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DUALITY, THE QUARK PARTON MODEL, AND INCLUSIVE PION ELECTROPRODUCTION

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A B S T R A C T

Consequences of deep inelastic duality and positivity are applied to single pion distributions within the framework of the quark parton model. The discussion is limited to the current fragmentation region. The distributions display new factorization features.

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There have been several attempts ^{1),2)} to apply the consequences of duality in deep inelastic interactions of hadrons with the weak and electromagnetic currents. In the work of Landshoff and Polkinghorne ¹⁾ and in Ref. 2), the Harari-Freund ³⁾ two-component duality idea and positivity constraints have been applied to the imaginary part of the forward current-nucleon amplitudes. It has been realized ²⁾ that the internal symmetry properties of the structure functions can be reduced to those of a quark-parton model with the quark-parton diagrams interpreted as duality diagrams ⁴⁾. In such a model ⁵⁾ the quark distribution within a nucleon is divided into two components: the neutral component ("sea" quarks) corresponding to the Pomeron-background part of the amplitude, and the quantum number carrying component ("valence" quarks) which corresponds to the Regge-resonance component. It is a typical duality element to assume separate positivity for these ²⁾, a property which is not implied in a mere parton approach. This enables one to obtain tight bounds on the neutrino structure functions by using experimental data of the electromagnetic ones.

The measurement of single hadron distributions in deep inelastic reactions can serve as a useful tool for the understanding of the structure of the nucleon. Some general consequences following from symmetry principles have been obtained by Lipkin and Paschos ⁶⁾ for the target fragmentation region. The light-cone approach has been applied to this region ⁷⁾, while the parton model has been applied to both this region and the current fragmentation region ^{8),9)}. Since the single hadron distributions in electroproduction are now in the process of measurement, it is worth while to look for possible tests of duality involving such cross-sections.

In this paper we study the current fragmentation region within the framework of the quark-parton model of Ref. 9), since in this region the model has its strongest predictive power. Moreover, at present, it is more easily accessible from the experimental point of view.

We consider the process $\gamma_V + N \rightarrow \pi + X$. The virtual photon carries four-momentum q , p is the nucleon momentum and h is the pion momentum. The current fragmentation region is defined by:

$$q^2 \rightarrow -\infty, \quad M\nu \equiv p \cdot q \rightarrow \infty, \quad x = -q^2/2M\nu \text{ finite},$$

$$z = \frac{h \cdot p}{M\nu} \text{ finite}, \quad h_T \text{ finite} \quad . \quad (1)$$

Here, h_T is the transverse momentum of the observed hadron.

It has been proposed by Feynmann ⁹⁾ that this process occurs in two steps : in the first one, the photon is absorbed by a parton with a momentum fraction x of the total proton momentum ; in the second step, this parton fragments into hadrons which are current fragments, while the rest of the partons fragments into target fragments. These two fragmentation processes are assumed to occur independently in the average sense (namely, not in individual events). As a consequence, simple forms for the structure functions are obtained :

$$\left. \begin{aligned} L_{1,\pi^+}^{ep} &= \frac{1}{2} \left[D_u^{\pi^+} \left(\frac{4}{9} u + \frac{1}{9} \bar{d} \right) + D_d^{\pi^+} \left(\frac{4}{9} \bar{u} + \frac{1}{9} d \right) + D_s^{\pi^+} \cdot \frac{1}{9} (s + \bar{s}) \right] \\ L_{1,\pi^-}^{ep} &= \frac{1}{2} \left[D_d^{\pi^-} \left(\frac{4}{9} u + \frac{1}{9} \bar{d} \right) + D_u^{\pi^-} \left(\frac{4}{9} \bar{u} + \frac{1}{9} d \right) + D_s^{\pi^-} \cdot \frac{1}{9} (s + \bar{s}) \right] \\ L_{1,\pi^+}^{en} &= \frac{1}{2} \left[D_u^{\pi^+} \left(\frac{4}{9} d + \frac{1}{9} \bar{u} \right) + D_d^{\pi^+} \left(\frac{4}{9} \bar{d} + \frac{1}{9} u \right) + D_s^{\pi^+} \cdot \frac{1}{9} (s + \bar{s}) \right] \\ L_{1,\pi^-}^{en} &= \frac{1}{2} \left[D_d^{\pi^-} \left(\frac{4}{9} d + \frac{1}{9} \bar{u} \right) + D_u^{\pi^-} \left(\frac{4}{9} \bar{d} + \frac{1}{9} u \right) + D_s^{\pi^-} \cdot \frac{1}{9} (s + \bar{s}) \right] \end{aligned} \right\} \quad (2)$$

