CM-P00058505

Ref.TH.1591-CERN

Archives

PROPERTIES OF HADRON STATES CONTAINING A CONDENSED
PHASE OF QUARK-ANTI-QUARK EXCITATIONS

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A B S T R A C T

We analyze how one-particle states can arise in a field theory of (three) quark triplets with current-current interaction which does not produce asymptotic quark states.

A condensed phase of quark-anti-quark pairs resembling the corresponding phase in a superconductor is responsible for the lack of stable states in the sectors with triality different from zero. The one-particle states are stable configurations of valence quarks in the presence of the surrounding condensed phase.

Ref.TH.1591-CERN

28 November 1972

1. INTRODUCTION

We propose to discuss the structure of one-particle (hadron) states within a field theory of basic quark fields which do not produce asymptotic quark states. The Hamiltonian is split into the kinetic energy, an $SU_3 \times SU_3$ symmetric current-current (four-Fermion) interaction as main components and residual interactions comprising mass terms treated as a perturbation.

The current-current interaction preserves the validity of the bilocal algebra of good-good components on a lightlike plane ^{1),2),3)}.

The sign of the corresponding Fermi coupling constant is chosen such that the interaction is attractive to quark-antiquark pairs, repulsive between two quarks.

We argue that the resulting pairing force together with the asymptotic condition for the basic fields generate a phase of virtual $q\bar{q}$ excitations within a one-particle state which resembles the BCS phase proposed for a superconductor ⁴⁾. This pairing force is a general feature of relativistic fermion systems with asymptotic chiral symmetry on the light cone ⁵⁾.

The limit to infinite volume which is related to renormalization can be performed for certain physical quantities as the energy gap, the dispersion law for virtual quarks within the condensed phase and under certain conditions for the particle mass. Independently of the above limit the self-consistency equation for the energy gap yields a momentum cut-off related to the Fermi coupling constant.

The residual interactions which modify the main, long range force presumably allow for valence quarks with very small momenta relative to the cut-off momentum. The one-particle states form the $36 = 1 + 35$ representation of SU_6 in the meson sector and the 56 representation in the baryon sector if three quark triplets distinguished by their colour are assumed ^{6),7)}.

It is shown that the so-constructed one-particle states are consistent with the absence of physical quarks. This fact is intimately related to the condensed phase and the mass gap it generates in complete analogy to the BCS phase.

In our opinion it is not excluded that the infinite set of renormalization constants associated with the four-Fermion interaction can be determined from a sufficiently large set of boundary asymptotic conditions including the validity of the light cone algebra of bilocal currents ⁸⁾.

The current-current interaction and the resulting pairing force are discussed in Section 2.

The characteristics of the condensed phase are worked out in Section 3. The mass spectrum in the meson, baryon, anti-baryon sectors and in the sector with triality one are treated in Section 4.

2. CURRENT-CURRENT INTERACTION, PAIRING FORCE

We disregard for the time being the colour quantum number and work with a triplet of left-handed and right-handed quark fields

$$\psi_b^{\dot{\beta}}, \quad u_{a,\alpha} \quad (2.1)$$

where $b, a = 1, 2, 3$, $\alpha, \dot{\beta} = 1, 2$ denote unitary spin and spin indices respectively. The relation of the above spinor notation to the four-component Dirac notation is given in the Appendix.

In close analogy with weak interactions of massless leptons ⁹⁾ the equations of motion take the form

$$\begin{aligned} p^{\dot{\beta}\alpha} u_{a\alpha}(x) &= g j_{ba}^{-\dot{\beta}\alpha}(x) u_{b\alpha}(x) \\ p_{\alpha\dot{\beta}} \psi_b^{\dot{\beta}}(x) &= g j_{ab/\alpha\dot{\beta}}^+(x) \psi_a^{\dot{\beta}}(x) \\ p_{\alpha\dot{\beta}} &= i \left(\partial_x^\mu \sigma_\mu \right)_{\alpha\dot{\beta}} ; p^{\dot{\beta}\alpha} = i \left(\partial_{x\mu} \sigma_\mu \right)^{\dot{\beta}\alpha} \end{aligned} \quad (2.2)$$

$$\sigma_\mu = (\sigma_0, \vec{\sigma}) : \text{Pauli matrices}$$

In Eq. (2.2) $j_{ab}^\pm$ denote the left-handed (+), and right-handed (-) chiral currents

$$\begin{aligned} j_{ab/\alpha\dot{\beta}}^+ &= \psi_a^{\dot{\beta}} \psi_{b\alpha} \sim \frac{1}{4} \left[(V_\mu + A_\mu)^{\alpha\dot{\beta}} \sigma_\mu \right]_{\alpha\dot{\beta}} \\ j_{ab}^{-\dot{\beta}\alpha} &= u_a^{\dot{\beta}} u_b^\alpha \sim \frac{1}{4} \left[(V_\mu - A_\mu)^{\alpha\dot{\beta}} \sigma_\mu \right]_{\beta\alpha} \end{aligned} \quad (2.3)$$

$g=1/m^2$ is the corresponding Fermi coupling constant with the dimension of an area.

Neglecting residual interactions Eqs. (2.2) and (2.3) describe a left-handed and a right-handed world without any forces between them. In this approximation the chiral $SU_3 \times SU_3$ group is an exact symmetry and the 18 chiral currents are conserved.

We assume that the many body phenomena relevant for the formation of one-particle states are insensitive to the residual interactions and restrict the following discussion to the left-handed world.

