CUT CONTRIBUTIONS IN THE TRIPLE-POMERON LIMIT

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A B S T R A C T

We investigate Pomeron cut contributions in the triple-Pomeron limit of one-particle inclusive cross-sections, near $t = 0$ where the triple-pole coupling vanishes. We find that at $t = 0$ the cuts themselves are suppressed, contributing factors $(\ln M^2)^{-2}$, rather than the single logarithms characteristic of cut contributions in two-body processes. We construct a simple Reggeon calculus model in which all important cuts near $t = 0$ can be calculated, and suggest a simple way of parametrizing the data in that region.

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1. - INTRODUCTION

It is now a well-established result that the triple-Pomeron pole coupling which appears in the one-particle inclusive cross-section in the limit $M^2 \rightarrow \infty$, $s/M^2 \rightarrow \infty$ must vanish at zero momentum transfer. This assumes that the Pomeron pole is a conventional Regge pole, with intercept one and finite slope at $t = 0$. If the pole coupling vanishes at $t = 0$, we naturally expect multi-Pomeron cuts to become important. It is the purpose of this paper to understand what form these contributions should take. Our main result is that, at $t = 0$, cut contributions are down by factors of $(\ln M^2)^{-2}$ and $(\ln s)^{-2}$ rather than the single logarithms generally characteristic of cuts. This is a consequence of the mechanism which produces the triple-Pomeron zero in the first place. Although the cleanest derivation of the zero follows from the inclusive sum rules ¹⁾, which operate in the s channel, it is necessary to look to the t channel for a dynamical explanation. In fact Bronzan ²⁾ has shown that a mechanism similar to that producing a zero in Gribov's triple-Pomeron vertex ^{3),4)} is responsible. The inclusive vertex and Gribov's vertex are defined in different ways - the former at $t_0 = 0$, $t_1 = t_2$; and the latter at $t_1 = t_2 = \frac{1}{4}t_0$, that is, on the fixed pole of the Reggeon production amplitude (see Fig. 1 for notation). However, Bronzan ²⁾ shows that, in a specific model, the two vertices are identical at $t_0 = t_1 = t_2 = 0$, and that the inclusive vertex is the natural continuation of Gribov's vertex off the fixed pole. The zero comes about because of the otherwise apparent impossibility of constructing a self-consistent theory with the cut and pole interacting in accordance with t_0 channel unitarity.

The lay-out of this paper is as follows. In Section 2 we briefly discuss Bronzan's results and show quite generally how cut effects in the t_0 channel give a $(\ln M^2)^{-2}$ behaviour at $t_0 = 0$. We then construct a simple Reggeon calculus with a minimum of parameters which explicitly satisfies Bronzan's consistency requirements, and has the triple-Pomeron zero built into it. In Section 3, we use this model to calculate all important cut contributions near $t = 0$, in terms of these few parameters. We conclude with a discussion of the present data in terms of this model. We shall throughout consider the effects of only two-Pomeron cuts. Multi-Pomeron cuts are thought to be weak and are usually neglected. However, we shall see that they can, in principle, be important here, and could spoil the quantitative, but not the qualitative results we derive.

2. - t_0 CHANNEL UNITARITY

We begin by outlining the arguments of Gribov and Migdal ³⁾ and Bronzan ^{2),4)} leading to the necessity of the triple-Pomeron zero. Gribov et al. ⁵⁾ have shown that the elastic partial wave amplitude has the form

$$f(t_0, j) = \frac{A}{D} = \frac{A(t_0, j)}{B(t_0, j) + [1/\alpha'(\frac{1}{4}t_0)] \ln [j - \alpha_c(t_0)]} \quad (1)$$

