CM-P00060999

Ref.TH.1614-CERN

CONSTITUENT QUARKS, CURRENT QUARKS AND PIONIC DECAYS OF RESONANCES

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A B S T R A C T

Predictions for pionic decays of meson and baryon resonances are derived from PCAC and the algebraic properties of the (Melosh) transformation between constituent and current quarks.

*) Research supported by a Royal Society European Fellowship.

It has been known for some time that two physically different sets of operators generate $SU(6)_W$ algebras. One of these, denoted $SU(6)_{W,\text{current}}$, was introduced by Dashen and Gell-Mann ¹⁾ as a possible extension of the chiral $SU(3) \times SU(3)$ algebra of vector and axial vector charges. The generators of $SU(6)_{W,\text{current}}$ are the so-called "good" charges in the infinite momentum limit and are defined as integrals over local densities which are directly or indirectly measurable in weak and electromagnetic processes. The second $SU(6)_W$ algebra, denoted $SU(6)_{W,\text{strong}}$ was introduced by Lipkin and Meshkov ²⁾ as an approximate "relativistic spin" symmetry of the strong interaction Hamiltonian. Except for their commutation relations, almost nothing is known about the generators of this algebra, but an impressive amount of empirical evidence supports the existence of a rather badly broken $SU(6)_{W,\text{strong}}$ symmetry. In particular, the low-lying hadrons do seem to fall in irreducible representations of $SU(6)_{W,\text{strong}}$: the mesons in 35's and 1's and the baryons in 56's and 70's. This successful particle classification leads naturally to an intuitive picture of mesons as made up of a "constituent" quark-antiquark pair, and of baryons as three "constituent" quarks. On the other hand, from the Adler-Weisberger sum rule ³⁾, it is equally clear that hadron states at infinite momentum cannot be classified in single irreducible representations of $SU(6)_{W,\text{current}}$ nor of its chiral $SU(3) \times SU(3)$ subgroup. There must be sizeable representation mixing, allowing for example, the axial vector charge to connect the nucleon state to higher mass N^* states. From this, and from the deep inelastic electron-proton scattering experiments ⁴⁾ one is led to view hadrons as very complex structures made up of many "current" quarks and "current" quark-antiquark pairs (with perhaps some neutral glue in addition). Hence, the generators of the two $SU(6)_W$ algebras, which in a quark model language roughly correspond to descriptions in terms of "constituent" and "current" quarks respectively, are not identical operators ⁵⁾.

Recently Melosh has shown ⁶⁾ that in the free quark model these two $SU(6)_W$ algebras are related by a unitary transformation V_{free} . It is the purpose of this letter to explore the consequences of the assumption that the algebraic structure of the correct unitary transformation, V , relating the $SU(3) \times SU(3)$ subalgebras of $SU(6)_{W,\text{strong}}$ and $SU(6)_{W,\text{current}}$ is very similar to that of V_{free} . In order to specify this assumption somewhat more, we go to the infinite momentum frame and restrict ourselves ab initio to the two $SU(3) \times SU(3)$ subalgebras. These are defined by the commutation relations ⁷⁾

$$[F_i, F_j] = i f_{ijk} F_k \quad [F_i, F_j^5] = i f_{ijk} F_k^5 \quad [F_i^5, F_j^5] = i f_{ijk} F_k^5 \quad (1)$$

where the indices i, j, k run from 1 to 8. As is well known, classification in $SU(3) \times SU(3)$ does not uniquely specify an infinite momentum state: at least one additional quantum number is required which is usually taken, in the quark model language, as the total constituent or current quark spin projection along the $\hat{z}$ axis, S_z . Since both of these operators commute with the total angular momentum operator J_z , one can then define "quark orbital angular momentum" operators L_z . Irreducible representations corresponding to particles with helicity J_z moving with infinite momentum along the $\hat{z}$ axis will then be labelled by $\{(A, B)_{S_z, L_z}\}$ where A and B are irreducible representations of $F_1 + F_1^5$ and $F_1 - F_1^5$ respectively; S_z is the eigenvalue of the total quark spin along the $\hat{z}$ axis and $L_z = J_z - S_z$. To avoid confusion, when specifically referring to the $SU(3) \times SU(3)_{\text{strong}}$ subgroup of $SU(6)_{W, \text{strong}}$, the generators will be denoted W_i and W_{iz} while when we refer to the chiral $SU(3) \times SU(3)_{\text{current}}$ subgroup of $SU(6)_{W, \text{current}}$, they will be denoted Q_i and Q_i^5 . (The CVC assumption thus implies $W_i \equiv Q_i$.) Furthermore, for the free quark model case, a subscript "free" will be added to all states and operators.