The boundary condition that no asymptotic free fields are associated with v_b^{β} can formally be incorporated into the retarded Yang-Feldman equations which become homogeneous

$$\begin{aligned} \tau_b^{\beta}(x) &= g \int d^4y \, d_{ret}^{\beta\alpha}(x-y) j_{ab/a\beta}^+(y) v_a^{\alpha}(y) \\ d_{ret}^{\beta\alpha}(z) &= P^{\beta\alpha} D_{ret}(z) = P^{\beta\alpha} f(z^0) \frac{1}{2\pi} \delta(z^2) \end{aligned} \quad (2.4)$$

Equation (2.2) follows from the Lagrangian density

$$\mathcal{L}(x) = \frac{1}{2} v_b^+(x) i \overleftrightarrow{\partial}_x^\mu \sigma_\mu v_b(x) - \frac{1}{2} g \left[v_{b\beta}^+ v_{a\alpha} v_a^{\beta} v_b^{\alpha} \right] \quad (2.5)$$

Adopting the canonical anti-commutation relations

$$\left[v_b^{\beta}(x), v_a^{\alpha}(y) \right]_{x^0=y^0}^+ = \delta_{ba} \delta^{\beta\alpha} \delta^3(\vec{x}-\vec{y}) \quad (2.6)$$

which are not affected by wave function renormalization one recovers the Hamiltonian density

$$\begin{aligned} \mathcal{H} &= \frac{1}{2} v_b^+ i \overleftrightarrow{\nabla} \sigma v_b + \frac{1}{2} g \left[v_{b\beta}^+ v_{a\alpha} v_a^{\beta} v_b^{\alpha} \right] \\ &= \mathcal{H}_{kin} + \frac{g}{16} (V^\mu + A^\mu)_{ba} (V_\mu + A_\mu)_{cb} \end{aligned} \quad (2.7)$$

From Eqs. (2.6) and (2.3) one of course obtains the equal time algebra of currents involving all Lorentz indices without Schwinger terms.

As in the case of a free field with mass zero Eq. (2.6) can be realized by setting

$$v_b^{\dot{\alpha}}(\vec{x}, 0) = \int \frac{d^3 q}{2|q|} e^{+i\vec{q}\cdot\vec{x}} \times \left\{ \varepsilon^{\dot{\alpha}\dot{\beta}}(q) a_b^{\dot{\beta}}(q) + \varepsilon^{\dot{\alpha}\dot{\beta}}(-q) b_b^{*\dot{\beta}}(-q) \right\} \quad (2.8)$$

if the anti-commutation relations

$$\begin{aligned} [a_b^{\dot{\beta}}(q), a_{b'}^{*\dot{\beta}}(q')]_+ &= [b_b^{\dot{\beta}}(q), b_{b'}^{*\dot{\beta}}(q')]_+ = \\ &= \frac{2|q|}{(2\pi)^3} \delta_{bb'} \delta^3(q-q') \end{aligned} \quad (2.9)$$

hold. The spinors $\varepsilon(q)$ in Eq. (2.8) are normalized as follows

$$\varepsilon^{\dot{\alpha}\dot{\beta}}(q) \varepsilon^{\alpha}(q) = q^{\dot{\beta}\alpha} = \begin{pmatrix} |q|-q_3 & -q_1+iq_2 \\ -q_1-iq_2 & |q|+q_3 \end{pmatrix} \quad (2.10)$$

In the limit $q \rightarrow 0$ the above relations are most sensitive to residual interactions and to the screening of the long range four-Fermion force in higher orders in g .

The operators a_b^{*L} , b_b^{*R} create left-handed quark excitations and right-handed anti-quark excitations respectively.

The interaction Hamiltonian is split into four parts

$$H_I = H_{vv} + H_{\bar{v}\bar{v}} + H_{v\bar{v}} + H_R \quad (2.11)$$

H_{vv} ($H_{\bar{v}\bar{v}}$) describes elastic scattering of two quarks (anti-quarks); $H_{v\bar{v}}$ describes elastic scattering of a quark-anti-quark pair and H_R contains all terms involving net creation or absorption.

For $g > 0$ $H_{v\bar{v}}$ gives rise to an attractive pairing force, H_{vv} ($H_{\bar{v}\bar{v}}$) to repulsive forces between two quarks (anti-quarks).

We assume at this point that the one-particle states in their rest frame are formed by a condensed phase of $q\bar{q}$ pairs which is determined by the pairing force alone except for very small internal momenta. For very small momenta as compared to $g^{-\frac{1}{2}}$ arbitrarily weak residual forces connecting left-handed and right-handed worlds dominate and modify the pairing force which generates the $q\bar{q}$ sea. H_{vv} , $H_{\bar{v}\bar{v}}$ and H_R are considered as parts of residual interactions which besides a stabilizing rôle do not affect the internal dynamics of one-particle states.

