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COMMENTS ON THE SYSTEMATICS OF MULTIPLICITY DISTRIBUTIONS

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A B S T R A C T

We combine the original arguments of Koba, Nielsen and Olesen with an assumption about isosinglet exchange dominance of the $N \rightarrow N$ forward scattering amplitude. Implications for the systematics of multiplicity distributions and correlations between the number of charged and neutral secondaries are pointed out and compared with available data. Further experiments are suggested.

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1. - INTRODUCTION AND COMMENTS ON DIFFERENT VERSIONS OF THE SHORT-RANGE ORDER HYPOTHESIS

There is evidence ¹⁾⁻⁴⁾ from the experimental data on charged multiplicities in pp collisions at $p_{lab} > 50 \text{ GeV/c}$, that the ratios $\langle n_c^q \rangle / \langle n_c \rangle^q$ are almost constant with p_{lab} and > 1 (this is true for not too big q , a restriction obvious for kinematic reasons). This behaviour has, in fact, been predicted by Koba, Nielsen and Olesen ⁵⁾ (KNO) who, using asymptotic arguments, derived the relation

$$\langle n_o^p n_c^q \rangle = C_{pq} \langle n_o \rangle^p \langle n_c \rangle^q \left[1 + O\left(\frac{1}{\langle n_o \rangle}, \frac{1}{\langle n_c \rangle}\right) \right] \quad (1)$$

where n_o and n_c are the number of neutral and charged secondaries, respectively. The early onset of the KNO scaling is very puzzling and I am unfortunately unable to add anything to its understanding. It might be that the observed empirical regularity has nothing to do with the KNO arguments. In this paper I shall assume the contrary and I shall extend KNO considerations to charge correlations.

Before entering the main subject of this paper, I will start with a few comments on the scaling hypothesis. Equation (1) has been derived assuming that the n particle ($n = p + q$) inclusive cross-section develops asymptotically a rapidity plateau in the central region :

$$\frac{d^n \sigma}{dy_1 \dots dy_n} \xrightarrow{Y_1 \ll y_1 \ll \dots \ll y_n \ll Y_2} \text{const.} \quad (2)$$

Y_1, Y_2 and y_j denote the longitudinal rapidities of the two incident particles and of the j^{th} secondary, respectively. A rationale for Eq. (2) is provided by the short range order (SRO) hypothesis ⁶⁾⁻⁸⁾ : at asymptotic energies, in an inclusive experiment, secondaries which are sufficiently fast with respect to each other become independent, in the sense of statistical independence. Hence

$$\frac{d^n \sigma}{dy_1 dy_{2q_1} \dots dy_n dy_{2q_n}} = a_n(Y_1, y_1, q_1, \dots, y_k, q_k) b_n(y_{k+1}, q_{k+1}, \dots, y_n, q_n, Y_2) \quad (3)$$

$Y_1, y_1, \dots, y_k \ll y_{k+1} \dots y_n, Y_2$

q_j denote transverse momenta which are assumed to be small. The now popular predictions about the existence of the rapidity plateau, limiting fragmentation (the concept introduced by Yang et al. ⁹⁾), etc., are conveniently summarized by Eq. (3). To realize that, it is sufficient to remember that only differences of rapidities are invariant with respect to boosts along the longitudinal direction ^{*}). Therefore an inclusive cross-section cannot contain factors depending on a single rapidity ^{**}). The factors in Eq. (3) can, in general, depend on n .

The SRO can also be formulated using Mueller's well-known generalization of the optical theorem ¹¹⁾. When $Y_1, y_1 \dots y_k \ll y_{k+1} \dots y_n, Y_2$ one can reasonably assume that the $(n+2) \rightarrow (n+2)$ forward scattering amplitude (more precisely its discontinuity across an appropriate cut) is dominated by Pomeron exchange. If it is furthermore postulated that the Pomeron is a Regge pole, the factorization properties of Regge poles lead to Eq. (3), but with factors independent of n ^{***}).

It might appear that by identifying the Pomeron with a Regge pole one has "derived" the SRO. This is probably an illusion of real progress since the difficulty has only been displaced. There is no theoretical reason for the Pomeron to be a pole, several serious arguments that it is not, and, more important, there is experimental data contradicting this assumption.

