NO-GHOST THEOREM FOR THE POMERON SECTOR OF THE DUAL MODEL

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A B S T R A C T

The Pomeron sector of the dual model comprises a spectrum of states obtained by factorizing the residues of poles occurring in a certain channel (with vacuum quantum numbers) of the orientable non-planar single loop graphs of the conventional dual model considered in 26 space-time dimensions. We show that the spectrum of Pomeron states which couple between external physical particle states are transverse and hence positive definite and thus ghost-free. The same states also factorize the Virasoro-Shapiro model which now appears very likely to be the scattering theory of Pomeron ground states.

The existing dual models seem to be more a self-consistent alternative to polynomial local field theories than a phenomenological approach to describe the real world of hadrons. The main advantage over field theories is that at each order of the perturbation expansion an infinite number of particle states of any spin is included, while maintaining the basic properties of duality, Regge behaviour, positive definiteness of the spectrum of resonances (absence of ghosts), and perturbative unitarity ¹⁾. The drawback is that this set of conditions is so constraining that it can be realized only for unphysical values of the number of dimensions of space-time, and at the expense of having, in general, tachyons. However, this critical dimension of space-time, D , varies from model to model ($D=26$ in the conventional model ^{2),3)}, 10 in the Neveu-Schwarz ^{4),5)} model) so that in the spirit of the bootstrap one may think that a realistic and unique dual model would yield critical dimension 4. Meanwhile, self-consistency has to be checked order by order in perturbation theory.

An interesting problem occurs in dual models in second order of perturbation theory : when one constructs the non-planar orientable single-loop graph so as to satisfy perturbative unitarity one finds that besides the normal two-body cuts, this graph exhibits additional cuts violating unitarity ⁶⁾. This cut appears in a channel being a singlet with respect to the symmetry group $SU(2)$ or $SU(3)$, and is dual to a background. Its slope is always half of the Regge trajectory slope. It is natural to associate it with the Pomeron. One can check that this cut does appear at high energy so as to ensure Regge behaviour ⁷⁾ and thus it cannot be subtracted without violating analyticity in the J plane. On the other hand since it violates perturbative unitarity it is indeed a disease.

Lovelace ⁸⁾ noticed, however, that this Pomeron singularity is not always a cut, but that if one chooses in the conventional model $\alpha(0) = 1$, and $D = 26$, and assumed that some gauge conditions decoupled two sets of oscillators, this singularity became a series of poles located on the Regge trajectory

$$\alpha_p(p^2) = 2 + \frac{\alpha'}{2} p^2 \quad (1)$$

together with its daughter trajectories displaced below (1) by integers.

It was later found that for $D = 26$ and $\alpha(0) = 1$ the conventional dual model is indeed ghost-free ^{2),3)} and that there is indeed a double (rather than a single) set of gauges at work. Just for this critical dimension Brink and Olive ⁹⁾ recently constructed the physical state projection operator which made it possible to compute the planar loop with only physical (ghost-free) states propagating internally. The result ¹⁰⁾ was that two partition functions were cancelled from the integrand and it now seems very likely that this will also happen for the non-planar loops, thereby confirming Lovelace's conjecture ⁸⁾ and guaranteeing that the Pomeron singularities are poles. Thus in order that the dual model provides an entirely self-consistent scheme, at least in second order, in the sense of satisfying perturbative unitarity, duality and Regge behaviour, the most urgent remaining properties to be checked are the factorization of the Pomeron poles residues and (if this is so) the positive definiteness of the resultant spectrum. (The problem of tachyons remains and probably cannot be eliminated with any of the self-consistent Pomeron models.)

The question of factorisability was answered by Cremmer and Scherk ¹¹⁾ for the conventional model and by Clavelli and Shapiro ¹²⁾ for more general cases, including the Neveu-Schwarz model. In the conventional case (which is all we shall consider in this note), one has the equation

$$F = \langle \phi | D_P(p^2) | \psi \rangle \quad (2)$$

which is represented graphically in Fig. 1a and Fig. 1b.

- Figure 1a -

- Figure 1b -

Figure 1a represents the left-hand side of the equation expressed in the form of a loop graph and Fig. 1b the right-hand side in the form of a tree graph. The wavy line corresponds to the Pomeron propagator

$$D_P(p^2) = \int_0^1 \frac{dz}{z} \prod_{n=1}^{\infty} (1 - z^{2n})^2 z^{L_0 + \bar{L}_0 - 2}$$

$$= \int_0^1 \frac{dz}{z} \prod_{n=1}^{\infty} (1 - z^{2n})^2 z^{H + \bar{H} - \alpha_P(p^2)} \quad (3)$$