Thus we retain only $H_{v\bar{v}}$. Using the normalized quantities

$$L_b [R_b] (q) = \left(\frac{(2\pi)^3}{2/q} \right)^{1/2} a_b [b_b] (q)$$

$$[L_b(q), L_{b'}^{v'}(q')]_+ = \delta_{bb'} \delta^3(q-q'), \dots$$

$$s_\omega(q) = (2/q)^{-1/2} \varepsilon_\omega(q)$$

$H_{v\bar{v}}$ takes the form

$$H_{v\bar{v}} = - \frac{g}{(2\pi)^3} \left\{ \int \frac{\pi^4}{i} d^3 q \cdot \delta^3(q_4 + q_2 - q_1 - q_3) \times \right. \\ \left. \left[\begin{array}{ccc} L^*(q_4) & R^*(-q_3) & R_q(-q_2) L_b(q_1) \\ + L^*(q_4) & R^*(-q_3) & R_q(-q_2) L_a(q_1) \end{array} \right] \right\} \quad (2.12)$$

Only the forward $q\bar{q}$ scattering diagrams with $q_4 \simeq q_1$, $q_3 \simeq q_2$ contribute with a well defined phase to the ground state (one-particle state) energy. The non-forward matrix element $h_{v\bar{v}}$ will exhibit a phase

$$h_{v\bar{v}}(p, r, \Delta) = |h_{v\bar{v}}| e^{i\phi} \quad (2.13)$$

$$p = q_1 - q_2 = q_4 - q_3, \quad r = \frac{q_1 + q_2}{2}, \quad \Delta = q_4 - q_1 = q_3 - q_2$$

If the phase ϕ varies with Δ much more rapidly than the modulus of $h_{v\bar{v}}$ the main contribution to the ground state energy will only contain

$$h_{v\bar{v}}(p, r, \Delta^*)$$

The region with $\phi \simeq 0$ will be characterized by

$$|\Delta| \leq 2\pi \Delta^* \quad (2.14)$$

where Δ^* depends on g and the residual interactions. Thus we replace in Eq. (2.12)

$$L^*(q_4) \dots L(q_1)$$

by

$$(2\pi)^3 (\Delta^*)^3 \delta^3(q_4 - q_1) L^*(q_1) \dots L(q_1)$$

which yields the reduced Hamiltonian. The fact that the above phase is undetermined is related to the existence of a local gauge group for massless fermion fields corresponding to the substitutions in momentum space

$$\psi(p) \rightarrow \psi(p) + \pi \chi \quad p^2 = 0$$

$$H_{v\bar{v}}^{\text{red}} = -\bar{m} \int d^3q d^3\bar{q} \cos^2\left(\frac{\Theta}{2}\right) \times$$

$$: \left[h_{bb}^L(q) h_{aa}^R(-\bar{q}) + h_{ba}^L(q) h_{ba}^R(-\bar{q}) \right] : \quad (2.15)$$

$$\bar{m} = g \cdot (\Delta^*)^3 = \frac{(\Delta^*)^3}{m^2}$$

$$h_{ba}^L(q) = L_b^*(q) L_a(q) \quad ; \quad h_{ba}^R(q) = R_b^*(q) R_a(q)$$

$$\cos \Theta = \frac{\vec{q} \cdot \vec{\bar{q}}}{|\vec{q}| |\vec{\bar{q}}|}$$

The pairing force in Eq. (2.15) tends to align a quark-anti-quark pair in the configuration shown in Fig. 1.

The approximation leading to $H_{v\bar{v}}^{\text{red}}$ implies

$$\bar{m} \gg \Delta^* \rightarrow \bar{m} \gg \Delta^* \gg m$$

From consideration of the transverse momentum cut-off in high energy hadron collisions we find as discussed in Section 4

$$\bar{m} \sim 230 \text{ MeV}$$

which yields for the ratio of g to the Fermi coupling constant

$$\frac{g}{G_F} \gg 10^5 \left(\frac{m_P}{\bar{m}} \right)^2 \sim 1.6 \cdot 10^6 \quad (2.16)$$

The effective Hamiltonian in Eq. (2.15) which will give rise to the condensed phase in the one-particle states is very similar to the pairing force in the BCS theory of superconductivity.

$$H_T = H_{kin} + H_{v\bar{v}}^{\text{red}}$$

$$H_{kin} = \int d^3q \frac{1}{q} \left[h_{bb}^L(q) + h_{bb}^R(q) \right] \quad (2.17)$$

3. CONDENSED PHASE IN ONE-PARTICLE STATES

In this Section we investigate the many-body phenomena due to the pairing force derived above in conjunction with the asymptotic condition on the quark fields.

The equations of motion are

$$\begin{aligned} [\bar{H}_1, L_a(q)] = & -1/q L_a(q) + \bar{m} \int d^3 \bar{q} \cos^2\left(\frac{\theta}{2}\right) \times \\ & : \left\{ R_b(-\bar{q}) L_b(q) R_a^*(-\bar{q}) + \right. \\ & \left. + R_b(-\bar{q}) L_a(q) R_b^*(-\bar{q}) \right\} : \end{aligned} \quad (3.1)$$

$$\begin{aligned} [\bar{H}_1, R_a^*(-\bar{q})] = & 1/\bar{q} R_a^*(-\bar{q}) - \bar{m} \int d^3 q \cos^2\left(\frac{\theta}{2}\right) \times \\ & : \left\{ L_b^*(q) L_a(q) R_b^*(-\bar{q}) + \right. \\ & \left. + L_b^*(q) L_b(q) R_a^*(-\bar{q}) \right\} : \end{aligned}$$

The one-particle states will be discussed in the rest frame only. A transformation to a frame in which the total momentum differs from zero cannot be performed maintaining the approximations made.