I think that it is wise to distinguish between a weak formulation of SRO (for convenience let me call it W-SRO), which permits the n dependence of factors in Eq. (3), and a strong hypothesis (S-SRO), which states that these factors are independent of n . I would like to emphasize that the successful verifications of SRO, including those provided by ISR experiments, confirm the W-SRO only.

^{*}) Pursuing this argument it is rather logical to tie SRO with the prediction that total cross-section $\sigma_T \rightarrow \text{const.}$ when $E \rightarrow \infty$. If the recent evidence for the increase of σ_T within the ISR energy range meant an indefinite, logarithmic increase of σ_T , one would be tempted to conclude that strict SRO is broken. However, present data are consistent with $\sigma_T \rightarrow \text{const.}$ from below, as, e.g., in models where the Pomeron is a Regge cut.

^{**}) This has been explained in a series of review papers. The most recent one is probably our Ref. 10).

^{***}) It is obvious that the assumption Pomeron = pole is a sufficient but not a necessary condition to get (3) from Mueller's theorem.

With S-SRO the constant on the right-hand side of Eq. (2) is a product of universal Pomeron coupling constants. This immediately implies that $C_{pq} = 1$. On the other hand W-SRO permits $C_{pq} \neq 1$. W-SRO also predicts that inclusive cross-sections factor in the central region as functions of transverse momenta. This is a strong statement which is, strictly speaking, unnecessary to derive the KNO result. This derivation requires Eq. (2) only. The purpose of the above discussion about the two versions of the SRO hypothesis was to make it clear that, contrary to what is often claimed, $C_{pq} \neq 1$ is not in variance with SRO. It is in variance with S-SRO which has a sloppy theoretical justification and no experimental support.

2. - SYSTEMATICS OF THE MOMENTS OF MULTIPLICITY DISTRIBUTIONS AND CHARGE CORRELATIONS

Let me come back to Eq. (2) and interpret it within the framework of Mueller's optical theorem. I shall assume that the $(n+2) \rightarrow (n+2)$ forward scattering amplitude is dominated by isosinglet exchanges ^{*}) but I will not postulate that the exchanged object is a Regge pole. Then the right-hand side of Eq. (2) does not depend on the number of positive (n_+), negative (n_-) and neutral (n_0) particles separately but only on the sum $n = n_+ + n_- + n_0$. Consequently C_{pq} depends only on the sum of its indices :

$$C_{pq} = C_{p+q} \quad (4)$$

Equation (4) is trivially satisfied when S-SRO is postulated, since one has then $C_{pq} = 1$. It is interesting that by adding the very plausible assumption about isosinglet exchange dominance of the $(n+2) \rightarrow (n+2)$ forward amplitude to the original KNO arguments one is led to a general prediction, Eq. (4), linking together different multiplicity moments, including those which involve information about charge correlations. Notice that the array of numbers $\langle n_0^p n_c^q \rangle$ contains the complete information about multiplicity distributions and correlations between neutral and charged secondaries. If Eq. (4) is true and if the early KNO scaling holds for arbitrary not too big

^{*}) I am indebted to Ed. Berger and to Ph. Salin for useful comments on this point.

p and q, then the structure of this array becomes particularly simple. It is tempting to inquire whether the striking empirical regularity observed for charged multiplicities can be extended to cases when both charged and neutral particles are observed. The tests of the appealing prediction, Eq. (4), will be presented in the next sections. Now, I would like to comment on the correlation between n_0 and n_- .

Consider a scatter plot of n_0 vs n_- , each event being represented by a point. The shape of this cloud of points can be to some extent described by two dispersion parameters: the average dispersion $D_{||}$ along the diagonal ($n_0 = n_-$) which is just proportional to the dispersion $D_c = [\langle n_c^2 \rangle - \langle n_c \rangle^2]^{\frac{1}{2}}$, and the average dispersion $D_{\perp}$ in the direction perpendicular to the diagonal

$$D_{\perp} = [\langle (n_0 - n_-)^2 \rangle]^{\frac{1}{2}} \quad (5)$$

One easily obtains from Eqs. (1), (4) that

$$D_{\perp} \propto \sqrt{\langle n_c \rangle} \quad (6)$$

From S-SRO it follows also that $D_{||} \propto \sqrt{\langle n_c \rangle}$, and one does not predict anything about the shape of the cloud of points in the (n_0, n_-) plane without making additional assumptions. The situation changes when one adopts a weaker hypothesis, since then $D_{||} \propto \langle n_c \rangle$ and one predicts

$$D_{\perp}/D_{||} \propto 1/\sqrt{\langle n_c \rangle} \xrightarrow{E \rightarrow \infty} 0 \quad (7)$$

