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LOW FREQUENCY PHOTON SCATTERING BY NUCLEI

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A B S T R A C T

We study the low frequency limit of elastic photon scattering by nuclei of spin zero. We calculate the energy independent cross-section (Thomson scattering) and the term quadratic in the frequency (Rayleigh scattering) in two ways; within classical electrodynamics and non-relativistic quantum mechanics. Particular emphasis is put on a correct description of the centre-of-mass motion. The origin of the various retardation and polarizability terms is discussed. They are related to polarizabilities in static external fields. We propose the existence of a dia-electric polarizability which is the electric counterpart of diamagnetism. Orders of magnitudes of the expected effects are given.

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1. INTRODUCTION

Low energy photon scattering, Thomson and Rayleigh scattering, has a considerable history. The energy independent Thomson scattering depends only on the charge and the mass of the target. This property holds true for any system and is proven within the framework of quantum field theory ¹⁾. The result has been verified for nuclear Compton scattering ²⁾. For a system without spin there is no term linear in the frequency ω . The Rayleigh amplitude, quadratic in ω is completely described by two constants, the "polarizabilities" ³⁾. Forward dispersion relation relates the sum of these constants to the energy weighted integral σ_{-2} over the total photo cross-section. A series of approaches has been made to the electric polarizability in the approximation that the cross-section is dominated by the giant dipole resonance while the magnetic polarizability is neglected [cf., Ref. 4].

Our present viewpoint is quite different from the previous ones, but has some connection with the recent work by Goldhaber ⁵⁾. We take the following attitude: the photon contains at low frequency a near static electric field $\underline{E}$ and a near static magnetic field $\underline{H}$. These fields induce electric and magnetic dipole moments which are proportional to the electric polarizability α and the magnetic susceptibility χ as measured by static external electric and magnetic fields. In this picture Rayleigh scattering of the photon is produced by the time variation of the induced dipole moments which act like antennas of a classical transmitter. Our task is to establish the exact relation between the Rayleigh scattering and the polarizabilities α and χ both in classical physics and in quantum theory. At the same time we shall obtain expressions for these structure constants in terms of nuclear matrix elements.

In order to expose the physics as clearly as possible, we first derive the photon scattering amplitude completely classically. Second we obtain the same expression within the frame of quantum mechanics by standard methods but concentrating on a strictly non-relativistic nucleus. The quantum mechanical result is discussed in some detail and a dia-electric polarizability (in analogy to the diamagnetic one) is proposed. Finally, we estimate the order of magnitude of the polarizability terms.

2. CLASSICAL DERIVATION

We take the system to have a spherical charge distribution, as the absence of spin implies, total charge Ze and mass Am . The electric field $\underline{E}(\underline{r}, t) = \underline{E}_0 \cos(\underline{k} \cdot \underline{r} - \omega t)$ of the projectile photon has two effects:

- i) It exerts a force on the total charge of the system

$$\begin{aligned} \underline{F}(t) &= Ze \underline{E}_0 \int d^3r \rho(|\underline{r}|) \cos(\underline{k} \cdot \underline{r} - \omega t) \\ &= Ze \underline{E}_0 \underline{F}_0(k') \cos \omega t \end{aligned} \quad (1)$$

with the nuclear charge form factor

$$\underline{F}_0(k^2) = 1 - \frac{1}{6} k^2 \langle r^2 \rangle + \dots \quad (2)$$

The effective force $\underline{F}(t)$ displaces the nuclear centre-of-mass periodically around the equilibrium position

$$Am \ddot{\underline{R}} = \underline{F}(t) \quad (3)$$

- ii) If the system is not rigid but polarizable, the electric field also induces a dipole moment

$$\underline{d}(t) = \alpha(\omega) \langle \underline{E}_0 \rangle \cos \omega t \quad (4)$$

Here, $\langle \underline{E}_0 \rangle = \underline{E}_0 (1 + O(k^2 \langle r^2 \rangle))$ is the average field over the nucleus and $\alpha(\omega)$ the frequency dependent polarizability.