Let us now consider the Melosh transformation V_{free} which relates the constituent quark representation [i.e., in $SU(3) \times SU(3)_{\text{strong}}$] of a hadron state H to its current quark representation [i.e., in $SU(3) \times SU(3)_{\text{current}}$]:

$$|H, \text{current quark state}\rangle_{\text{free}} = V_{\text{free}} |H, \text{constituent quark state}\rangle_{\text{free}} \quad (2)$$

Measurable matrix elements such as, for example

$${}_{\text{free}} \langle H', \text{current quark state} | Q_{i\text{free}}^5 | H, \text{current quark state} \rangle_{\text{free}}$$

can be written as

$${}_{\text{free}} \langle H', \text{constituent quark state} | V_{\text{free}}^{-1} Q_{i\text{free}}^5 V_{\text{free}} | H, \text{constituent quark state} \rangle_{\text{free}}$$

This demonstrates that the Melosh operator, V_{free} , contains in a compact and unified form, all the current quark representation mixing for all hadronic states. In the free quark model where V_{free} can be explicitly constructed, Melosh finds indeed that acting with V_{free} on an irreducible representation of $SU(3) \times SU(3)_{\text{strong}}$ generates an infinite sequence of $SU(3) \times SU(3)_{\text{current}}$ representations with increasing orbital angular momentum. It turns out, however, that the transformed axial charge $V_{\text{free}}^{-1} Q_1^5 V_{\text{free}}$ has an extremely simple algebraic structure: it transforms as the sum of the $\{(8,1)_{0,0}\}_{\text{free}} - \{(1,8)_{0,0}\}_{\text{free}}$ and $\{(3,3)_{1,-1}\}_{\text{free}} - \{(\bar{3},\bar{3})_{-1,1}\}_{\text{free}}$ representations of $SU(3) \times SU(3)_{\text{strong}}$. Our assumption in this letter is that this remarkable algebraic structure of the axial charge is preserved for the correct V transformation i.e., that $V^{-1} Q_1^5 V$ still transforms under $SU(3) \times SU(3)_{\text{strong}}$ as a sum of $\{(8,1)_{0,0}\} - \{(1,8)_{0,0}\}$ and $\{(3,3)_{1,-1}\} - \{(\bar{3},\bar{3})_{-1,1}\}$ representations⁸⁾. Our assumption may be stated more precisely as follows: if one could construct physical states by acting with some operator U on free quark states, then this operator U , at least for one-particle states, is assumed to transform as an $SU(3) \times SU(3)$ singlet⁹⁾.

With the standard $SU(6)_{W,\text{strong}}$ classification of hadronic states, written in $SU(3) \times SU(3)_{\text{strong}}$ form, our assumption leads, via PCAC, to the following predictions for pionic decays of resonances.

1) For the "ground state" baryons [i.e., N octet and $\Delta(1236)$ decimet] only the $\Delta L_z = 0$ part of $V^{-1} Q_1^5 V$, i.e., the $\{(8,1)_{0,0}\} - \{(1,8)_{0,0}\}$ representation, can contribute. With the helicity $\frac{1}{2}$ octet and decimet states classified in the same $\{(6,3)_{\frac{1}{2},0}\}$ representation of $SU(3) \times SU(3)_{\text{strong}}$ one obtains

$$\frac{3}{4} g^* = \frac{1}{1+\alpha} g_A \quad (3)$$

where g^* is the matrix element $\langle N | Q_1^5 | \Delta \rangle$ which can be related, via PCAC, to the $\Delta(1236) \rightarrow N \pi$ amplitude, and α is the F/D ratio of the nucleon octet coupling to the axial charges. Within the rather large uncertainties on g^* and α this prediction is in agreement with experiment.

2) Similarly, if one classifies the Roper resonance $R(1470)$ together with a decimet Δ_R , in a pure $\{(6,3)_{\frac{1}{2},0}\}$ representation of $SU(3) \times SU(3)_{\text{strong}}$

one obtains

$$\frac{3}{4} g(\Delta_R \rightarrow N\pi) = \frac{1}{1+\alpha} g(R \rightarrow N\pi) \quad (4)$$

$$g(R \rightarrow \Delta\pi) = -\sqrt{2} g(\Delta_R \rightarrow N\pi) \quad (5)$$

$$\sqrt{2} g(R \rightarrow N\pi) + g(R \rightarrow \Delta\pi) = \sqrt{\frac{2}{3}} g_{\frac{1}{2}}(\Delta_R \rightarrow \Delta\pi) \quad (6)$$

where $g_{\frac{1}{2}}(\Delta_R \rightarrow \Delta\pi)$ refers to the helicity $\frac{1}{2}$ coupling of Δ_R to $\Delta(1236)\pi$.

3) For mesons with $L=1$ our scheme predicts that both decays $1^{+-} \rightarrow 1^{--} + 0^{-+}$ (e.g., $B \rightarrow \omega\pi$) and $1^{++} \rightarrow 1^{--} + 0^{-+}$ (e.g., $A_1 \rightarrow \rho\pi$) occur in both helicity states. This is to be contrasted with the $SU(6)_W$ result which forbids the helicity one decay $1^{+-} \rightarrow 1^{--} + 0^{-+}$ and the helicity zero decay $1^{++} \rightarrow 1^{--} + 0^{-+}$.