Following the idea of a condensed (superconducting) phase in the ground state $|\psi^0\rangle$ we assume that an amplitude for virtually bound quark-anti-quark pairs exists

$$\langle \psi^0 | R_b^*(-\bar{q}) L_a^*(q) | \psi^0 \rangle = \delta^3(q - \bar{q}) h_{ba}(q) \quad (3.2)$$

Replacing the operator expressions

$$R_b(-\bar{q}) L_a(q) \quad , \quad R_b^*(-\bar{q}) L_a^*(q)$$

in Eq. (3.1) by the corresponding ground state matrix elements one finds

$$[H_1, L(q)] = -|q| L(q) - \bar{m} H^+(q) R^*(-q) \quad (3.3)$$

$$[H_1, R^*(q)] = |q| R^*(q) - \bar{m} H(q) L(q)$$

In Eq. (3.3) we have used matrix notation with respect to SU3 indices with

$$H_{ab} = h_{ab} + \delta_{ab} \sum_c h_{cc}$$

H, H^+ can always be reduced to diagonal form through the substitutions

$$L(q) \rightarrow U(q) L(q), \quad R^*(-q) \rightarrow U'(q) R^*(-q)$$

$$H(q) \rightarrow U'(q) H(q) U^{-1}(q)$$

$$H^+(q) \rightarrow U(q) H^+(q) (U')^{-1}(q)$$

Without loss of generality we can write

$$H = H^+ = \begin{pmatrix} p_u(q) & & 0 \\ & p_d(q) & \\ 0 & & p_s(q) \end{pmatrix} \quad (3.4)$$

$$p_i(q) \geq 0 \quad ; \quad i = u, d, s$$

The Bogolubov-Valatin transformation yields the eigen-frequency operators

$$\delta_{i1}(q) = u_i(q) L_i(q) + v_i(q) R_i^*(-q)$$

$$\delta_{i2}(-q) = u_i(q) R_i(q) - v_i(q) L_i^*(q) \quad (3.5)$$

$$|u_i|^2 + |v_i|^2 = 1$$

which obey the anti-commutation relations

$$[\delta_{i\alpha}(q), \delta_{i'\alpha'}(q')] = \delta_{ii'} \delta_{\alpha\alpha'} \delta^3(q-q') \quad (3.6)$$

$\alpha, \alpha' = 1, 2$

The functions $u_i(q)$, $v_i(q)$ are determined up to a common phase from the equations of motion

$$[H_1, \delta_{i\alpha}(q)] = -E_{q,i} \delta_{i\alpha}(q) \quad (3.7)$$

which imply

$$\begin{aligned} E_{q,i} u_i(q) &= |q| u_i(q) + \bar{m} p_i(q) v_i(q) \\ E_{q,i} v_i(q) &= \bar{m} p_i(q) u_i(q) - |q| v_i(q) \end{aligned} \quad (3.8)$$

From Eq. (3.8) follows the dispersion law

$$(E_{q,i})^2 = q^2 + \bar{m}^2 p_i^2(q) \quad (3.9)$$

$\bar{m} p_i$ represents the energy gap which will provide a means to satisfy the boundary condition on the quark fields.

Furthermore, one obtains

$$\begin{aligned} |u_i(q)|^2 &= \frac{1}{2} \left(1 + \frac{|q|}{E_{q,i}} \right) \\ |v_i(q)|^2 &= \frac{1}{2} \left(1 - \frac{|q|}{E_{q,i}} \right) \end{aligned} \quad (3.10)$$

In order to obtain the self-consistency requirement for the energy gap $\bar{m} \int_i$ we are forced at this point to introduce discrete momenta defining the operators

$$L[R](q^*) = \frac{1}{\sqrt{V_0}} \int d^3 q \chi_{V_0}(q-q^*) L[R](q)$$

$$\chi_{V_0}(q) = \begin{cases} 1 & -\frac{q}{2} \leq q_i < \frac{q}{2} \quad i=1,2,3 \\ 0 & \text{otherwise} \end{cases} \quad (3.11)$$

$$q^* = a (n_1, n_2, n_3); \quad n_i = 0, \pm 1, \pm 2, \dots$$

$$V_0 = a^3$$

This procedure allows to construct explicitly normalized one-particle states for which, however, the limit $a \rightarrow 0$ cannot be performed in a simple way. Nevertheless the above limit exists for the self-consistency equation for $\int_i$.

Similarly to Eq. (3.11) we define

$$\delta_{i\alpha}(q^*) = \frac{1}{\sqrt{V_0}} \int d^3 q \chi_{V_0}(q-q^*) \delta_{i\alpha}(q) \quad (3.12)$$

and use the mean value theorem to approximate $\delta_{i\alpha}(q^*)$ by

$$\begin{aligned} \delta_{i1}(q^*) &\simeq u_i(q^*) L_i(q^*) + v_i(q^*) R_i^*(-q^*) \\ \delta_{i2}(-q^*) &\simeq u_i(q^*) R_i(-q^*) - v_i(q^*) L_i^*(q^*) \end{aligned} \quad (3.13)$$

The approximation in Eq. (3.13) is presumably correct relative to the ground state if we choose

$$V_0 / \bar{m}^3 \ll 1$$

Following Bardeen, Cooper and Schrieffer ⁴⁾ the ground state is given by

$$|\psi_{V_0}^L\rangle = \prod_{q^*} \prod_i \{u_i(q^*) + v_i(q^*) R_i^*(q^*) L_i^*(q^*)\} |\Omega\rangle \quad (3.14)$$

$|\Omega\rangle$ denotes the vacuum states for the excitations $L_i^*(q^*)$, $R_i^*(q^*)$.