This is expressed in terms of two mutually commuting infinite sets of 26 dimensional oscillators a_n^μ and $\bar{a}_n^\mu$, each set like that of the conventional dual model. $H, \bar{H}, L_0$ and $\bar{L}_0$ are constructed out of these in the obvious way but with one proviso : whenever p_0 (the momentum operator) appears it is replaced by $p_0/2$ (and whenever q_0 , the displacement operator, appears in later expressions it will be replaced by $2q_0$ in order to preserve the usual commutation relations). As indicated in Fig. 1, the states $|\phi\rangle$ and $|\psi\rangle$ represent the coupling of the Pomeron states to the external scalar states via a Pomeron-Reggeon coupling [which now incorporates the factor $\int_0^{2\pi} d\theta e^{2i\theta(H-\bar{H})}$ as explained in the footnote on p. 235 of Ref. 11], followed by a Reggeon propagator and then a normal dual model tree. The precise form of these states $|\psi\rangle$ and $|\phi\rangle$ is given in Ref. 11). The only property we need is the gauge property, which follows from that found by Cremmer and Scherk ¹¹⁾, if we take account of the modification just mentioned :

$$(L_n - \bar{L}_{-n})|\phi\rangle = 0 \quad \text{for all } n \quad (4)$$

(once again L_n and $\bar{L}_n$ are constructed out of a_n^μ and $\bar{a}_n^\mu$ respectively in the usual way with the proviso that p_0 is replaced by $p_0/2$). For $n = 0$ Eq. (4) states that the number of a modes excited always equals the number of $\bar{a}$ modes.

The result just quoted shows that the Pomeron poles do indeed have factorizable residues and hence a resonance interpretation. In addition we see a very interesting feature of the dual model : unitarity corrections are equivalent to the mixing of the Reggeon model with another model through a direct coupling. This feature strikingly distinguishes the dual model from field theory since divergences are replaced by the appearance of new particles.

The remaining question then is whether the spectrum of the propagator (3) is ghost free when the gauge conditions (4) are taken into account. The appearance of the peculiar factor $\prod_{n=1}^{\infty} (1-q^{2n})^2$ looks unfavourable but when Neveu ¹³⁾ carefully analyzed the residue at the second level he found that only transverse (and hence positive norm) states contribute there. The even spin wave contains only a purely transverse (massless) graviton together with a scalar while the odd spin wave contains a transverse antisymmetric tensor of rank 2. So it seems that at this level the Pomeron spectrum is identical to the physical spectrum of the Virasoro-Shapiro (VS) model ¹⁴⁾ [which has the same leading trajectory (1)], up to the fact that due to the sum over permutations of the external legs in the VS model odd spin states decouple, while they do not in the Pomeron sector.

We shall show that this result - the identity of the Pomeron and VS spectra - generalizes to all levels. In fact the same transverse states, constructed by analogy with those of the ordinary dual model, will saturate the residues of both theories. Since these transverse states have positive norm the no-ghost theorem follows. To make our result more definite we recall the factorization of the VS model ¹⁵⁾. This also involves two precisely similar sets of oscillators a_n^μ and $\bar{a}_n^\mu$:

$$F_{VS} = \langle \phi_{VS} | D_{VS}(P^2) | \psi_{VS} \rangle \quad (5)$$

where

$$D_{VS}(P^2) = \int_0^1 \frac{dq}{q} q^{L_0 + \bar{L}_0 - 2} \quad (6)$$

is the propagator, and the states $|\psi_{VS}\rangle$ describing the coupling to external VS ground states satisfy the gauge conditions :

$$\left. \begin{aligned} L_n |\psi_{VS}\rangle &= (L_0 + n - 1) |\psi'_{VS}\rangle & n \geq 1 \\ \bar{L}_n |\psi_{VS}\rangle &= (\bar{L}_0 + n - 1) |\psi''_{VS}\rangle & n \geq 1 \\ (L_0 - \bar{L}_0) |\psi_{VS}\rangle &= 0 \end{aligned} \right\} \quad (7)$$

(ψ' and ψ'' denote states differing from ψ in a way we need not specify).

Notice that the propagator (6) is simpler than the Pomeron propagator (3) in that it lacks the factor $\prod_{n=1}^{\infty} (1-q^{2n})^2$. Thus when we equate the two spectra we shall be showing that this difference is precisely compensated for by the difference between the gauge conditions (4) and (7).

