 1c originally derived in the $\mathcal{F}_1$ picture which is the one in which duality works easily. We now see, by Eq. (24) that the amplitude can also be written down in the $\mathcal{F}_2$ picture. Amazingly enough the vertices for fermion emission in the two pictures differ only by the factor $-\sqrt{2}m\gamma_5$. The $\mathcal{F}_2$ picture has two advantages; the gauge conditions and the structure of the physical states are simpler, and, what will be more important for us, the limit of zero fermion mass m is well defined (and non-vanishing). This is easily checked from the structure of the left-hand side of (20). It follows that the right-hand side has a non-singular limit, despite the singular factor $1/m$. The problem is that the operator formalism in the $\mathcal{F}_1$ space is singular in the limit since, e.g.,

$$\frac{1}{\sqrt{2}m} \langle 0 | V_0(1) | 0 \rangle = \frac{1}{\sqrt{2}m}$$

Eventually we shall be interested just in the case $m=0$ and so we shall avoid this difficulty by henceforth working in the $\mathcal{F}_2$ space where the factor m is explicitly divided out.

First let us record the gauge conditions involving the fermion emission vertex in the $\mathcal{F}_2$ picture. Using Eqs. (6), (9) and (21) we see

$$\begin{aligned} \langle R2 | V_0(1) \gamma_5 u(\kappa) G_{\phi_n} &= - \langle R2 | \left[F_{\phi_n} - \frac{im}{\sqrt{2}} \right] V_0(1) u(\kappa) \\ &= \begin{cases} 0 & \text{if } n=0 \\ + \frac{im}{\sqrt{2}} \langle R2 | V_0(1) u(\kappa) & \text{if } n \geq 1 \end{cases} \end{aligned} \quad (25)$$

while using (6), (9) and (20)

$$\begin{aligned} F_{\phi_n} V_0(1) \gamma_5 u(\kappa) | NS2 \rangle &= V_0(1) \left(-G_{\phi_n} - \frac{im}{\sqrt{2}} \gamma_5 \right) u(\kappa) | NS2 \rangle \\ &= - \frac{im}{\sqrt{2}} V_0(1) \gamma_5 u(\kappa) | NS2 \rangle \quad \text{if } n \geq 1 \end{aligned} \quad (26)$$

Notice how the right-hand sides of these equations vanish if $m=0$. In other words if the fermion has zero mass our gauges will annihilate the trees above.

We now consider the question of whether the transverse meson states saturate the internal meson line just to the right of the fermion emission vertex in the Fig. 1c. To check this we must prove that

$$\langle R_2 | V_0(1) \gamma_5 u(k) | X \rangle = 0 \quad (27)$$

where

$$|X\rangle = (\mathcal{T}_{NS} - 1) \oint \frac{dx}{4\pi i x} x^{L_0 - \frac{1}{2}} V_{NS} \frac{1}{L_0 - \frac{1}{2}} \dots |0\rangle$$

Now, from the structure of $\mathcal{T}_{NS}$ ⁶⁾ which is the physical state projection operator known in ten dimension one can deduce by arguments parallel to those in the Appendix of the second paper in Ref. 10) that

$$|X\rangle = G_{-\frac{1}{2}} |Y_1\rangle + (G_{-\frac{3}{2}} + 2 L_{-1} G_{-\frac{1}{2}}) |Y_2\rangle \quad (28)$$

where $|Y_1\rangle$ and $|Y_2\rangle$ satisfy

$$G_r |Y_i\rangle = 0 \quad L_0 |Y_i\rangle = (L_0 + 1) |Y_i\rangle = 0 \quad (29)$$

$r \geq \frac{1}{2} \quad i = 1, 2.$

Since by (29)

$$G_{-\frac{1}{2}} |Y_1\rangle = -i G_{\phi_0} |Y_1\rangle$$

$$G_{-\frac{3}{2}} |Y_2\rangle = -i (G_{\phi_1} + \frac{1}{2} G_{\phi_0}) |Y_2\rangle$$

we have, using (28), (29), (25) and (8) with $\phi_1 \phi_2 = 1 - 1/x$, that expression (27) equals

$$\begin{aligned} & -i \langle R_2 | V_0(1) \gamma_5 u(k) G_{\phi_1} |Y_2\rangle \\ & = + \frac{m}{\sqrt{2}} \langle R_2 | V_0(1) u(k) |Y_2\rangle \end{aligned}$$

Thus the desired result Eq. (27) follows if $m=0$.

Thus we have verified an important consistency requirement, namely that the same positive definite resonance spectrum that factorizes purely mesonic amplitudes also factorises amplitudes involving a single fermion. That this should be so is extremely non-trivial in view of the different forms of Ward identity valid in the two cases, and in fact is only so if the ground state fermion has zero mass. Actually, we have not verified the result in all possible meson channels, but only in the one indicated in Fig. 1c, but feel it should not be difficult to generalize the result.

The conclusion that $m=0$ for the self consistency of the theory has now been reached by several different routes ^{11),12)}: Thorn was the first to suggest to us that it could lead to the decoupling proved above ¹²⁾. In our view the importance of the result above is that it is a necessary condition for the successful computation of amplitudes for two fermions (see Fig. 1b). In fact we can now write down a new candidate for the fermion-fermion scattering amplitude by using the $\mathcal{F}_2$ formalism that we have discovered:

$$\bar{u} \langle 0 | V_0(i) \gamma_5 u \frac{1}{L_0 - \frac{1}{2}} \bar{u} \gamma_5 V_0^+ | 0 \rangle u$$

It certainly lacks the most obvious unwanted states which plagued the amplitudes previously written down on the basis of the $\mathcal{F}_1$ formalism. Whether it is correct remains to be checked by seeing whether it is altered by inserting into its residues the physical state projection operator $\mathcal{T}_{NS}$ ⁶⁾. If it is altered one must check if the projected residues are Lorentz covariant when our new found gauge conditions are used.

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fig 1

fig 2