There is a difference between the ground state in Eq. (3.14) and the BCS ground state because the individual pairs $R_i^*(-q^*)$, $L_i^*(q^*)$ carry unit spin along the direction of $-q^*$.

A Lorentz covariant description would imply the replacements

$$u_i(q^*) + v_i(q^*) R_i^*(q^*) L_i^*(q^*) \rightarrow \quad (3.15)$$

$$u_i(q^*) \frac{q^3 \alpha}{191} + v_i(q^*) \frac{q^3 \alpha}{191} R_i^*(q^*) L_i^*(q^*)$$

and the introduction of a spin correlation function

$$C_{\beta_1 \alpha_1, \beta_2 \alpha_2, \dots} (q_1^*, q_2^*, \dots)$$

All the pairs with momenta q^* , $-q^*$ can combine to yield zero total spin except for $q^*=0$ pairs which will demand special attention.

We note that $|\psi_{V_0}^L\rangle$ is not an eigenstate of the parity operator. The latter maps the Fock space of left-handed quark (right-handed anti-quark) excitations

$$L_i^*(q), R_i^*(q)$$

onto a corresponding orthogonal Fock space of right-handed quark (left-handed anti-quark) excitations generated by the reflected operators

$$R_i'^*(q) = \hat{P} L_i^*(-q) \hat{P}^{-1}$$

$$L_i'^*(q) = \hat{P} R_i^*(q) \hat{P}^{-1}$$

$\hat{P}$ denotes the parity operator.

The complete ground state is then given by

$$| \psi_{V_0}^{L+R} \rangle = \left[\frac{\pi}{q_{i,i}^*} \{ u_i(q^*) + v_i(q^*) R_i^*(-q^*) L_i^*(q^*) \} \times \right. \\ \left. \frac{\pi}{q_{i,i}^*} \{ u_i'(q^*) + v_i'(q^*) L_i'^*(-q^*) R_i'^*(q^*) \} \right] | \Omega \rangle \quad (3.16)$$

The self-consistency equations for the six quantities ρ_i , ρ_i' take the form

$$2 \rho_i^{(1)} = \frac{\rho_i^{(1)}}{\sqrt{(\rho_i^{(1)})^2 + \frac{q^2}{m^2}}} + \sum_{\alpha=1}^3 \frac{\rho_{\alpha}^{(1)}}{\sqrt{(\rho_{\alpha}^{(1)})^2 + \frac{q^2}{m^2}}} \quad (3.17)$$

We prove that there are no solutions to (3.17) unless $\rho_i \equiv \rho_j$. Assuming the contrary, e.g., $\rho_1 \neq \rho_2$ it follows from Eq. (3.17)

$$2(x-y) = f(x) - f(y) \quad \rho_1, \rho_2 \rightarrow x, y \quad (3.18)$$

$$f(x) = \frac{x}{\sqrt{x^2 + a^2}} \quad ; \quad a = \frac{\hbar}{m}$$

Since $f(x)$ is a convex function $[f''(x) < 0, x > 0]$ it follows from Eq. (3.18) for $x \neq y$

$$x, y \leq x^*$$

where x^* is the root of the equation

$$2 x^* = f(x^*) \implies x^* = \frac{1}{4} - a^2 (\geq 0)$$

On the other hand, Eq. (3.17) implies

$$2x \geq f(x), \quad 2y \geq f(y)$$

which yields

$$x, y \geq x^*$$

and thus

$$x = y$$

contrary to the assumption.

The general solution of Eq. (3.17) is of the form

$$\rho_1^{(1)} = \rho_2^{(1)} = \rho_3^{(1)} = \rho^{(1)}$$

$$(\rho^{(1)})^2 = \begin{cases} 4 - \frac{q^2}{\bar{m}^2} & |q| \leq q_c^{(1)} \leq 2\bar{m} \\ 0 & |q| > q_c^{(1)} \end{cases} \quad (3.19)$$

We choose the continuous solutions

$$q_c = q_c' = 2\bar{m} \quad (3.20)$$

The equality of q_c and q_c' will be enforced by terms in the total Hamiltonian which combine right and left-handed Fock spaces.

We have assumed that the self-masses of the excitations L , R , are negligible within a large condensed phase whereas the vacuum expectation value of e.g., the anti-commutator of the basic quark fields must be of the form

$$\langle \Omega | [\bar{\psi}_a(x), \bar{\psi}_b(y)] | \Omega \rangle =$$

$$\delta_{ab} \left[i \gamma^4 \partial_z \int_{M^2}^{\infty} dm^2 \rho_2(m^2) \Delta(z, m^2) + \int_{M^2}^{\infty} dm^2 \rho_1(m^2) \Delta(z, m^2) \right] \quad (3.21)$$

$$z = x - y, \quad M^2 > 0$$

in order to satisfy the boundary conditions on ψ . In the limit of a large average number of excitations in $|\psi^{L+R}\rangle$ one obtains for the expectation value of H_1

$$\langle \psi^{L+R} | H | \psi^{L+R} \rangle \sim -2\bar{m} \left(\frac{N}{5} + \frac{N^2}{24} \right) \quad (3.22)$$

$$N \gg 1$$

where N denotes the average total number of excitations L, R, L', R' in $|\psi^{L+R}\rangle$.

In establishing (3.22) one encounters the difficulty that there is no continuous momentum limit for $|\psi^{L+R}\rangle$.