In the absence of a static magnetic moment of the target the magnetic component of the photon only induces a magnetic dipole moment $\underline{m}(t)$, which is proportional to the susceptibility χ

$$\underline{m}(t) = \chi(\omega) \langle \underline{H}_0 \rangle \cos \omega t \quad (5)$$

According to the formula for the classical antenna⁶⁾, the magnetic field of the emitted radiation is

$$\underline{H}(t) = \frac{1}{R_0} \left\{ \ddot{\underline{D}}(t) \times \underline{n} + (\ddot{\underline{m}}(t) \times \underline{n}) \times \underline{n} \right\}, \quad (6)$$

where R_0 is the distance from the source. The total electric dipole moment $\underline{p}(t)$ is the sum of the induced one and the one arising from the c.m. motion

$$\ddot{\underline{p}}(t) = Ze \ddot{\underline{R}}(t) \left(1 - \frac{1}{6} k^2 \langle r^2 \rangle\right) + \ddot{\underline{d}}(t) \quad (7a)$$

$$= \left[\frac{(Ze)^2}{Am} \left(1 - \frac{1}{3} k^2 \langle r^2 \rangle\right) - \omega^2 \alpha \right] \underline{E}_0 \cos \omega t + O(k^3). \quad (7b)$$

The vector $\underline{n}$ in Eq. (6) points in the direction of the outgoing radiation. The retardation correction $-\langle r^2 \rangle k^2/6$ in Eq. (7a) for the outgoing wave arises from the finite size of the source in complete analogy to the corresponding term in Eqs. (1) and (2). The intensity radiated into an angular element $d\Omega$ is

$$dI = \frac{H^2(t)}{4\pi} R_0^2 d\Omega. \quad (8)$$

The time average of this quantity divided by the time averaged incident flux gives the total cross-section

$$\frac{d\sigma}{d\Omega} = \frac{\overline{H^2}^t}{4\pi} R_0^2 \cdot \left(\frac{1}{4\pi} \overline{E^2}^t \right)^{-1}. \quad (9)$$

For unpolarized photons this expression should be averaged with the condition that the electric and magnetic fields remain orthogonal to each other and to the direction of propagation. Then, to order ω^2 , the cross-section is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{1 + \cos^2 \theta}{2} \left[\frac{(Ze)^2}{Am} \left(1 - \frac{1}{3} k^2 \langle r^2 \rangle\right) - \omega^2 \alpha(0) \right]^2 \\ & - 2 \omega^2 \cos \theta \cdot \frac{(Ze)^2}{Am} \chi(0). \end{aligned} \quad (10)$$

For $\omega \rightarrow 0$ one obtains the familiar Thomson limit

$$\left(\frac{d\sigma}{d\Omega} \right)_{Th} = \frac{1 + \cos^2 \theta}{2} \left(\frac{(Ze)^2}{Am} \right)^2. \quad (11)$$

Several points should be noted. First, Thomson and electric Rayleigh scattering cannot be separated by measuring the angular distribution, Eq. (10), nor by measuring the polarization, cf. Eqs. (6) and (7b), but only by the energy dependence. The retardation term and the electric polarizability cannot even be distinguished by varying the photon energy. Second, the magnetic Rayleigh term has a different angular dependence from the electric one, which permits a direct determination of χ , at least in principle. Finally, our derivation makes it quite clear, that the polarizabilities α and χ could have been functions of ω without restrictions, provided the retardation effect is small ($k^2 r^2 \ll 1$).

3. QUANTUM MECHANICAL DERIVATION

We derive the Compton scattering amplitude for a nucleus of non-relativistic nucleons which interact via a potential V . We assume that this potential commutes with any function $f(\underline{x}_i)$, which depends on the spatial co-ordinates of the nucleons. This excludes exchange and velocity dependent forces but otherwise the potential V is completely general. The main purpose of this assumption is to safeguard against those discontinuities in currents, which appear when meson degrees of freedom are eliminated. In more technical language: in the absence of exchange and velocity dependent forces, the vector potential of the radiation field is introduced only into the kinetic energies via minimal coupling. Furthermore, during our derivation we frequently use a generalized Thomas-Reiche-Kuhn (TRK) sum rule

$$\sum_n (E_n - E_0) \langle 0 | G(\underline{x}_1, \dots, \underline{x}_A) | n \rangle \langle n | H(\underline{x}_1, \dots, \underline{x}_A) | 0 \rangle = \frac{1}{2m} \langle 0 | \sum_i \underline{p}_i \cdot \underline{G} \cdot \underline{p}_i H | 0 \rangle, \quad (12)$$

which in this form holds only in the absence of velocity dependent and exchange forces. The two hermitian operators G and H depend on the co-ordinates only.