where $\alpha_c(t_0) = 2\alpha(\frac{1}{4}t_0) - 1$ and A and B are real analytic in the neighbourhood of $j = \alpha_c(t_0)$. They also show that the Reggeon production amplitude (Fig. 2) and the Reggeon-Reggeon amplitude (Fig. 3) have the forms $A^{\frac{1}{2}}/D$ and $1/D$ respectively. This means that the Pomeron pole $j = \alpha(t_0)$ must arise from a zero in D , rather than a pole in A . This is the self-consistency requirement of Bronzan ⁴⁾. Neglecting weak and fixed cuts in B , we assume it is meromorphic near $t_0 = 0$, $j = 1$. If B has no poles in this region one is led to a solution of the self-consistency condition in which the pole decouples from the elastic amplitude at $t_0 = 0$ and the cut contributes positively. Also the trajectory $\alpha(t) = 1 + O(t \ln t)$ near $t = 0$. We shall discount this possibility ^{*}).

To avoid this decoupling it is necessary to assume that A and B carry a pole which passes through $j = 1$ at $t_0 = 0$. To avoid a square-root singularity in the Reggeon production amplitude, this must be a double pole. We then assume A and B are rational functions :

$$A = \frac{\tilde{A}(t_0, j)}{C(t_0, j)^2} \quad B = \frac{\tilde{B}(t_0, j)}{C(t_0, j)^2} \quad \text{with } [C(0, 1) = 0] \quad (2)$$

The partial wave amplitude then has the form

$$f(t_0, j) = \frac{\tilde{A}(t_0, j)}{\tilde{B}(t_0, j) + C(t_0, j)^2 [1/\alpha'(\frac{1}{4}t_0)] \ln [j - \alpha_c(t_0)]} \quad (3)$$

^{*}) Note that this is not the t channel analogue of the alleged Pomeron decoupling results in the s channel ⁶⁾, since the latter assume that $\alpha'(0)$ is finite.

If $\tilde{B}$ has a linear zero passing through $j = 1$ at $t_0 = 0$, $f(t_0, j)$ will have a pole there. The self-consistent trajectory has the reasonable form $\alpha(t) = 1 + \alpha' t + O(t^2 \ln t)$.

The Reggeon production amplitude will have the form

$$\frac{\tilde{A}C}{\tilde{B} + C^2 [1/\alpha'] \ln[j - \alpha_c]} \quad (4)$$

At $t_0 = 0$ both $\tilde{B}$ and C vanish linearly in $(j-1)$. Therefore this amplitude does not have a pole, or, rather, the triple-Pomeron coupling vanishes. The remaining contribution has the form

$$\frac{a}{b + (c/\alpha')(j-1) \ln(j-1)} \quad (5)$$

where a, b, c are constants. It is then straightforward to perform the Sommerfeld-Watson integral to obtain the asymptotic behaviour $M^2 [\ln M^2]^{-2}$. In the above we have considered only the Gribov vertex. Bronzan²⁾ has extended the analysis off the fixed pole, making reasonable assumptions on analyticity in t_1 and t_2 , to obtain a similar result for the inclusive vertex. It is interesting to note that the inclusive sum rule argument¹⁾ requires the coupling to vanish only as fast as $(\ln t)^{-1}$ ⁷⁾. However, a linear vanishing is more easily seen to be consistent with analyticity.

We now show how the expression (3) can be understood in terms of a Reggeon calculus model. Expanding (3) in a power series in α'^{-1}

$$f(t_0, j) = \sum_{n=0}^{\infty} \frac{(-1)^n \tilde{A} C^{2n}}{\alpha'^n \tilde{B}^{n+1}} [\ln(j - \alpha_c)]^n \quad (6)$$

This series can be understood in terms of a decomposition of the amplitude into terms with 0, 1, 2, 3, ... two-Pomeron cuts coupled together by amplitudes with no two-Pomeron cuts:

$$\text{---}\bigcirc\text{---} = \text{---}\textcircled{2}\text{---} + \text{---}\textcircled{2}\text{---}\textcircled{2}\text{---} + \text{---}\textcircled{2}\text{---}\textcircled{2}\text{---}\textcircled{2}\text{---} + \dots \quad (7)$$