The energy eigen-value and the detailed form of the density of excitations within the one-particle states in momentum space $n(q)$ with $N = \int d^3q n(q)$ are determined entirely by the residual interactions. The energy difference between stable and excited states are well defined.

The ground state constructed according to Eqs. (3.16), (3.19) and (3.20) has the quantum numbers of the vacuum. We identify this state with the quark-anti-quark sea which gives rise in the quark parton model to the Pomeron part of deep inelastic structure functions. In the present model valence quarks are confined to the region of small momenta q . Thus we adopt the ground state according to Eq. (3.16) except for the cell with $q^* = 0$. The present model does not explain the existence of valence quarks. If valence quarks are taken for granted, however, the surrounding $q\bar{q}$ sea admits simple momentum space wave functions in contrast with the corresponding situation in a potential well. The baryon spectrum demands a modification of the current-current interaction discussed so far if Fermi statistics are to be maintained. These play an essential rôle in the dynamics of the condensed phase. Following Gell-Mann ^{6), 7)} we introduce three triplets of quark fields distinguished by a new quantum number, e.g., red, green and blue colour. These modifications amount to the following substitutions

$$L_a(q), \dots \rightarrow L_a^i(q), \dots \quad i = \text{red, green, blue}$$

$$j + \beta_a^\alpha(x) \rightarrow \sum_i v_b^{\alpha i}(x) v_a^{\beta i}(x)$$

$$\rho^2 \rightarrow \rho_c^2 = g - \frac{g^2}{\bar{m}^2}$$

$$E_{q,a} = E = 2\bar{m} \rightarrow E_{q,a}^i = E = 3\bar{m}$$

$$|\psi^{L+R}\rangle \rightarrow \left\{ \begin{array}{l} \pi_{q,a,i} \{ u(q^*) + v(q^*) P_a^*(-q^*) L_a^i(q^*) \} \\ \pi_{q,a,i}^* \{ u(q^*) + v(q^*) L_a^{i*}(q^*) P_a^*(q^*) \} \end{array} \right\} |\Omega\rangle \quad (3.23)$$

$$u^2[v^2] \rightarrow u^2[v^2] = \frac{1}{2} \left(1 \mp \frac{19}{3\bar{m}} \right)$$

$$\langle \psi^{L+R} | H_1 | \psi^{L+R} \rangle \rightarrow \langle \psi_c^{L+R} | H_1 | \psi_c^{L+R} \rangle = -3\bar{m} \left(\frac{N}{5} + \frac{N^2}{72} \right)$$

The breakdown of the condensed phase can be traced back to the build up of a large enough effective mass $\mu(q)$ of the primary excitations in the vicinity of $q=0$. Then, the self-consistency equation (3.17) takes the form

$$\rho_c^2 = g \frac{\rho_c^2}{\rho_c^2 + \frac{g^2 + \mu^2(q)}{\bar{m}^2}} \Rightarrow \rho_c^2 = g - \frac{g^2 + \mu^2(q)}{\bar{m}^2} \quad (3.24)$$

The BCS condensation cannot occur whenever

$$\sqrt{\mu^2(q) + g^2} \geq 3\bar{m} \quad (3.25)$$

At the same time the right and left-handed operators $L_a^i(q)$, $R_a^i(q)$ loose their meaning for $\mu(q) \neq 0$ and have to be replaced by the appropriate absorption operators for a massive spin $\frac{1}{2}$ field

$$a_{a\sigma}^i(q), \quad b_{a\sigma}^i(q) \quad (3.26)$$

where $\sigma = 1, 2$ denotes the spin orientation with respect to a fixed axis. The operators in Eq. (3.26) satisfy the anti-commutation relations

$$\begin{aligned} [a_{\alpha}^i(q), a_{\alpha'}^{*i'}(q')] &= [b_{\alpha}^i(q), b_{\alpha'}^{*i'}(q')] = \\ &= \delta_{ii'} \delta_{\alpha\alpha'} \delta^3(q-q') \end{aligned} \quad (3.27)$$

$$\alpha = (\alpha, \sigma) ; \alpha' = (\alpha', \sigma') \quad \alpha, \alpha' = 1, \dots, 6$$

with all other anti-commutators vanishing.

The actual presence of valence quarks in the state with minimal energy is readily accommodated by an over-all positive energy representing the action of the neutral $q\bar{q}$ sea which is independent of the occupation numbers $(0,1)$ of the 36 possible excitations in the cell $q^*=0$. The present model gives no indication however why the complete one-particle states consists of just one $q\bar{q}$ pair (with neutral colour) in the meson sector and of just three quark excitations with neutral colour in the baryon sector.

We are led to consider a phenomenological Hamiltonian for $q^*=0$, e.g.,

$$\begin{aligned} H(q^*=0) &= r + \mu (v + \bar{v}) - \frac{c}{3} M_{\alpha\beta}^* M_{\alpha\beta} (3 - v - \bar{v}) \\ &- \frac{c'}{36} (B_{\alpha\beta\gamma}^* B_{\alpha\beta\gamma} + \bar{B}_{\alpha\beta\gamma}^* \bar{B}_{\alpha\beta\gamma}) (4 - v - \bar{v}) \end{aligned}$$

$$v = \sum_{i,\alpha} a_{\alpha}^{*i} a_{\alpha}^i, \quad \bar{v} = \sum_{i,\alpha} b_{\alpha}^{*i} b_{\alpha}^i \quad (3.28)$$