We start the derivation from the general scattering amplitude ⁷⁾

$$f_{fi} = 2\sqrt{\omega_f \omega_i} \left\{ \sum_n \left[\frac{\langle f | \int d^3x \underline{j}(\underline{x}) \cdot \underline{A}_f^*(\underline{x}) | n \rangle \langle n | \int d^3x \underline{j}(\underline{x}) \cdot \underline{A}_i(\underline{x}) | i \rangle}{E_i - E_n + \omega_i} + \frac{\langle f | \int d^3x \underline{j}(\underline{x}) \cdot \underline{A}_i | n \rangle \langle n | \int d^3x \underline{j}(\underline{x}) \cdot \underline{A}_f^* | i \rangle}{E_i - E_n - \omega_f} \right] + \frac{1}{m} \langle f | \int d^3x \underline{A}_f^*(\underline{x}) \cdot \underline{A}_i(\underline{x}) \rho(\underline{x}) | i \rangle \right\} \quad (13)$$

The vector potentials

$$\underline{A}_{i,f}(\underline{x}) = \frac{\underline{\xi}_{i,f}}{\sqrt{\omega_{i,f}}} \exp(i \underline{x} \cdot \underline{k}_{i,f}) \quad (14)$$

describe photons of direction $\underline{k}$, energy ω and polarization vector $\underline{\xi}$. The nuclear current operator $\underline{j}(\underline{x})$ contains a convection term and a spin term. It satisfies the continuity equation

$$\nabla \cdot \underline{j}(\underline{x}) = -\frac{d}{dt} \rho(\underline{x}) = -i[H, \rho(\underline{x})] \quad (15)$$

with the charge density

$$\rho(\underline{x}) = \sum_i e_i \delta(\underline{x} - \underline{x}_i) \quad (16)$$

The states $|i\rangle$, $|n\rangle$ and $|f\rangle$ in Eq. (13) describe the motion of the centre-of-mass with wave vector $\underline{K}$ and the intrinsic co-ordinates e.g., for elastic scattering:

$$|i\rangle = |\underline{K}_i, 0\rangle; |f\rangle = |\underline{K}_f, 0\rangle \quad (17)$$

The scattering amplitude f_{fi} is evaluated in the laboratory frame, i.e. $\underline{K}_i = 0$.

We discuss the low frequency limit of the various terms in Eq. (13). The last term in the curly brackets, often called the gauge or contact term, is proportional to the nuclear charge form factor

$$\begin{aligned} \frac{1}{m} \langle f | \int d^3x \tilde{A}_f^*(x) \cdot \tilde{A}_i(x) | i \rangle &= \\ &= \langle \underline{k}_f | e^{i(\underline{k}_i - \underline{k}_f) \cdot \underline{R}} | \underline{k}_i \rangle \cdot \left[\frac{\underline{\epsilon}_f \cdot \underline{\epsilon}_i}{m} Z e^2 F_{00}((\underline{k}_i - \underline{k}_f)^2) \right] \cdot (4\omega_i \omega_f)^{-\frac{1}{2}} \end{aligned} \quad (18)$$

The first factor on the right-hand side contains the over-all momentum conservation. The form factor $F_{00}(q^2)$ is expanded in powers of the momentum transfer according to Eq. (2) where the mean square radius $\langle r^2 \rangle = \langle 0 | (\underline{x}_p - \underline{R})^2 | 0 \rangle$ is measured with respect to the centre-of-mass $\underline{R}$.