$L_{1,\pi^\pm}^{ep}(x, z, h_T^2)$, the structure function for $\pi^\pm$ electroproduction on a proton, for example, is the analogue of $F_1^{ep}(x)$ in the total deep inelastic cross-section ⁹⁾. $u(x)$, $d(x)$, ... are the distribution functions of the various types of quarks in the proton. $D_u^{\pi^+}(z, h_T^2)$, for example, is the fragmentation function of a u quark into π^+ + anything. Isospin and charge conjugation invariance have been already applied to the D functions.

Since the same parton distribution functions $[u(x), d(x), \dots]$ also appear in the total deep inelastic cross-sections, we can use the results of Landshoff and Polkinghorne ¹⁾ and Chaichian et al. ²⁾, even before embarking on the more ambitious task of a complete analysis of a six-point function duality ^{*}). In the framework of the parton model this amounts to ²⁾ :

- 1) the s channel resonance part is limited to non-exotic quantum numbers ;
- 2) the Pomanchuk contribution is a t channel $SU(3)$ singlet ;
- 3) currents with the ϕ meson quantum numbers decouple from nucleons.

^{*}) This problem is now being investigated.

The resulting solution for the parton distribution functions is ²⁾ :

$$\left. \begin{aligned} \bar{u} &= \bar{d} = \bar{s} = \frac{1}{3} \\ u &= v_u + \frac{1}{3} \\ d &= v_d + \frac{1}{3} \end{aligned} \right\} \quad (3)$$

Notice that one is not forced to assume a relation between v_u and v_d as in Kuti and Weisskopf ⁵⁾.

Using these results one can obtain new relations among the structure functions in single pion distributions. For example, limiting oneself to the parton model of Ref. 9), one finds after integration over x in Eq. (2) and using well-known parton model sum rules :

$$D_u^{\pi^+} - D_d^{\pi^+} = \frac{18}{7} \int (L_{1,\pi^+}^{ep} - L_{1,\pi^-}^{ep}) dx \quad (4a)$$

$$D_u^{\pi^+} + D_d^{\pi^+} = \frac{6}{2 \int (u(x) - d(x)) dx - 1} \int [L_{1,\pi^+}^{ep} + L_{1,\pi^-}^{ep} - L_{1,\pi^+}^{en} - L_{1,\pi^-}^{en}] dx \quad (4b)$$

$$\begin{aligned} (D_u^{\pi^+} + D_d^{\pi^+}) \left[\frac{2}{9} \int u(x) dx + \frac{2}{9} \int d(x) dx - 1 \right] + \frac{4}{9} D_s^{\pi^+} \int s(x) dx \\ = 2 \int [L_{1,\pi^+}^{ep} + L_{1,\pi^-}^{ep}] dx \end{aligned} \quad (4c)$$

Thus, the difference $D_u^{\pi^+}(z, h_T^2) - D_d^{\pi^+}(z, h_T^2)$ is completely determined from electroproduction data ; $D_u^{\pi^+}(z, h_T^2)$ and $D_d^{\pi^+}(z, h_T^2)$ can be separately determined up to one free parameter $(\int [u(x) - d(x)] dx)$; and $D_s^{\pi^+}$ is determined up to two additional parameters. As is well known, duality implies :

$$\left. \begin{aligned} \int [u(x) - d(x)] dx &= 1 \\ \int [u(x) - s(x)] dx &= 2 \\ \int [d(x) - s(x)] dx &= 1 \end{aligned} \right\} \quad (5)$$

Thus by adding duality constraints $D_u^{\pi^+}$ and $D_d^{\pi^+}$ are completely determined by Eqs. (4a), (4b), and $D_s^{\pi^+}$ now depends on one free parameter only ($\int s(x) dx$).