4) More specifically for helicity zero decays one has the following prediction ¹⁰⁾, assuming the η to be a pure $SU(3)$ octet member

$$\sqrt{2} g_0(\delta \rightarrow \eta\pi) + g_0(A_2 \rightarrow \eta\pi) = \frac{1}{\sqrt{2}} g_0(A_1 \rightarrow \rho\pi) \quad (7)$$

5) Assuming ¹¹⁾ the ω to be a pure non-strange quark state and the validity of "Zweig's rule" (i.e., $B \not\rightarrow \varphi + \pi$) one predicts ¹²⁾ in addition

$$\sqrt{2} g_0(A_2 \rightarrow \eta\pi) - g_0(\delta \rightarrow \eta\pi) = g_0(B \rightarrow \omega\pi) \quad (8)$$

6) For helicity one decays of the $L=1$ mesonic states, one has the relation ¹³⁾

$$g_1(A_1 \rightarrow \rho\pi) - g_1(A_2 \rightarrow \rho\pi) = 2 g_1(B \rightarrow \omega\pi) \quad (9)$$

7) Finally, there are several predictions for the $L=1$ baryon resonances which, however, are not easily tested since most of the observed physical states appear to be mixtures ¹⁴⁾ of constituent quark spin doublet and quartet states. Assuming the non-strange members of the $\underline{10}^{\frac{3}{2}-}$ and $\underline{10}^{\frac{1}{2}-}$ decimets to be unmixed states, one has the following relation

$$g_{\frac{1}{2}}(\frac{3}{2}^- \rightarrow N\pi) - \sqrt{2} g_{\frac{1}{2}}(\frac{1}{2}^- \rightarrow N\pi) = \frac{1}{\sqrt{5}} \{ g_{\frac{1}{2}}(\frac{3}{2}^- \rightarrow \Delta\pi) - \sqrt{2} g_{\frac{1}{2}}(\frac{1}{2}^- \rightarrow \Delta\pi) \} \quad (10)$$

Many predictions can similarly be derived for the pionic decays of $L=2$ meson and baryon resonances. Details of these calculations will be presented in a forthcoming publication.

Although some of our predictions have been derived elsewhere, in phenomenological mixing schemes, ¹⁵⁾ we feel that the approach adopted here, besides obvious advantages such as systematizing all chiral symmetry representation mixing for all hadrons, may eventually lead to an understanding of the breaking patterns of $SU(6)_{W, strong}$.

It is a pleasure to thank M. Gell-Mann and J.L. Rosner for stimulating discussions on the subject of this letter.

R E F E R E N C E S

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- 5) Except, of course, for the eight $SU(3)$ generators which are identified through CVC.
- 6) H.J. Melosh, Cal.Tech. Ph.D. Thesis (1973), unpublished.
- 7) M. Gell-Mann, Physics 1, 63 (1964).
- 8) F. Gilman and M. Kugler, SLAC-PUB 1157 (to be published), have made a similar assumption but impose in addition that the $\{(8,1)_0,0\}$ - $\{(1,8)_0,0\}$ part of the axial charge be proportional to a generator of $SU(3) \times SU(3)$. This last property, as remarked by the authors themselves is not verified in the free quark case. We thank F. Gilman and M. Kugler for sending us their preprint prior to publication.
- 9) We thank M. Gell-Mann for a discussion on this point.
- 10) With $\Gamma(\delta \rightarrow \gamma \pi) \simeq 30$ MeV and $\Gamma(A_2 \rightarrow \gamma \pi) \simeq 20$ MeV, Eq. (7) leads to $\frac{1}{3} \Gamma_0(A_1 \rightarrow \rho \pi) \simeq 135$ MeV or 2.5 MeV depending on the relative signs of $g_0(\delta \rightarrow \gamma \pi)$ and $g_0(A_2 \rightarrow \gamma \pi)$.
- 11) Without any extra assumption one can derive a relation similar to Eq. (8) for the decays of the strangeness ± 1 members of the appropriate octets. We thank J.L. Rosner for pointing this out to us.
- 12) This prediction leads to $\frac{1}{3} \Gamma_0(B \rightarrow \omega \pi) \simeq 17$ or 95 MeV depending again on the relative signs of $g_0(A_2 \rightarrow \gamma \pi)$ and $g_0(\delta \rightarrow \gamma \pi)$.
- 13) Assuming $\Gamma(B \rightarrow \omega \pi) \simeq 100$ MeV, and that $\Gamma_1(B \rightarrow \omega \pi)$ is appreciably different from zero, footnote 12) suggests $\frac{2}{3} \Gamma_1(B \rightarrow \omega \pi) \simeq 85$ MeV from which we deduce $\frac{2}{3} \Gamma_1(A_1 \rightarrow \rho \pi) \simeq 750$ MeV(!) or 50 MeV depending once again on relative signs of helicity couplings.
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