The low frequency limit of the dispersive term [sum over excited states in Eq. (13)] has contributions from the expansion of the vector potential

$$\sqrt{2\omega} \tilde{A}(\underline{x}) = e^{i\underline{k} \cdot \underline{R}} \cdot \underline{\epsilon} \left(1 + i \underline{k} \cdot (\underline{x} - \underline{R}) - [\underline{k} \cdot (\underline{x} - \underline{R})]^2 / 2! + \dots \right) \quad (19)$$

and from an expansion of the denominator in powers of $\omega / (E_n - E_0)$. The intermediate states $|n\rangle$ have the structure $|\underline{K}_n = \underline{k}_i, n^{(i)}\rangle$ in the first sum and $|\underline{K}_n = -\underline{k}_f, n^{(i)}\rangle$ in the second sum, where $|n^{(i)}\rangle$ denotes the wave function of the intrinsic excited state. Like in Eq. (18), the proper treatment of the centre-of-mass variables leads to the over-all momentum conservation. Since the initial c.m. momentum $\underline{K}_i = 0$, the c.m. part of the nuclear current in the matrix elements does not contribute. Therefore, after having taken care of the factors $\exp(i \underline{k}_{i,f} \cdot \underline{R})$, the matrix elements which appear in the dispersive terms depend only on intrinsic variables. Only these are considered in the following when we discuss the various contributions to the lowest orders in frequency.

- i) Set $\underline{A}(\underline{x}-\underline{R}) = \underline{\xi} / \sqrt{2\omega}$ and $\omega = 0$ in the denominators: we use the continuity equation (15) for the matrix elements

$$\begin{aligned} \langle 0 | \int d^3(\underline{x}-\underline{R}) \underline{\xi} \cdot \underline{j}(\underline{x}-\underline{R}) | n \rangle &= i \langle 0 | [H, \int d^3(\underline{x}-\underline{R}) \underline{\xi} \cdot (\underline{x}-\underline{R}) \rho(\underline{x}-\underline{R})] | n \rangle \\ &= i(E_0 - E_n) \langle 0 | \underline{\xi} \cdot \underline{D} | n \rangle \end{aligned} \quad (20)$$

$$\underline{D} = \sum_i e_i (\underline{x}_i - \underline{R}) = \sum_i e_i^{eff} \cdot \underline{x}_i$$

The electric dipole operator $\underline{D}$ contains the effective charges (eN/A for protons and $-eZ/A$ for neutrons). The TRK sum rule enables us to evaluate the sum over the excited states

$$\sum_n \left[\right] = - \frac{\underline{\xi}_f \cdot \underline{\xi}_i}{m} \frac{Ze^2}{A} (4\omega_f \omega_i)^{-\frac{1}{2}} \quad (21)$$

This term and the momentum independent expression of Eq. (18) combine to the Thomson amplitude

$$f_{Th} = \underline{\xi}_f \cdot \underline{\xi}_i \frac{(Ze)^2}{Am} \quad , \quad (22)$$

which depends on the total charge Ze and the total mass Am of the nucleus.

- ii) Set $\underline{A} = \underline{\xi} / \sqrt{2\omega}$ and expand the energy denominator of the dispersive terms: the energy denominators are:

$$E_i - E_n + \omega_i = E_0^{(i)} - \left(E_n^{(i)} - \frac{k_i^2}{2Am} \right) + \omega \quad (23)$$

for the first (direct term) and

$$E_i - E'_n - \omega_f = E_0^{(i)} - \left(E_n^{(i)} - \frac{k_f^2}{2Am} \right) - \left(\omega - \frac{(k_i - k_f)^2}{2Am} \right) \quad (24)$$

for the second term (crossed graph). The notation $E_n^{(i)}$ refers to intrinsic energies. We expand the two terms to order ω^2

$$(E_i - E_n + \omega_i)^{-1} + (E_i - E'_n - \omega_f)^{-1} = (E_0^{(i)} - E_n^{(i)})^{-1} \left[2 + \frac{k_i \cdot k_f}{Am} (E_0^{(i)} - E_n^{(i)})^{-1} + 2\omega^2 (E_0^{(i)} - E_n^{(i)})^{-2} \right] \quad (25)$$

After using Eq. (20) closure is performed in the term proportional to $\underline{k}_i \cdot \underline{k}_f$ and the ω^2 dependent expression leads to the electric polarizability α

$$f_R^{(E)} = \sum_n [] = \underline{\xi}_f \cdot \underline{\xi}_i \left(\frac{\underline{k}_f \cdot \underline{k}_i}{2Am} \frac{1}{3} \langle 0 | D^2 | 0 \rangle - \omega^2 \alpha \right) \quad (26)$$

with

$$\alpha = 2 \sum_n \frac{\langle 0 | D_z | n \rangle^2}{E_n^{(i)} - E_0^{(i)}} \quad (27)$$

iii) Set $A(\underline{x}-\underline{R}) = i \underline{\xi}(\underline{k} \cdot (\underline{x}-\underline{R})) / \sqrt{2\omega}$ and neglect the ω and k dependences in the denominators: we decompose the terms $(\underline{\xi} \cdot \underline{j}(\underline{x})) \cdot (\underline{k} \cdot \underline{x})$ into irreducible tensors in the photon and nuclear co-ordinates (note $\underline{\xi} \cdot \underline{k} = 0$).