In this picture the irreducible amplitudes have the forms

$$\begin{aligned}
 \text{---}\textcircled{2}\text{---} &= \tilde{A}/\tilde{B} \\
 \text{---}\textcircled{2}\text{---} &= \tilde{A}^{\frac{1}{2}}C/\tilde{B} \\
 \text{---}\textcircled{2}\text{---} &= C^2/\tilde{B}
 \end{aligned} \tag{8}$$

Thus all these amplitudes carry the "unrenormalized" pole produced by the zero of $\tilde{B}$, which couples to the two-Pomeron cut via C , which has a zero. The important feature is that the two-Pomeron cuts can only couple to anything via the pole, which itself couples to the cuts via the triple-Pomeron zero. This is what Gribov³⁾ calls the quasi-stability of the Pomeron.

In the rest of this paper we shall discuss a model with the simplest possible parametrization for the functions $\tilde{A}$, $\tilde{B}$ and C . It is clear from Eq. (8) that the pole, two-particle/Pomeron vertex, and the triple-Pomeron vertex are only determined up to an arbitrary scale factor :

$$\begin{aligned}
 \text{---}\text{---} &= \lambda^{-\frac{1}{2}} \tilde{A}^{\frac{1}{2}} \\
 \text{---} &= \lambda/\tilde{B} \\
 \text{---}\text{---} &= \lambda^{-\frac{1}{2}}C
 \end{aligned} \tag{9}$$

We can choose λ so that the pole becomes simply $(j-\alpha)^{-1}$. We also define

$$\text{---}\text{---} = a \quad \text{---}\text{---} = b[j-\beta(t_0)] + c[t_1+t_2-\frac{1}{2}t_0] \tag{10}$$

where $\beta(t_0)=1+\beta't_0$. We have defined the triple-Pomeron vertex off the fixed pole, assuming a linear vanishing²⁾. In formulating a Reggeon calculus there will also be an unknown normalization constant multiplying each two-dimensional momentum loop integration, as a consequence of our specific choice of λ above. For example, the single ununitarized two-Pomeron cut is :

$$\text{---}\text{---} = \frac{2\alpha'\tilde{\lambda}}{\pi} \int \frac{d^2k}{j-\alpha[(k+\frac{1}{2}q)^2]-\alpha[(k-\frac{1}{2}q)^2]+1} \quad (q^2=-t_0) \tag{11}$$

$$\sim -\tilde{\lambda} \ln [j-\alpha_c(t_0)] \tag{12}$$

The series in Eq. (7) can now be summed to give :

$$f(t_0, j) = \frac{a^2}{[j - \alpha(t_0)] + \tilde{\lambda} b c^2 [j - \beta(t_0)]^2 \ln [j - \alpha(t_0)]} \quad (13)$$

3. - THE INCLUSIVE CROSS-SECTION

We shall now use the model described above to calculate all leading contributions to the triple-Pomeron limit near $t = 0$. To do this we must also consider cuts in the t_1 and t_2 channels, and also "mixed" singularities which have no simple factorization properties. We make the usual assumption of a Reggeon calculus model that vertices which look the same, are the same, no matter in which channel they appear. There is also the question of the M^2 discontinuity which must be taken to obtain the inclusive cross-section. We shall appeal to the Feynman diagram model underlying the Reggeon calculus whenever necessary.

First, a simplification. We note that in Eq. (5), at $t_0 = 0$, the leading $M^2 (\ln M^2)^{-2}$ contribution comes from the single logarithm term in the expansion. Away from $t_0 = 0$ we obtain at infinite energy the $M^2 (\ln M^2)^{-1} \times (\ln \ln M^2)^{-2}$ behaviour characteristic of unitarized cuts⁸⁾. However, because of the triple-Pomeron zero, each term in the series is damped by higher and higher powers of t_0 , and so, neglecting terms $O(t_0^2)$, the important contribution comes only from the single logarithm term, which, away from $t_0 = 0$, behaves like $(-t_0) M^2 (\ln M^2)^{-1}$. The consequence of the triple-Pomeron zero is that near $t_0, t_1, t_2 = 0$ only lowest order diagrams need be considered.