Moreover, one obtains an interesting factorization property which is not predicted by the parton model of Ref. 9). Namely,

$$\left. \begin{aligned} L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en} &= \frac{1}{2} (v_u - v_d) \left[\frac{4}{9} D_u^{\pi^+}(z, h_T^2) - \frac{1}{9} D_d^{\pi^+}(z, h_T^2) \right] \\ L_{1,\pi^-}^{ep} - L_{1,\pi^-}^{en} &= \frac{1}{2} (v_u - v_d) \left[\frac{4}{9} D_d^{\pi^+}(z, h_T^2) - \frac{1}{9} D_u^{\pi^+}(z, h_T^2) \right] \end{aligned} \right\} \quad (6)$$

Using

$$\begin{aligned} \int (v_u(x) - v_d(x)) dx &= 1 \\ v_u(x) - v_d(x) &= 6 (F_1^{ep}(x) - F_1^{en}(x)) \end{aligned}$$

one finds :

$$\frac{L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en}}{\int [L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en}] dx} = 6 (F_1^{ep}(x) - F_1^{en}(x)) \quad (7)$$

We see that the left-hand side becomes independent of the pion altogether. More important, it is easy to show that a similar relation is true for any type of current fragment, independent of its internal quantum numbers ($\pi, K, N, \dots$). The x dependence is determined by the total inelastic structure functions. Equation (6) also implies that

$$\frac{L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en}}{L_{1,\pi^-}^{ep} - L_{1,\pi^-}^{en}}$$

is independent of x .

It is worth noting that in the quark parton model ⁹⁾ one already obtains another type of factorization :

$$L_{1,\pi^+}^{ep} - L_{1,\pi^-}^{ep} = \frac{1}{2} (D_u^{\pi^+} - D_d^{\pi^+}) \left[\frac{4}{9} (u - \bar{u}) - \frac{1}{9} (d - \bar{d}) \right] . \quad (8)$$

This implies that in the quantity

$$\frac{L_{1,\pi^+}^{ep} - L_{1,\pi^-}^{ep}}{\int [L_{1,\pi^+}^{ep} - L_{1,\pi^-}^{ep}] dz dh_T^2} = \frac{D_u^{\pi^+} - D_d^{\pi^+}}{\int (D_u^{\pi^+} - D_d^{\pi^+}) dz dh_T^2} \quad (9)$$

the presence of the proton is not felt at all. Thus, similar relations should hold for any target ($\pi, N, d, \Sigma, \dots$). It is amusing to note the striking symmetry between Eqs. (6), (7) and Eqs. (8), (9).

In addition, various inequalities can be derived. The basic inequalities are :

$$D_u^{\pi^+} \geq \frac{1}{3} D_d^{\pi^+} \quad (10)$$

which follows from isospin invariance and positivity constraints ; the well-known inequalities ¹⁰⁾

$$u \geq d/2, \quad \bar{d} \geq \bar{u}/2, \quad (11)$$

and their duality analogue :

$$v_u \geq v_d/2 \quad (12)$$

The SLAC-MIT ¹¹⁾ measurements of $F_2^{\text{en}}/F_2^{\text{ep}}$ imply :

$$u + \bar{u} \geq d + \bar{d} \quad (13)$$

and a duality analogue

$$v_u \geq v_d \quad (14)$$

In the Table, we give a few examples of bounds obtained at various stages. In the "Model" column, "Partons" denotes the quark parton model of Ref. 9) ; "Partons + Experiment" denotes the same model with Eq. (13) as a constraint ; "Partons + Duality" denotes the same model with duality constraints [Eqs. (3), (12)] ; "Partons + Duality + Experiment" denotes the above with Eq. (14) superimposed.

It is amusing to note that there is an inequality whose bounds do not depend on the choice of "Model" in the Table. This is :

$$0 \leq \frac{L_{\nu p}^{\pi^+} + L_{\nu p}^{\pi^-} + L_{\nu n}^{\pi^+} + L_{\nu n}^{\pi^-}}{L_{e p}^{\pi^+} + L_{e p}^{\pi^-} + L_{e n}^{\pi^+} + L_{e n}^{\pi^-}} \leq \frac{1}{5} \cdot 18 \quad (15)$$

All the neutrino bounds refer to the strangeness conserving amplitudes (i.e., can be applied to the full amplitudes in the zero approximation for the Cabibbo angle).

We would like to conclude with the following remarks.