$$\begin{aligned} (\underline{\xi} \cdot \underline{j}(\underline{x})) (\underline{k} \cdot \underline{x}) &= (\underline{\xi} \times \underline{k}) \cdot \frac{1}{2} (\underline{j} \times \underline{x}) \\ &+ \sum_{m,n=1}^3 \frac{1}{2} (\epsilon_m k_n + \epsilon_n k_m) \left[\frac{1}{2} (j_m x_n + j_n x_m) - \frac{1}{3} \delta_{nm} \underline{j} \cdot \underline{x} \right]. \end{aligned} \quad (28)$$

The operator $\frac{1}{2} \underline{j} \times \underline{x}$ is just the magnetic dipole operator (M1). Therefore, the first term in Eq. (28) inserted into the dispersive sum leads to the Rayleigh amplitude involving the "paramagnetic" susceptibility χ_p

$$f_R^{(M1)} = - (\underline{\xi}_f \times \underline{k}_f) \cdot (\underline{\xi}_i \times \underline{k}_i) \cdot \chi_p \omega^2 \quad (29)$$

$$\chi_p = 2 \sum_n \frac{\langle 0 | (M1)_z | n \rangle^2}{E_n^{(i)} - E_0^{(i)}} \quad (30)$$

The second term in Eq. (28) is of electric quadrupole character (E2). For a spin zero ground state, there are no interference terms between the M1 and E2 matrix elements in the sum over intermediate states Eq. (13). The continuity equation can be applied to the E2 matrix elements

$$\int d^3(x-R) \left((x-R)_n \int_m (x-R)_n + (n \leftrightarrow m) \right) = - \int d^3(x-R) (x-R)_m (x-R)_n \nabla \cdot \mathbf{j}(x-R). \quad (31)$$

With the help of the TRK sum rule one derives

$$f_R^{(E^2)} = - \left[2(\underline{\xi}_f \cdot \underline{\xi}_i) (\hat{k}_f \cdot \hat{k}_i) - (\underline{\xi}_f \times \hat{k}_f) (\underline{\xi}_i \times \hat{k}_i) \right] \left(\frac{Ze^2}{6m} \langle r^2 \rangle - \frac{\langle D^2 \rangle}{6Am} \right) \cdot k^2. \quad (32)$$

The term which contains $\langle D^2 \rangle$, the average value of the dipole operator squared, arises from the fact that the quadrupole operator in Eq. (31) is written in intrinsic co-ordinates. This term is of order $1/A$ compared to the one proportional to $\langle r^2 \rangle$.

iv) Approximate one of the two vector potentials by $A_1 = \underline{\xi} / \sqrt{2\omega}$ and use for the other one, A_2 , the quadratic retardation

$A_2 = -\underline{\xi} (\underline{k} \cdot \underline{x})^2 / 2\sqrt{2\omega}$: the term which involves A_1 can be treated according to Eq. (20). The excitation energies $E_0 - E_n$ can be divided out and closure can be performed

$$\begin{aligned} f_R' &= -\frac{i}{2} \langle 0 | \left[D \cdot \underline{\xi}_f, \sum_j (\underline{\xi}_i \cdot \mathbf{j}_j(x_i-R)) (\hat{k}_i \cdot (x_i-R))^2 \right] + (i \leftrightarrow f) | 0 \rangle \\ &= \underline{\xi}_f \cdot \underline{\xi}_i \frac{NZe^2}{Am} \frac{e^2}{3} \langle r^2 \rangle k^2. \end{aligned} \quad (33)$$