Thus the cloud is expected to become more and more elongated along the diagonal as energy increases, its transverse dimensions (although increasing) becoming asymptotically negligible compared to its longitudinal dimensions. This picture is compatible with the present data in the sense that $\langle n_0 \rangle_{n_-}$ rises with n_- in the way I would expect. However, one has to be cautious before making any firm conclusion. In this respect I would like to give an example which seems instructive.

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Consider the particular version of the "two-component model" where one assumes that two non-interfering mechanisms contribute to channel cross-sections : diffraction dissociation (let it be energy independent, for simplicity) and pionization described by a multiperipheral model [cf. references in 10) and especially Ref. 12)]. In such a model :

$$\begin{aligned} \langle n_c^2 \rangle &\propto \frac{1}{1-\varepsilon} \langle n_c \rangle^2 \\ \langle n_o^2 \rangle &\propto \frac{1}{1-\varepsilon} \langle n_o \rangle^2 \\ \langle (n_o + n_c)^2 \rangle &\propto \frac{1}{1-\varepsilon} \langle n_o + n_c \rangle^2 \end{aligned} \quad (8)$$

where $\varepsilon = \sigma_D / \sigma_{in}$ is the fraction of the total inelastic cross-section which corresponds to diffraction dissociation. Of course one also has

$$\langle n_c n_o \rangle \propto \frac{1}{1-\varepsilon} \langle n_c \rangle \langle n_o \rangle \quad (9)$$

and Eqs. (1), (4) are satisfied for $p+q=2$. Therefore one also predicts (7), but in fact it is realized in a somewhat trivial manner. At asymptotic energies the cloud of points in the (n_o, n_-) scatter plot splits into two clouds. The first, corresponding to diffraction dissociation, has an energy independent shape while the second, corresponding to multiperipheralism, has both $D_{||}$ and $D_{\perp} \propto \sqrt{\langle n_c \rangle}$ as expected from S-SRO. Nevertheless the overall $D_{||} \propto \langle n_c \rangle$, simply because the two clouds get farther and farther from each other when the collision energy increases.

3. - COMPARISON WITH EXPERIMENT

The results of a test of Eq. (4) using recent NAL data ¹³⁾⁻¹⁵⁾ are presented in Table I. All experimental errors were assumed purely statistical since I am not in a position to speculate about possible systematic errors. It is seen that, within present errors, Eq. (4) seems to be confirmed. The asymptotic limit seems to be reached from below. For the second moment, Wroblewski ¹⁶⁾ presented a fit which takes care of the non-leading terms in Eq. (1). He finds $D_c = 0.586 (\langle n_c \rangle - 1)$ which can be rewritten as

$$\langle n_c^2 \rangle = 1.34 \langle n_c \rangle^2 \left[1 - 0.51 / \langle n_c \rangle + 0.25 / \langle n_c \rangle^2 \right] \quad (10)$$

Let me remark that the negative sign of the next to the leading term in (10), if taken seriously, can be considered as an argument against S-SRO. The ratio $\langle n_c^2 \rangle / \langle n_c \rangle^2$ has rather a tendency to increase slightly instead of decreasing toward unity. One can easily show, using Wroblewski's fit, the positivity of $\langle (n_0 - n_-)^2 \rangle$ and, assuming $\langle n_0 \rangle = \langle n_- \rangle$, that the negative sign of the next to the leading term in Eq. (10) implies that the corresponding term in Eq. (1) with $p=q=1$ is also negative. A thorough analysis of data, including a discussion of systematic errors, in order to find the magnitude of the non-leading terms for higher moments would be very useful.