A) - Pole contribution

To establish relative normalizations we first evaluate the triple-Pomeron pole contribution. Introducing complex angular momenta j_1, j_2 in the t_1, t_2 channels respectively, the multiple partial wave amplitude is

$$a^3 [j - \alpha(t_0)]^{-1} [j_1 - \alpha(t_1)]^{-1} [j_2 - \alpha(t_2)]^{-1} \{ b [j - \beta(t_0)] + c [t_1 + t_2 - \frac{1}{2} t_0] \} \quad (14)$$

Performing the Sommerfeld-Watson integral and specializing to $t_0 = 0$, $t_1 = t_2 = t$, we obtain the contribution

$$2a^3 ct (M^2) (s/M^2)^{2\alpha(t)} \quad (15)$$

b) - Cuts in the t_0 channel

As explained above, the leading contribution is that of Fig. 4. The partial wave amplitude is

$$\begin{aligned} & a^3 [j - \alpha(t_0)]^{-1} b [j - \beta(t_0)] (-\tilde{\lambda}) \ln [j - \alpha(t_0)] b [j - \beta(t_0)] [j - \alpha(t_0)]^{-1} \\ & \times \{ b [j - \beta(t_0) + c [t_1 + t_2 - \frac{1}{2} t_0]] \} [j_1 - \alpha(t_1)]^{-1} [j_2 - \alpha(t_2)]^{-1} \end{aligned} \quad (16)$$

$$= -a^3 b^2 \tilde{\lambda} [b(j-1) + 2ct] \ln(j-1) [j_1 - \alpha(t)]^{-1} [j_2 - \alpha(t)]^{-1} \quad (17)$$

This gives two contributions

$$a^3 b^2 \tilde{\lambda} \left[\frac{b}{(\ln M^2)^2} - \frac{2ct}{(\ln M^2)} \right] (M^2) (s/M^2)^{2\alpha(t)} \quad (18)$$

c) - Cuts in the t_1 and t_2 channels.

A cut in the t_1 channel is assumed to couple in the same manner as those in the t_0 channel (Fig. 5). The partial wave amplitude is

$$\begin{aligned} & a [j - \alpha(t_0)]^{-1} \{ b [j - \beta(t_0)] + c [t_1 + t_2 - \frac{1}{2} t_0] \} [j_2 - \alpha(t_2)]^{-1} a \\ & \times [j_1 - \alpha(t_1)]^{-1} b [j_1 - \beta(t_1)] (-\tilde{\lambda}) \ln [j_1 - \alpha(t_1)] b [j_1 - \beta(t_1)] [j_1 - \alpha(t_1)]^{-1} a \end{aligned} \quad (19)$$

$$= -2a^3 b^2 \tilde{\lambda} ct (j-1)^{-1} [j_2 - \alpha(t)]^{-1} [j_1 - \beta(t)]^2 [j_1 - \alpha(t)]^{-2} \ln [j_1 - \alpha(t_1)] \quad (20)$$

The pole terms in the t_1 channel are negligible, being $O(t^2)$, so (20) simplifies to

$$-2a^3 b^2 \tilde{\lambda} ct (j-1)^{-1} [j_2 - \alpha(t)]^{-1} (1 - 2\beta'/\alpha')^2 \ln [j_1 - \alpha(t)] \quad (21)$$

which gives the contribution

$$-2\alpha^3 b^2 \tilde{\lambda}_{ct} (1-2\beta'/\alpha')^2 \frac{M^2}{\ln(s/M^2)} (s/M^2)^{\alpha(t)+\alpha_c(t)} \quad (22)$$

The cut in the t_2 channel will give the complex conjugate contribution.