We collect all terms, Eqs. (22), (26), (29) and (33), and rearrange them:

$$\begin{aligned} f_{fi} &= \underline{\xi}_f \cdot \underline{\xi}_i \left[\frac{(Ze)^2}{Am} \left(1 - \frac{k^2 \langle r^2 \rangle}{3} \right) - \alpha \omega^2 \right] \\ &\quad - (\underline{\xi}_f \times \hat{k}_f) \cdot (\underline{\xi}_i \times \hat{k}_i) [\chi_p + \chi_D] \omega^2 \end{aligned} \quad (34)$$

with the "diamagnetic" susceptibility

$$\chi_D = -\frac{1}{6} \frac{Ze^2}{mc^2} \langle r^2 \rangle + \frac{\langle D^2 \rangle}{6Amc^2}. \quad (35)$$

The Compton amplitude Eq. (34) satisfies the general low energy theorems³⁾: two structure constants

$$C_1 = \alpha + \frac{(Ze)^2}{Am} \frac{\langle r^2 \rangle}{3c^2} \quad \text{and} \quad C_2 = \chi_p + \chi_D \quad (36)$$

enter in addition to the Thomson amplitude. Our derivation leads to explicit expressions for c_1 and c_2 in terms of nuclear operators. The cross-section for unpolarized radiation derived from Eq. (34) is identical to the one obtained in the classical case, Eq. (10).

4. DISCUSSION OF THE QUANTUM MECHANICAL DERIVATION

The quantum mechanical derivation of the nuclear Thomson and Rayleigh amplitudes, as presented in the previous section, is much more complicated than the classical derivation. In particular, the dispersive and the gauge terms contribute both to the Thomson and Rayleigh amplitudes in a way which is not very transparent. The following remarks may clarify the situation somewhat.

The Thomson amplitude gets a contribution Ze^2/m from the gauge term and a correction $-NZe^2/Am$ from the sum over intermediate states. The physical origin of the second term lies in the fact that the photons pull only on the protons. The force has to be transmitted to the neutrons, the internal rearrangement in the nucleus is associated with the presence of excited states and leads to a correction term, which is proportional to the number of neutrons.

The electric polarizability α and the two magnetic susceptibilities, χ_p and χ_D originate from virtual intrinsic excitations of the system as is to be expected.

The classical retardation factor $-Ze^2\langle r^2 \rangle/3Amc^2$ originates partly in the gauge term and partly in the sum over excited states. The contribution of the excited states to the mean square radius may be surprising. But in non-relativistic quantum mechanics the size of an object is intimately connected to the existence of excited states as is evident from the following theorem:

Theorem: A system which has no excited states and which is symmetric under space reflection, has no size.

Proof: For a system without excited states, a super-selection rule ensures that any non-diagonal matrix element of a physically realizable operator Z vanishes. For these operators the closure relation reduces to

$$\langle 0 | Z^n | 0 \rangle = \langle 0 | Z | 0 \rangle^n \quad (37)$$

If Z is a component of an intrinsic length, $\langle 0 | Z | 0 \rangle = 0$ for a system which is symmetric under space reflections so that $\langle 0 | Z^n | 0 \rangle = 0$, q.e.d. However, a system without excited states has a number of intricacies: for instance, intrinsic momenta are infinite and a non-relativistic treatment is not appropriate.

The magnetic structure constant $c_2 = \chi_p + \chi_D$ separates formally into a paramagnetic and diamagnetic term, as is expected from classical physics. The two contributions, however, cannot be separated by any experimental set-up of elastic photon nucleus scattering. They lead to the same angular, polarization and energy dependence. Furthermore, the magnetic susceptibility χ which determines the energy shift

$$\Delta \epsilon_M = - \frac{1}{2} \chi H^2 \quad (38)$$

in a static external magnetic field H is also the sum of χ_p and χ_D ⁸⁾. Therefore, as long as we consider ground state properties only (and not the thermal behaviour) there is only one magnetic susceptibility.