- 1) We see that duality ideas impose strong constraints on single hadron distributions in the current fragmentation region.
- 2) It is worth noting that the Pomeron contribution is cancelled out in Eqs. (4a), (4b) and (6). Therefore, these equations and the conclusions they imply do not depend on the Pomeron being an SU(2) or SU(3) singlet.
- 3) The determination of D_s^{π} from Eq. (4c) needs one more parameter if the Pomeron is an SU(2) singlet only.
- 4) One expects SU(3) to be a reasonable approximate symmetry. In the electroproduction amplitudes, the deviation of the Pomeranchuk contribution from that of an SU(3) singlet is damped by a factor of $\sim 1:6$ in comparison with the SU(3) singlet contribution [see, e.g., Gronau et al. in Ref. 9]. Thus, we expect Eq. (4c) and the inequalities in the Table to be good approximations to exact results.
- 5) The parton model enables one to find quantities which are independent of the target [Eq. (9)]. Duality, when added, provides analogous quantities which do not depend on the observed hadron [Eq. (7)].

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Model \ Ratio	$\frac{L_{1,\pi^+}^{ep} + L_{1,\pi^+}^{en}}{L_{1,\pi^-}^{ep} + L_{1,\pi^-}^{en}}$	$\frac{L_{1,\pi^+}^{ep} + L_{1,\pi^-}^{ep}}{L_{1,\pi^+}^{en} + L_{1,\pi^-}^{en}}$	$\frac{L_{1,\pi^+}^{\nu p} + L_{1,\pi^+}^{\nu n}}{L_{1,\pi^+}^{ep} + L_{1,\pi^+}^{en}}$
Partons	$\frac{1}{4} \leq r \leq 4$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 18$
Partons + Experiment	$\frac{1}{4} \leq r \leq 4$	$1 \leq r \leq 4$	$0 \leq r \leq 18$
Partons + Duality	$\frac{7}{13} \leq r \leq 4$	$\frac{2}{3} \leq r \leq 4$	$0 \leq r \leq \frac{2}{5} \cdot 18$
Partons + Duality + Experiment	$\frac{7}{13} \leq r \leq 4$	$1 \leq r \leq 4$	$0 \leq r \leq \frac{2}{5} \cdot 18$

TABLE : Bounds for single pion distribution functions.
r denotes the ratio involved.

REFERENCES

- 1) H. Leutwyler and J. Stern - Phys.Letters 31B, 458 (1970) ;
E.D. Bloom and F.J. Gilman - Phys.Rev.Letters 25, 1140 (1970) ;
V. Rittenberg and H.R. Rubinstein - Phys.Letters 35B, 50 (1971) ;
P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B28, 225 (1971) ;
S. Pallua and B. Renner - Nuclear Phys. B43, 331 (1972).
- 2) M. Chaichian, S. Kitakado, S. Pallua, B. Renner and J. De Azcárraga -
DESY 72/50 (to be published in Nuclear Phys.).
- 3) H. Harari - Phys.Rev.Letters 22, 1395 (1968) ;
P.G.O. Freund - Phys.Rev.Letters 22, 235 (1968).
- 4) H. Harari - Phys.Rev.Letters 22, 562 (1969) ;
J.L. Rosner - Phys.Rev.Letters 22, 689 (1969).
- 5) C.H. Llewellyn Smith - Nuclear Phys. B17, 277 (1970) ;
J. Kuti and V.F. Weisskopf - Phys.Rev. D4, 3418 (1971).
- 6) H.J. Lipkin and E.A. Paschos - Phys.Rev.Letters 29, 525 (1972).
- 7) J. Ellis - Phys.Letters 35B, 537 (1971) ;
J.D. Stack - Phys.Rev.Letters 28, 57 (1972) ;
O. Nachtmann - IAS Princeton Preprint (1972) ;
H. Fritzsch and P. Minkowski - CERN Preprint TH. 1525 (1972).
- 8) S.D. Drell and T.M. Yan - Phys.Rev.Letters 24, 855 (1970) ;
P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B33, 221 (1971).
- 9) R.P. Feynmann - "Photon-Hadron Interactions", to be published by
W.A. Benjamin, New York (1972), and talk presented at the
Neutrino 1972 Conference, Balatonfüred, Hungary (June 1972) ;
M. Gronau, F. Ravndal and Y. Zarmi - CALTECH Preprint CALT-68-367
(to be published in Nuclear Phys.) ;
J.D. Bjorken - SLAC Preprint 1114 (September 1972).
- 10) O. Nachtmann - Nuclear Phys. B38, 397 (1972).
- 11) H. Kendall - in Proceedings of the 1971 International Symposium on
Electron and Photon Interactions at High Energies, Cornell Uni-
versity, Ithaca, N.Y. (1972) ; the corrected version of these
data is due to S.D. Drell (private communication).