$$M_{\alpha\beta} = \sum_i b_{\alpha}^i a_{\beta}^i$$

$$B_{\alpha\beta\gamma} = \sum_{i,j,k} \varepsilon_{ijk} a_{\alpha}^i a_{\beta}^j a_{\gamma}^k$$

$$\bar{B}_{\alpha\beta\gamma} = \sum_{i,j,k} \varepsilon_{ijk} b_{\alpha}^i b_{\beta}^j b_{\gamma}^k$$

$$\mu = \mu(q=0)$$

$r > 0$ stands for the energy due to the condensed phase and c, c' denote positive coupling constants with the dimension of mass. Choosing

$$\begin{aligned} c &= 2\mu + r - m_M \\ c' &= 3\mu + r - m_B \end{aligned} \quad 0 < m_M, m_B < r \quad (3.29)$$

there exist three sets of one-particle states with minimal total energy, all with neutral colour, one each in the meson, baryon, and anti-baryon sector

$$|P_{\alpha\beta}\rangle = \frac{1}{\sqrt{3}} M_{\alpha\beta}^* |\psi'\rangle \quad E_P = m_M \quad (3.30)$$

$$|X_{\alpha\beta\gamma}\rangle = \frac{1}{6} B_{\alpha\beta\gamma}^* |\psi'\rangle \quad E_X = m_B$$

$$|\bar{X}_{\alpha\beta\gamma}\rangle = \frac{1}{6} \bar{B}_{\alpha\beta\gamma}^* |\psi'\rangle \quad E_{\bar{X}} = m_B$$

$|\psi'\rangle$ denotes the neutral state as given in Eq. (3.23) with $q^* = 0$ omitted.

The meson and baryon states form a 36 (1+35) and a 56 representation of SU6 respectively.

4. QUARK EXCITATION SPECTRUM

We discuss the spectrum of excitations of mesonic one-particle states. The corresponding spectra from baryon and anti-baryon states are analogous.

The dispersion law for quark (anti-quark) excitations in a one-particle state is given by

$$E(q) = \begin{cases} \mu & q^* = 0 \\ x & q^* \neq 0 \quad |q^*| \leq x \\ |q^*| & |q^*| > x \end{cases} \quad (4.1)$$

$$x = 3\bar{m}, \quad \mu \geq x$$

We assume that in the limit of continuous momenta $E(q)$ approaches a smooth function of the qualitative shape shown in Fig. 2.

The lowest excitations in the triality one sector display the energy momentum spectrum

$$(E_1, q) = (m_M + E(q), q) \quad (4.2)$$

and the break up of one bound pair gives rise to excited meson states with

$$(E_2, q) = (m_M + E(q_1) + E(q_2), q_1 + q_2) \quad (4.3)$$

The above sets of states are not Lorentz invariant, because they are all constructed from a one-particle state at rest. Together with given points $(E_{1,2}, q)$ corresponding mass shells

$$(m_{1,2})^2 = (E_{1,2})^2 - q^2$$

lie in the spectrum of the mass square operator. The augmented set of states forms continua in the triality one and in the meson sector with

$$(m_1)^2 \geq (m_M)^2 + 2\pi m_M \quad (4.4)$$

$$(m_2)^2 \geq (m_M)^2 + 4\pi m_M$$

In the case of excited baryons one finds

$$(m_{2,B})^2 \geq (m_B)^2 + 4\pi m_B \quad (4.5)$$

In order that these continuum states can decay into stable particles one has to impose

$$\frac{3}{4} m_M \leq \pi \quad (4.6)$$

In the case of unequal masses the above criterion leads to

$$\frac{3}{4} m_\pi \leq \pi \quad (4.7)$$

The stability of the meson states against single quark excitation demands that the spectral functions $\rho_{1,2}^{(m)}$ defined in Eq. (3.21) vanish unless

$$m^2 \geq m_M^2 + 2 \mathcal{E} m_M \quad (4.8)$$

An estimate for $\mathcal{E}$ may be obtained from the observed transverse momentum distribution of secondaries in a high energy collision demanding that the average transverse momentum square per particle after the collision should be the same as the corresponding quantity per quark excitation before the collision

$$\frac{2}{\sqrt{15}} \mathcal{E} \sim 350 \text{ MeV} \quad (4.9)$$

$$\mathcal{E} \sim 680 \text{ MeV}$$

Finally, we note how the individual excitations within e.g., a nucleon transform to the $p_z \rightarrow \infty$ frame. The wave function will not show the simple factorization into pairs with opposite momentum and helicity. Nevertheless a given excitation with momentum q_R in the nucleon rest frame will have a four momentum approximately equal to

$$q' (p_z \rightarrow \infty) \sim (p_z \xi, q_\perp^R, p_z \xi) \quad (4.10)$$

$$\xi(q_R) = \frac{\sqrt{p^2(q_R) + q_R^2} + (q_R)_z}{m_B}$$

According to the qualitative picture in Fig. 2 we have for the valence quarks

$$0 \leq \xi \leq \text{Min} \left(1, \frac{\mu^{(0)}}{m_B} \right) \quad |q_\perp| \leq \mathcal{E} \quad (4.11)$$

and for the $q\bar{q}$ sea

$$0 \leq \xi \leq \text{Min} \left(1, \frac{2\mathcal{E}}{m_B} \right); \quad \langle q_\perp^2 \rangle_{av} = \frac{4}{15} \mathcal{E}^2 \quad (4.12)$$