By slightly generalizing the recent work of Brink and Olive for the conventional dual model ⁹⁾ we shall construct the positive definite transverse state projection operator $\mathcal{T}_{VS}$ for the VS model. To show that it saturates residues we shall check :

$$\langle \phi_{VS} | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} | \psi_{VS} \rangle = \langle \phi_{VS} | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} \mathcal{T}_{VS} | \psi_{VS} \rangle_{(8)}$$

We have selected a momentum lying at the pole $p^2 = 8(N-1)$ ($N=0,1,2,\dots$) of (5) and evaluated the residue of this pole by taking the q integration to encircle the origin. The equation (8) states that this residue is factorized by the states lying in the positive definite subspace onto which $\mathcal{T}_{VS}$ projects and possessing the eigenvalues N under the operation of both H and $\bar{H}$. Our main result is that precisely the same statement applies to the corresponding residue of the amplitude (7) for the Pomeron exchange :

$$\langle \phi | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} \prod_{n=1}^{\infty} (1-q^{2n})^2 | \psi \rangle = \langle \phi | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} \mathcal{T}_{VS} | \psi \rangle_{(9)}$$

To construct $\mathcal{T}_{VS}$ we form the quantities

$$E = L_0 - H \quad \bar{E} = \bar{L}_0 - \bar{H}$$

where $L_0 - H$ (and $\bar{L}_0 - \bar{H}$) are the same functions of a_n^μ (and $\bar{a}_n^\mu$) as in Ref. 9), but with the proviso mentioned before that we replace $p \rightarrow p/2$ and $q \rightarrow 2q$. Then by the same argument as in 9)

$$\mathcal{T}_{VS} = \oint \frac{dy}{2\pi i y} y^{E + \bar{E}} \quad (10)$$

projects onto the two Fock spaces of DDF transverse states ¹⁶⁾ formed in the two $(a_n^\mu$ and $\bar{a}_n^\mu)$ Fock spaces. Just as before ⁹⁾ we can rewrite

$$E = (D_0 - 1)(L_0 - 1) + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n) \quad (11)$$

and similarly for $\bar{E}$ and this enables us to prove (8) if we use the algebraic properties (valid only for $D = 26$)

$$[L_n, D_m] = -(2n+m) D_{n+m} \quad L_n y^E = y^{E-n} L_n \quad (12)$$

For we can write

$$\begin{aligned} & \langle \phi_{VS} | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} (y^{E + \bar{E}} - 1) | \psi_{VS} \rangle \\ &= \langle \phi_{VS} | \oint \frac{dq}{2\pi i q} q^{L_0 + \bar{L}_0 - 2} (E + \bar{E}) \int_1^y dy' y'^{E + \bar{E} - 1} | \psi_{VS} \rangle \end{aligned}$$

and check that this vanishes using the gauge properties (7), the mass-shell conditions, and the commutators (12). Division by y , followed by integration around the origin then gives (8) and completes the proof that VS residues are completely factorized by transverse and hence ghost-free states.

Now we check the corresponding result for the Pomeron residue, Eq. (9). The analysis follows rather closely the loop analysis of Ref. 10) but is, fortunately simpler. Since it is our main result we shall give it in some detail. We shall form a differential equation for the integrand of the right-hand side of (9) by considering :

$$\begin{aligned} & y \frac{\partial}{\partial y} \langle \phi | q^{L_0 + \bar{L}_0 - 2} y^{E + \bar{E}} | \psi \rangle \\ &= \frac{1}{2} q \frac{\partial}{\partial q} \langle \phi | q^{2(L_0 - 1)} y^{E + \bar{E}} (D_0 + \bar{D}_0 - 2) | \psi \rangle \quad (13) \\ &+ \sum_{n=1}^{\infty} \langle \phi | q^{2(L_0 - 1)} y^{E + \bar{E}} (L_{-n} D_n + D_{-n} L_n + \bar{L}_{-n} \bar{D}_n + \bar{D}_{-n} \bar{L}_n) | \psi \rangle \end{aligned}$$

We have already used Eq. (4) for $n = 0$ several times and replaced $L_0 - 1$ or $\bar{L}_0 - 1$ by a differentiation everytime it occurs. Now we shall evaluate the following quantity by using the gauge condition (4) and the commutators (12)

$$\begin{aligned}
 \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} D_n L_n | \psi \rangle &= \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} D_n \bar{L}_{-n} | \psi \rangle \\
 &= y^{-n} \langle \phi | \bar{L}_{-n} q^{2(L_0-1)} y^{E+\bar{E}} D_n | \psi \rangle \\
 &= y^{-n} \langle \phi | L_n q^{2(L_0-1)} y^{E+\bar{E}} D_n | \psi \rangle \\
 &= \left(\frac{q}{y}\right)^{2n} \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} L_n D_n | \psi \rangle \\
 &= \left(\frac{q}{y}\right)^{2n} \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} (D_n L_n - n D_0) | \psi \rangle
 \end{aligned}$$

Finally, solving this equation

$$= \frac{-n \left(\frac{q}{y}\right)^{2n}}{1 - \left(\frac{q}{y}\right)^{2n}} \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} D_0 | \psi \rangle$$

