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MODIFIED MESON-NUCLEUS OPTICAL POTENTIALS

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A B S T R A C T

We argue that the energy dependence of the meson-nucleus optical potential should be treated in a more dynamical fashion. When this is done, some of the difficulties found in phenomenological applications are eliminated.

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It is customary to analyze the mass of experimental data on mesic atoms ¹⁾ and meson-nucleus scattering ²⁾ in terms of simple optical potentials. In many cases there remain serious anomalies ; the values of the potential parameters derived from a microscopic theory often differ significantly from the "best-fit" values. It is the object of this note to suggest that this might be due to the neglect of the energy dependence of these potentials.

In momentum space, the one-body optical potential has the form ³⁾

$$2\mu \tilde{V}_{opt} = -4\pi [Z f_p + N f_n] S(q) \quad (1)$$

where $S(q)$ is the nuclear form factor for momentum transfer q , f_p and f_n the amplitudes for the scattering from protons and neutrons of a particle of reduced mass μ . This can be derived in many different ways, but the basic argument is that for very weak scattering from the individual nucleons, the Born approximation for this potential should yield the simple impulse approximation. The correction terms to V are therefore of order f^2 . Unfortunately this criterion by itself is not sufficient to specify the equation into which V must be substituted. A familiar example of this ambiguity is met when the Klein-Gordon equation is used. If V is introduced in the same way as the Coulomb potential, then the equation will contain V^2 terms. Should these be kept or not ? It is impossible to answer unless one feeds in extra physical information, such as the scattering amplitude to order f^2 . The specific problem that we have to face is that the f are energy dependent and how does one introduce an energy dependent potential into quantum mechanics. Apart from the obvious answer of putting it on the right-hand side of the Schrödinger equation, the most consistent procedure is to assume that wherever the energy occurs in the free equation it should be replaced by the local energy in that potential. This minimal coupling scheme is very reasonable for the introduction of the Coulomb potential into a problem where there already exists an energy dependent potential ⁴⁾. When the apple hit Newton's head, it was the coefficient of restitution at the local energy that was relevant. In terms of a multiple scattering approach, this ansatz can only be made plausible in the absence of strong correlations between the nucleons in the nucleus. If we build up the optical potential by putting one nucleon at a time into the nuclear volume, then it is quite likely that it is the local energy in that region which is the important quantity. In our ansatz though, we are assuming that the local potential energy coincides with the average potential. There

is also a problem of the self-correlation kind ; it should be only the average potential from the other nucleons which is included for the scattering from a particular nucleon. In the opposite limit of strong repulsion between the nucleons, the nuclear amplitude only depends upon the physical nucleon amplitudes at the incident energy ⁵⁾.

Assuming that the ansatz is a valid approximation, then the simplest way of implementing it is to solve the implicit equation for V^M obtained by substituting into the functional energy dependence of V

$$V^M = V \{ E - V^M \} \quad (2)$$

before inserting V^M in the usual way into the Schrödinger equation.

We now investigate the effects of our prescription for the case of low energy pion-nucleus scattering. Up to a few hundred MeV, the nucleon amplitudes are dominated by s and p waves. We can thus take the isoscalar amplitude as

$$f = b + c \vec{k} \cdot \vec{k}' \quad (3)$$

where k (k') is the initial (final) pion momentum. If this form is used both on and off the energy shell, then the resulting optical potential defined by Eq. (1) is the Kisslinger potential ⁶⁾,

$$2\mu V_K = q(r) - \vec{\nabla} \alpha(r) \vec{\nabla} \quad (4)$$

We have here employed the standard notation

$$\begin{aligned} q(r) &= -4\pi A b \rho(r) \\ \alpha(r) &= -4\pi A c \rho(r) \end{aligned} \quad (5)$$

with ρ the nuclear density. If we neglect the energy variation of the parameters b and c , this form is energy independent, but velocity dependent. It could therefore be introduced directly into the Schrödinger equation.

An obvious alternative to this may be obtained by extrapolating off-shell with the amplitude

$$F = b + c (2E_\mu - \frac{1}{2} q^2) \quad (6)$$

Of course for physical pion-nucleon scattering, Eqs. (3) and (6) are completely equivalent. In this case the V depends only upon q , so that a local but energy dependent potential is realized ⁷⁾

$$2\mu V_L = q(r) + (2E_\mu + \frac{1}{2} \nabla^2) \alpha(r) \quad (7)$$

where the Laplacian acts only upon the nuclear density.

These two (out of the many possible) may be compared very conveniently ⁸⁾ by introducing an auxiliary function $\mathcal{G}$, defined in terms of the Kisslinger wave function by

$$\psi_K(r) = \mathcal{G}(r) \times (1 + \alpha(r))^{-1/2} \quad (8)$$

Then $\mathcal{G}$ satisfies a Schrödinger equation with the local potential

$$2\mu V_K^L = (1 + \alpha)^{-1} \left[q + (2E_\mu + \frac{1}{2} \nabla^2) \alpha - \frac{1}{4} (1 + \alpha)^{-1} (\nabla \alpha)^2 \right] \quad (9)$$

For the purpose of calculating the pion-nucleus scattering amplitude, $\mathcal{G}$ is as good a wave function as ψ_K , since they have the same asymptotic behaviour.