I have done analogous calculations using the preliminary results of the CERN-Hamburg-Vienna ISR collaboration ¹⁷⁾. These data are not yet ready for publication and I cannot present here the numerical results of my calculation. Let me only say that numbers corresponding to those listed in columns 3 and 4 of Table I differ by less than 10%. However, these numbers are different from those of Table I, which is not astonishing since the CERN-Hamburg-Vienna set-up covers only a restricted solid angle. What is remarkable is that Eq. (4) seems to be nevertheless satisfied. In Table II, I present the results of a calculation using π^-p data ^{18),19)}. The energies are lower and the agreement with the prediction is poorer. It is amusing (and not at all obvious once S-SRO has been abandoned) that at 40 GeV/c the ratios $\langle n_c^q \rangle / \langle n_c \rangle^q$ are very similar to what one finds in pp collisions (this has already been noticed by Buras and Koba ⁴⁾).

4. - SUMMARY AND CONCLUSION

It has been pointed out that by combining the original KNO arguments with the assumption about the isosinglet exchange dominance of the forward $(n+2) \rightarrow (n+2)$ amplitude one predicts a simple systematics of multiplicity distributions and correlations. The predictions have been tested with existing data and the agreement is, in my opinion at least, significant. Of course, more and better data are needed to reach a firm conclusion.

Let me remark that Eqs. (1), (4) are subject to direct experimental test. The different moments $\langle n_o^p n_c^q \rangle$ are, according to them, linked together. The relatively simplest new experiment would consist in measuring the double-inclusive distribution for neutral pions, for different numbers of charged secondaries. Equations (1), (4) provide an immediate prediction

$$\sum_{n_c} n_c^q \sigma_{n_c} \left\{ \int d^2 \sigma_{\pi^0 \pi^0} + \int d\sigma_{\pi^0} \right\} \equiv \langle n_o^2 n_c^q \rangle \approx \frac{\langle n_o^2 \rangle^2}{\langle n_c \rangle^2} \langle n_c^{q+2} \rangle \quad (11)$$

for not too big values of q . In view of the fact that preliminary results of the CERN-Hamburg-Vienna collaboration are compatible with Eq. (4) in spite of the incomplete geometry of this experiment, I hope that a test of Eq. (11) with their set-up can be meaningful.

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p_{lab}	q	$\langle n_o n_c^{q-1} \rangle / \langle n_o \rangle \langle n_c \rangle^{q-1}$	$\langle n_c^q \rangle / \langle n_c \rangle^q$	average $\langle n_c^q \rangle / \langle n_c \rangle^q$ from Ref. 4)
205 GeV/c $\langle n_c \rangle = 7.65$ Ref. 13)	2	1.13 ± 0.11	1.26 ± 0.03	1.244 ± 0.006
	3	1.47 ± 0.32	1.85 ± 0.07	1.813 ± 0.020
	4	2.11 ± 0.63	3.06 ± 0.21	2.973 ± 0.057
	5	3.23 ± 1.22	5.55 ± 0.55	5.36 ± 0.15
303 GeV/c $\langle n_c \rangle = 8.86$ Refs. 14), 15)	2	1.18 ± 0.12	1.25 ± 0.03	see above
	3	1.63 ± 0.41	1.82 ± 0.05	
	4	2.54 ± 0.97	2.99 ± 0.24	
	5	4.35 ± 2.40	5.43 ± 0.64	

TABLE I : Systematics of moments of multiplicity distributions for pp collisions

p_{lab}	q	$\langle n_o n_c^{q-1} \rangle / \langle n_o \rangle \langle n_c \rangle^{q-1}$	$\langle n_c^q \rangle / \langle n_c \rangle^q$
25 GeV/c $\langle n_c \rangle = 4.86$ Ref. 18)	2	1.08 ± 0.05	1.19 ± 0.02
	3	1.38 ± 0.14	1.61 ± 0.02
	4	1.96 ± 0.29	2.44 ± 0.10
	5	3.05 ± 0.58	4.05 ± 0.24
	6	5.11 ± 1.22	7.29 ± 0.54
40 GeV/c $\langle n_c \rangle = 5.62$ Ref. 19)	2	1.06 ± 0.03	1.24 ± 0.02
	3	1.37 ± 0.14	1.82 ± 0.03
	4	2.03 ± 0.31	3.07 ± 0.16
	5	3.36 ± 0.67	5.85 ± 0.48

TABLE II : Systematics of moments of multiplicity distributions for π^-p collisions

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