All the other terms in the n summation of Eq. (13) can be expressed similarly and so (13) becomes

$$\begin{aligned}
 y \frac{\partial}{\partial y} \langle \phi | q^{L_0 + \bar{L}_0 - 2} y^{E+\bar{E}} | \psi \rangle &= \frac{1}{2} q \frac{\partial}{\partial q} \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} (D_0 + \bar{D}_0 - 2) | \psi \rangle \\
 - \sum_{n=1}^{\infty} \frac{2n \left(\frac{q}{y}\right)^{2n}}{1 - \left(\frac{q}{y}\right)^{2n}} \langle \phi | q^{2(L_0-1)} y^{E+\bar{E}} (D_0 + \bar{D}_0) | \psi \rangle & \quad (14)
 \end{aligned}$$

We are going to use the relations ⁹⁾

$$(D_0 - 1) \mathcal{T}_{VS} = 0 = \mathcal{T}_{VS} (D_0 - 1) \quad (15)$$

and so we split $D_0 = (D_0 - 1) + 1$ wherever D_0 (and $\bar{D}_0$) occur. Also since

$$\begin{aligned}
 -\sum_{n=1}^{\infty} \frac{2n (q/y)^{2n}}{1 - (q/y)^{2n}} &= \frac{\frac{1}{2} q \frac{\partial}{\partial q} \prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2}{\prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2} \\
 &= \frac{-\frac{1}{2} y \frac{\partial}{\partial y} \prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2}{\prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2} \quad (16)
 \end{aligned}$$

we can rewrite Eq. (14) as

$$\begin{aligned}
 &y \frac{\partial}{\partial y} \left\{ \prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2 \langle \phi | q^{L_0 + \bar{L}_0 - 2} y^{E + \bar{E}} | \psi \rangle \right\} \\
 &= \frac{1}{2} q \frac{\partial}{\partial q} \left\{ \prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2 \langle \phi | q^{2(L_0 - 1)} y^{E + \bar{E}} (D_0 + \bar{D}_0 - 2) | \psi \rangle \right\}
 \end{aligned}$$

This we integrate $\int_1^y (dy/y)$, choosing the lower limit 1 since it will give rise to a term with $\mathcal{T}_{VS}$ replaced by 1 :

$$\begin{aligned}
 &\prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2 \langle \phi | q^{L_0 + \bar{L}_0 - 2} y^{E + \bar{E}} | \psi \rangle = \prod_{n=1}^{\infty} (1 - q^{2n})^2 \langle \phi | q^{L_0 + \bar{L}_0 - 2} | \psi \rangle \\
 &+ \frac{1}{2} q \frac{\partial}{\partial q} \left\{ \int_1^y \frac{dy'}{y'} \prod_{n=1}^{\infty} (1 - (q/y')^{2n})^2 \langle \phi | q^{2(L_0 - 1)} y'^{E + \bar{E}} (D_0 + \bar{D}_0 - 2) | \psi \rangle \right\}
 \end{aligned}$$

Now we divide by $\prod_{n=1}^{\infty} (1 - (q/y)^{2n})^2$ and integrate $\oint (dy/y) \oint (dq/q)$. The y integration of the first term on the right-hand side can then be performed trivially by deforming the y contour into an infinitely large circle by using the analyticity of the infinite product for $|y| > |q|$. (Note that since $E \leq 0, H \geq 0$ the contours should originally be arranged so $|q| < 1, |y| > 1$.) In the second term we integrate by parts with respect to q , use (16) again, and then integrate by parts with respect to y . The result is

$$\begin{aligned}
 &\oint \frac{dq}{q} \langle \phi | q^{L_0 + \bar{L}_0 - 2} \mathcal{T}_{VS} | \psi \rangle = \oint \frac{dq}{q} \prod_{n=1}^{\infty} (1 - q^{2n})^2 \langle \phi | q^{L_0 + \bar{L}_0 - 2} | \psi \rangle \\
 &- \frac{1}{2} \oint \frac{dq}{q} \langle \phi | q^{2(L_0 - 1)} \mathcal{T}_{VS} (D_0 + \bar{D}_0 - 2) | \psi \rangle
 \end{aligned}$$

The last term vanishes by (15) leaving the desired result (9) thereby establishing the completeness of the positive definite transverse states in the Pomeron pole residues.

Note that because of the condition

$$(L_0 - \bar{L}_0) |\phi\rangle = 0 \quad (17)$$

satisfied in both the Pomeron and VS cases, only transverse states satisfying this condition (17) can be excited and so the physical state projection operator is more correctly

$$\mathcal{T}_{VS} \oint \frac{dz}{2\pi i z} z^{L_0 - \bar{L}_0}$$

For some time it has been felt that the VS model may provide a scattering theory of Pomeron ground states¹⁷⁾, particularly since it is a theory so difficult to modify. Our results make this much more plausible and suggest that many simplifications will occur when we learn to make the necessary multi-Pomeron factorizations.

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