The situation seems different in the electric case. Although the electric polarizability α cannot be separated from the retardation term $Ze^2 \langle r^2 \rangle / 3Amc^2$ by the scattering of low energy photons, the energy shift in a static external electric field E is usually given as

$$\Delta \epsilon_E = - \frac{1}{2} \alpha E^2, \quad (39)$$

where the $\langle r^2 \rangle$ term does not appear. We do not think that this asymmetry between the magnetic and electric polarizabilities is real. Therefore, we propose to introduce a dia-electric polarizability

$$\alpha_D = \frac{(Ze)^2 \langle r^2 \rangle}{3Amc^2} \quad (40)$$

in analogy to the diamagnetic case. Low energy photon scattering depends only on $\alpha + \alpha_D$. Is there also a shift $\Delta \epsilon$ in an external field which arises from α_D ? Not within a strictly non-relativistic Schrödinger equation, where the electric field appears linearly in the form $\underline{D}, \underline{E}$. The factor c^2 in the denominator of α_D shows the relativistic origin, however. If we include the lowest order relativistic corrections by starting from a single particle Klein-Gordon or Dirac equation we find a term ^{8),9)}

$$H' = - \frac{e^2 (\underline{x} \cdot \underline{E})^2}{2 m c^2} \quad (41)$$

whose expectation value has the anticipated structure

$$\langle H' \rangle = - \frac{1}{2} \left(\frac{e^2 \langle r^2 \rangle}{3 m c^2} \right) E^2. \quad (42)$$

For a nucleus which consists only of protons ($A=Z$) the sum of the terms in Eqs. (41) and (42) leads exactly to the dia-electric polarizability, Eq. (40). If neutrons are also present, the sum of the terms Eq. (41) still includes the protons only and α_D essentially remains unchanged and does not agree with Eq. (40). We suspect that this deficiency is connected with the fact that the correction term Eq. (41) is appropriate for a single particle equation, but it is not clear how the centre-of-mass corrections should be made in this additional relativistic term. The dia-electric polarizability, Eq. (40), is a 5% correction to the normal polarizability for a nucleus. For an atom the correction is of order 10^{-4} and therefore negligible.

Forward dispersion relations relate the sum of the structure constants c_1 and c_2 , Eq. (36), to σ_{-2} , the energy weighted integral over the total cross-section

$$c_1 + c_2 = \frac{1}{2\pi^2} \int_0^\infty \frac{\sigma_{tot}(\omega)}{\omega^2} d\omega \quad (43)$$

The dia-electric and diamagnetic polarizabilities in the sum c_1 and c_2 can be combined and one obtains

$$\alpha + \chi_p + \frac{1}{6 A m c^2} [\langle \underline{D}^2 \rangle - Z(N-Z) \langle r^2 \rangle] = \frac{\sigma_{-2}}{2\pi^2} \quad (44)$$

For nuclei with $N=Z$ the term in square brackets is positive but for heavy nuclei the second term proportional to $\langle r^2 \rangle$ is much larger (by a factor of 10^5 in the case of Pb). Nevertheless the sum never becomes negative as it should (for Pb the electric polarizability α is about a hundred times larger than the term in square brackets).

5. ORDER OF MAGNITUDE ESTIMATES

Since the nuclear Thomson scattering scales with the nuclear mass, it is a very small cross-section (fractions of nb/sr), see the Table. The interference term between Thomson and Rayleigh scattering increases with ω^2 . Even for photon energies of a few MeV, which can still be considered small compared to the frequencies of the strong E1 and M1 nuclear modes, the interference term is a fraction of the Thomson cross-section. Estimates for the nuclear electric and magnetic polarizabilities, on which the values in the Table are based, can be found in Ref. 8). One generally accepts that the magnetic susceptibility χ is small compared to the electric polarizability, and then has

$$\alpha \approx \frac{1}{2\pi^2} G^{-2} \quad (45)$$

from the forward dispersion relation. Our estimate in the Table is based on

$$\alpha = 3.2 \cdot 10^{-3} A^{5/3} [\text{fm}^3], \quad (46)$$

but the present uncertainty is still large: values for α as low ¹⁰⁾ as $\alpha = 2.2 \cdot 10^{-3} A^{5/3} \text{ fm}^3$ and as high as ¹¹⁾ $3.75 \cdot 10^{-3} A^{5/3} \text{ fm}^3$ are derived from different measurements of the total photo-nuclear cross-section. Very little is known about the susceptibility χ . While the diamagnetic contribution is readily evaluated, the distribution of M1 strength in nuclei must be known for the paramagnetic term. We base our estimate ⁸⁾ for χ_p on the independent particle model. For a nucleus in which the $j = l + \frac{1}{2}$ shell is completely filled and the $j = l - \frac{1}{2}$ shell completely empty (e.g., ¹²C) we find