d) - Mixed singularities

The leading diagram of this type is shown in Fig. 6. It is the analogue of the mixed contributions considered by Drummond⁹⁾ for the five-point function, and Botke¹⁰⁾ for the six-point function in the pionization region. In this diagram there arises the problem of taking the discontinuity. However, a consideration of Feynman diagrams like Fig. 7 convinces one that the only allowable discontinuity cuts must pass through the triple-Pomeron vertex, which can thus be identified with the inclusive vertex continued to non-forward values of t_1 and t_2 . Figure 6 can be evaluated using the parametrization already used, but we can see immediately that at $t = 0$ it must be down by a factor $(\ln s)^{-2}$ - one factor coming from the loop integration, the other from a suppression due to the triple-Pomeron zero. Away from $t = 0$ it will behave like $t(\ln s)^{-1}$.

We conclude with a brief discussion of the present data in terms of this model. The power behaviour $(M^2)(s/M^2)^2 \alpha(t)$ of the triple-Pomeron pole contribution, written in terms of Feynman's parameter $x = 1 - M^2/s$, has the form $s(1-x)^{1-2\alpha(t)}$. This exhibits scaling, and a strong peaking near $x = 1$. This peaking is observed in the data¹¹⁾. It could be explained by a Reggeon-Pomeron-Pomeron coupling, which would lead to a non-scaling behaviour $s^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ near $t = 0$. However, data at two energies, $s = 930$ and 1995 GeV^2 , seem to indicate scaling and so rule this possibility out. Unfortunately, the present data lie only in $t < -0.27 (\text{GeV}/c)^2$, and, while it seems to follow the exponential behaviour in t suggested by the formula $(1-x)^{1-2\alpha(t)}$, there is no indication of the coupling vanishing at $t = 0$. However, our analysis has shown that one must include cuts near $t = 0$. At sufficiently high energies only the t_0 channel cuts will be important, the others being down by factors of $\ln s$ or $\ln s/M^2$ rather than $\ln M^2$. These cuts will have the same trajectory dependence on t as the pole contributions. So at small t we expect the x distribution to have roughly the $(1-x)^{-1}$ shape, but one should observe a breakdown in scaling, becoming more marked at higher

energies. For larger values of t , the t_1 and t_2 channel cuts will dominate because of their flatter trajectories. In this region of t one should see a flattening of the $(1-x)$ peak as the cuts become less important with higher energies, and a fast approach to scaling, as these cuts themselves will scale, only the mixed cuts of Fig. 6 spoiling the scaling.

It is interesting to note that, within the model, the total cross-section is given by

$$\sigma_{\text{tot}} \sim (\text{const})^2 \left[a^2 - \frac{a^2 b^2 \tilde{\lambda}}{\ln s} \right] \quad (23)$$

Thus, in principle, a^2 and $b^2 \tilde{\lambda}$ are measurable. If c can also be measured from the triple-Pomeron pole coupling, then the cuts $O(t(\ln M^2)^{-1})$ are determined, but not the cuts at $t = 0$, $O((\ln M^2)^{-2})$. However, our work suggests that the way the triple-Pomeron limit should be parametrized near $t = 0$ is

$$\left[A \sigma_{\text{tot}}(M^2)(-t) + \frac{B}{(\ln M^2)^2} \right] (M^2) (s/M^2)^{2\alpha(t)} \quad (24)$$

where A and B are positive constants. The first term will then automatically take into account the effects of $t \neq 0$ cuts. Finally we note that the $t = 0$ suppression of the two-Pomeron cut to a form like $(j-1)\ln(j-1)$ makes it comparable with the three-Pomeron cut, unless this is also suppressed for some reason. In that case the detailed predictions of our model would be invalid, but not, however, the general form of Eq. (24).

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FIGURES

Fig 1 Definition of variables

Fig 2 Reggeon production amplitude

Fig 3 Reggeon-Reggeon amplitude

Fig 4 Leading t_0 -channel cut

Fig 5 Leading t_1 -channel cut

Fig 6 Leading mixed cut

Fig 7 A typical Feynman diagram contributing to Fig 6, with an allowable discontinuity cut