In general in scattering processes, the Kisslinger and local potentials give rather different answers when used with the same values of the parameters b and c . They also suffer from different diseases. Taking for ρ a uniform sphere model, the local potential (7) contains derivatives of delta functions on the nuclear surface. On the other hand, if the central nuclear density is too high, the denominator in the transformed Kisslinger potential (9) might vanish, and the potential diverge. This does not actually happen in the scattering region because of the imaginary part of the nucleon amplitude, but this effective mass singularity causes much

difficulty in the fitting of experimental data⁹⁾. This comparison of the two potentials has been made assuming that the local one could also be inserted linearly into the Schrödinger equation. The situation is changed if the energy dependence is treated as was argued previously. When the local potential is renormalized by Eq. (2), the modified potential develops the same effective mass singularity as the Kisslinger

$$2\mu V_L^\eta = (1+\alpha)^{-1} \left[q + (2E\mu + \frac{1}{2}\nabla^2)\alpha \right] \quad (10)$$

In uniform nuclear matter they become identical. This is also true for more complicated momentum dependences of the potential in the constant density limit. They differ by the surface term $(\nabla\alpha)^2$, but at high energies ($kR_{\text{nucleus}} \gg 1$) this is unlikely to be important. It is, however, all that remains of the different extrapolations (3) and (6). The large differences usually found are due to the local potential being put into the "wrong" equation?

Now that we have seen how the singularity arises, we are in a position to show that it is spurious, at least in the local case. In fitting data, the nucleon amplitudes b and c are taken to be constant at the values appropriate to the incident energy E_0 . Consequently if we vary E (not E_0) in Eq. (7), the potential behaves like E . The singularity comes when the coefficient is larger than unity, so that it dominates the eigenvalue in the Schrödinger equation. But we know that the energy dependence of the nucleon amplitude bears no resemblance to E anywhere except close to threshold. We should of course be using the full energy dependence of the functions b and c . To illustrate the effect of this, suppose for simplicity that b is constant and

$$c(E) = c_0 (1 + 2\mu \epsilon E)^{-1} \quad (11)$$

This is not terribly realistic for the π -N amplitude since the resonance structure has not been built in. Also we know that the asymptotic behaviour non-relativistically of the p wave term is even better than this (perhaps $E^{-\frac{3}{2}}$). Calculating the modified potential from Eqs. (2) and (7),

$$2\mu V_L^\eta = q(r) + \frac{[2E\mu + \frac{1}{2}\nabla^2 - 2\mu V_L^\eta] \alpha_0(r)}{1 + 2\mu \epsilon (E - V_L^\eta)} \quad (12)$$

This can be solved by iteration or resolving the quadratic equation. Since the coefficient of the leading term is just ϵ , the resulting modified local potential is everywhere finite for non-zero ϵ . It can therefore have neither the effective mass, nor the surface singularity.

The results obtained here are rather similar to those of Moniz ¹⁰⁾, who started from a sum of separable potentials. Taking account only of the first energy derivatives of b and c , and then keeping only the first iterant of Eq. (2), the resulting first order correction to the optical potential is quite close to his. This is to be contrasted with those methods which damp the potential in the momentum variable. For example in the exhaustive work of Landau et al. ¹¹⁾, the pion-nucleon phase shifts are fit to separable potentials, so that an off-shell behaviour of the form

$$c(k, k', E_0) = c(k_0, k_0, E_0) g(k) g(k') / [g(k_0)]^2 \quad (13)$$

could be used with calculable values for $g(k)$. (This parametrization was not restricted to the p wave.) This also eliminates the effective-mass singularity, but if the prescription of the present note is useful, then the energy behaviour of $c(k_0, k_0, E_0)$ should also be taken into account.

The approach may be profitable also in the case of K mesic atoms ¹⁾. The difficulty in understanding these results stems in part from the $I = 0$ bound state in the K^-p system. Since this is only a little way below threshold, the amplitude is very energy-dependent, which might make the induced second order terms significant even out on the edge of the nucleus.

In conclusion, when the single scattering approximation gives rise to an energy-dependent potential, we have to decide into which equation to substitute it. At best Eq. (2) is a plausible hypothesis. Even when there is nothing to stop the potentials from the different nucleons overlapping (a condition which may not be realized in real nuclei), it may represent some overcounting of the effect, and if the nucleons are disjoint there should be no effect of this kind at all. Model calculations are required to test it. On the other hand, we noticed a close parallel between the transformed Kisslinger and modified local potentials [Eqs. (9) and (10)]. In nuclear matter they become the same. This is caused by the k^2 in the Kisslinger equation acting like a local energy, and the minimal

substitution (2) mocking up this feature. This suggests strongly that any criticism of this substitution is implicitly a criticism of the Kisslinger approach as well. If we accept it as plausible, then we can argue as to whether the Kisslinger or the local potential reproduces the surface contribution better. Before we can actually compare with data though, we showed that it was important to investigate the energy or momentum dependence of b and c . This is especially true if the effective mass denominators cause trouble.

A full calculation along the lines suggested here may involve no extra free parameters. Though in principle we need to know the physical pion-nucleon amplitudes at all energies, in practice it is likely that only the energy variation in the neighbourhood of E_0 is important. This needs not, however, appear linearly in the result. With relativistic kinematics the problem is changed slightly. In order to give cross-sections which approach constants asymptotically, the potential going into the Klein-Gordon equation becomes independent of energy at high energy. There is thus no effect of an energy shift in this region.

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