$$\chi_p = \frac{1}{3} \left(\frac{e}{2m} \right)^2 \frac{(g_s - g_l)^2}{10 \text{ MeV}} A^{2/3}, \quad (47)$$

where g_s and g_l are the gyromagnetic ratios for spin and orbital motion of the nucleons which fill the $j = l + \frac{1}{2}$ shell. This part of the paramagnetic polarizability is zero for closed l shells, e.g., ^{16}O or ^{40}Ca . It is expected, however, that certain collective excitations add to the values Eq. (47) so that the magnetic estimates in the Table should be considered as a lower limit. The paramagnetic, Eq. (47), and diamagnetic susceptibilities are about equally large, more precisely $\chi_p/\chi_D \simeq -40/Z$.

While we think that a measurement of the nuclear polarizabilities would be highly desirable, the experiments will not be simple. In addition to difficulties associated with the small cross-sections of nuclear Thomson and Rayleigh scattering, the elastic photon scattering by the electrons of the atomic cloud and the Delbrück scattering add coherently to the nuclear Thomson and Rayleigh amplitudes. Photon energies of a few MeV are large compared to electronic frequencies, so that one may neglect the dispersive term in the scattering amplitude, Eq. (13), when applied to the electronic case. Then the form factor term survives and the elastic cross-section for the atomic cloud is

$$\left(\frac{d\sigma}{d\Omega}\right)_{A.C.} = \left(\frac{e^2}{m_e}\right)^2 \cdot |F_{A.C.}(q^2)|^2; \quad F_{A.C.}(0) = Z, \quad (48)$$

which scales with the electron mass. In forward direction the atomic cross-section dominates over the nuclear one by a factor 10^7 . Fortunately, the atomic form factor $F_{A.C.}(q^2)$ drops rapidly with increasing momentum transfer. For instance one has

$$F_{A.C.}(q^2) = \frac{2}{(1 + \alpha^2 q^2)^2}, \quad \alpha = (Ze^2 m_e)^{-1} \quad (49)$$

for the two K electrons, whose form factor is expected to decrease slower than the contribution of the other electrons. If one performs elastic photon scattering experiments with energies of a few MeV and $\theta > 90^\circ$, the atomic scattering may be neglected compared to the Rayleigh scattering.

The behaviour of nuclear Delbrück scattering is less clear. In forward direction the cross-section can be calculated analytically ¹²⁾ and is (normalized to the nuclear Thomson value):

$$\left(\frac{d\sigma}{d\Omega}\right)_D \cdot \left(\frac{d\sigma}{d\Omega}\right)^{-1}_{Th} = 10^{-5} \left(\frac{\omega}{m_e}\right)^4 A^2 . \quad (50)$$

Angular distributions have been computed numerically ¹³⁾ for various energies and $\theta \leq 120^\circ$, and are reproduced in the Table.

The accuracy of these calculations have not yet been tested experimentally. Furthermore, there are no values in the backward direction, where any experiment on nuclear Rayleigh scattering would have to be performed.

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NUCLEUS	E_γ	NUCLEAR THOMSON	NUCLEAR RAYLEIGH		NUCLEAR DELBRÜCK	ATOMIC CLOUD
			ELECTRIC	MAGNETIC		
	$[\text{MeV}]$	$[\text{nb/sr}]$				
^{12}C	2.5	$6 \cdot 10^{-3}$	1.2%	1%	$\sim 0.1\%$	10^{-6}
	6	$6 \cdot 10^{-3}$	7.2%	6%	$\sim 3\%$	$3 \cdot 10^{-8}$
^{208}Pb	2.5	0.8	14%	$\sim 1\%$	$\sim 1.2\%$	0.4%
	6	0.8	95%	$\sim 6\%$	$\sim 30\%$	0.01%

TABLE: The various contributions to the elastic scattering of 2.5 and 6 MeV γ -rays by ^{12}C and ^{208}Pb at an angle of $\theta = 120^\circ$. Absolute values are given for the nuclear Thomson scattering. The nuclear Rayleigh terms, the Delbrück scattering¹³⁾ and the contribution from the atomic cloud are calculated relative to the nuclear Thomson cross-section. The estimates are described in the text.