$$|q_\perp| \leq \mathcal{E}$$

The above kinematical regions have to be weighted by appropriate distribution functions inherent to the quark parton model¹⁰⁾. If the picture obtained in the particle rest frame remains true in the infinite momentum frame, i.e., that a hadron is composed of valence quarks (not necessarily SU3 symmetric) surrounded by a SU3 symmetric sea of quark-anti-quark pairs one naturally arrives at the following ansatz for the scaling functions in deep inelastic electron, and neutrino scattering off nucleons

$$f_1^{eP} = \frac{4}{9} u(\xi) + \frac{1}{9} d(\xi) + \frac{4}{3} \lambda(\xi)$$

$$f_1^{en} = \frac{1}{9} u(\xi) + \frac{4}{9} d(\xi) + \frac{4}{3} \lambda(\xi)$$

$$f_1^{\nu P} = 2 d(\xi) + 4 \lambda(\xi)$$

$$f_1^{\bar{\nu} P} = 2 u(\xi) + 4 \lambda(\xi)$$

$$f_3^{\nu P} = -2 d(\xi)$$

$$f_3^{\bar{\nu} P} = -2 u(\xi)$$

$$\xi = -\frac{q^2}{2\nu}$$

$$f_1(\xi) = \lim_{-q^2, \nu \rightarrow \infty} 2 W_1(q^2, \nu)$$

$$f_3(\xi) = \lim_{-q^2, \nu \rightarrow \infty} \nu W_3(q^2, \nu)$$

u , d denote the longitudinal momentum distribution of valence u and d type quarks in the proton. λ denotes the corresponding distribution of any kind of quarks or anti-quarks in the $q\bar{q}$ sea.

Sum rules follow from the normalization conditions

$$\int_0^1 d\xi u(\xi) = 2, \quad \int_0^1 d\xi d(\xi) = 1$$

The above relations are identical to the solution obtained from requirements of duality in Ref. 11).

After this work was completed we became aware that Domokos¹²⁾ had pursued similar ideas for a basic four fermion interaction on a light-like plane. We note that the stability of one-particle states cannot be discussed in such a frame. The present analysis however confirms the results stated in Ref. 12).

ACKNOWLEDGEMENTS

It is a pleasure to thank Y. Zarmi for his continued encouragement and interest in the present work. We also thank K. Gottfried for discussions and for a careful reading of the manuscript.

A P P E N D I X

We use the following representation of the γ matrices to reduce the Dirac equation in the massless case to the two-component Weyl equation

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma_k = -\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}; k=1,2,3 \quad (\text{A.1})$$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = i\gamma_0\gamma_1\gamma_2\gamma_3 \quad \sigma_{1,2,3}: \text{Pauli matrices}$$

The projection operators $\frac{1}{2}(1 \pm \gamma_5)$ yield left (+) and right (-) handed two-component spinors

$$\begin{aligned} \frac{1+\gamma_5}{2} \psi &= \begin{pmatrix} 0 \\ v \end{pmatrix} \quad v = v^{\dot{\alpha}}; \dot{\alpha}=1,2 \\ \frac{1-\gamma_5}{2} \psi &= \begin{pmatrix} u \\ 0 \end{pmatrix} \quad u = u_{\alpha}; \alpha=1,2 \end{aligned} \quad (\text{A.2})$$

which satisfy the Weyl equations

$$\begin{aligned} p_{\alpha\dot{\beta}} v^{\dot{\beta}} = 0 &\Rightarrow (i\partial_0 - i\vec{\nabla} \cdot \vec{\sigma}) v = 0 \\ p^{\dot{\alpha}\alpha} u_{\alpha} = 0 &\Rightarrow (i\partial_0 + i\vec{\nabla} \cdot \vec{\sigma}) u = 0 \end{aligned} \quad (\text{A.3})$$

A chiral current has the representation

$$\begin{aligned} V_{\mu} + A_{\mu} &= 2 v^{\dagger\dot{\beta}} (\sigma_{\mu})_{\dot{\beta}\alpha} v^{\alpha} \\ V^{\mu} - A^{\mu} &= 2 u^{\dagger}_{\dot{\beta}} (\sigma^{\mu})^{\dot{\beta}\alpha} u_{\alpha} \end{aligned} \quad (\text{A.4})$$

Defining the spinor components of a vector or axial vector ξ

$$(\xi^{\mu} \sigma_{\mu})_{\alpha\dot{\beta}} = \xi_{\alpha\dot{\beta}}; \quad (\xi_{\mu} \sigma^{\mu})^{\dot{\alpha}\beta} = \xi^{\dot{\alpha}\beta} \quad (\text{A.5})$$

and using the completeness relation

$$\sum_{\mu} (\sigma_{\mu})_{\alpha\beta} (\sigma_{\mu})_{\gamma\delta} = 2 \delta_{\alpha\gamma} \delta_{\beta\delta} \quad (\text{A.6})$$

one obtains the factorized expressions for chiral currents

$$(V+A)_{\alpha\beta} = 4 u_{\alpha}^{\dagger} u_{\beta} \quad (\text{A.7})$$

$$(V-A)_{\alpha\beta} = 4 u_{\alpha}^{\dagger} u_{\beta}$$

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FIGURE CAPTIONS

Figure 1 :

Energetically most favourable alignment of a $q\bar{q}$ pair in the left-handed world.

Figure 2 :

Qualitative shape of the dispersion law for single quark- (anti-quark) excitations.

FIG. 1

FIG. 2