THE PHYSICAL STATE PROJECTION OPERATOR IN DUAL RESONANCE
MODELS FOR THE CRITICAL DIMENSION OF SPACE - TIME

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ABSTRACT

We present the physical state projection operator for the simple dual resonance model in twenty-six dimensions of space-time and unit intercept in two forms (which are proved to be equivalent). One form obviously projects onto the so-called transverse states and the other obviously gives unity when inserted into any residue of a pole in a dual amplitude. Thus we explicitly verify the completeness of transverse states in 26 dimensions and have a weapon for constructing genuine unitary dual loop amplitudes (as is shown in a second paper). We also present the corresponding results for the Neveu-Schwarz model.

1. INTRODUCTION

The dual resonance model can be formulated in terms of diagrammatic rules which assign to the vertices and lines of tree diagrams well-defined vertices and propagators which are expressed in terms of an infinite set of d dimensional annihilation and creation operators ¹⁾ (d = dimension of space-time) :

$$\begin{aligned} [a_n^\mu, a_m^{\nu\dagger}] &= -g^{\mu\nu} \delta_{nm} & \mu, \nu &= 0, 1, 2, \dots, d-1 \\ & & m, n &= 1, 2, 3, \dots \end{aligned} \quad (1.1)$$

This formalism explicitly exhibits the factorization properties necessary for the particle interpretation of the theory via the factorization of the residues of poles occurring in the tree amplitudes. The physical particle spectrum implied by the theory is actually the minimum spectrum needed to factorize these residues and, owing to possible symmetries of the theory, this could be much less than the spectrum implied by the Fock space of the above oscillators. (This is a familiar situation in QED and Yang-Mills theories where four-component fields are used to describe quantities with only two physical states.) In fact, such a distinction between what we shall call the "full" spectrum and the "physical" spectrum of the theory would be very important since the full spectrum necessarily includes undesirable negative norm (or "ghost") states, which could be absent from the physical spectrum. That this is indeed so was shown recently ^{2),3)} in the case when the leading trajectory intercept (α) is unity and $d \leq 26$. Further if $d=26$ precisely, the structure of the physical spectrum simplifies since it is given by the positive norm Fock space generated by the so-called "transverse" creation operators $A_n^{i\dagger}$ satisfying ⁴⁾

$$\begin{aligned} [A_n^i, A_m^{j\dagger}] &= n \delta^{ij} \delta_{nm} & i, j &= 1, 2, \dots, d-2 \\ & & n, m &= 1, 2, \dots \end{aligned} \quad (1.2)$$

These A_n^i are themselves complicated functions of the a_n^μ above. The special significance of $d=26$ is also suggested by the connection with the quantized string model ⁵⁾, and the conjectured simplicity of the Pomeron singularity ⁶⁾.

The completeness of the "transverse states" for $d=26$, when inserted into residues, followed only from indirect arguments. It has never been shown

explicitly and the purpose of this paper is to remedy this by constructing the projection operator which projects from the full space [generated by the operators in Eq. (1.1)] to the transverse space [generated by the operators in Eq. (1.2)] and showing that it is effectively unity when placed in a residue, if $d=26$. This projection operator is also necessary for the construction of genuine unitary dual loop amplitudes in which only the physical spectrum propagates internally. [This we do for the so-called planar single loop amplitudes in a second paper ⁷⁾.]

In the second section of this paper, we present the projection operator in two forms. In the first form, it is obviously the desired projector and in the second form, it is expressed in terms of those operators which generate the symmetries of the dual model and it is this which enables us to verify the completeness, at least on the mass shell. The equivalence of the two forms is verified (in $d=26$) by means of an identity, Eqs. (2.13) and (2.14) of the next section, which is the main result of this paper. This identity expresses the mode number operator for the "full" Fock space as being equal to the mode number operator for the "transverse" Fock space, plus a spurious part which can be transformed away by the gauge symmetries of the couplings. Certain other elementary properties of the projection operators are demonstrated. Actually, our projection operator appears to be well defined off the mass shell and we have some reason to think that it is still effectively unity then, but we have been unable to construct a satisfactory proof.

In the third section we present a systematic derivation of the second form from the first form. The analysis is relatively complicated but we feel it may have other applications. This is illustrated by the fourth section, in which we present the corresponding results for the Neveu-Schwarz model for $d=10$, also showing that the corresponding projection operator is unity when placed in a residue.

2. THE PROJECTION OPERATOR

Select an internal line of a tree graph and put it on the mass shell corresponding to the N^{th} level of the dual spectrum. Since the mass shell condition is $L_0 - 1 = 0$ where

$$L_0 = H - p^2/2 = - \sum_{n=1}^{\infty} n a_n^+ \cdot a_n - p^2/2 \quad (2.1)$$

we have, if p_N is the momentum of the line,

$$p_N^2 = 2(N-1) \quad (2.2)$$

Then the projection operator acting from the full Fock space onto this level is

$$m_N = \oint \frac{dx}{2\pi i} x^{H-N-1} = \oint \frac{dx}{2\pi i} x^{L_0(p_N)-2} \quad (2.3)$$

where the contour encircles the origin in the x -plane. Replacement of the propagator

$$D(p) = \int_0^1 dx x^{L_0(p)-2} \quad (2.4)$$

by the projection operator (2.2) in the expression for the tree amplitude gives the complete residue at this N^{th} level. Notice that the mass shell condition (2.1) is necessary to make the second integrand in (2.3) single valued and hence the integral well defined.

We shall now see that the further projection onto the transverse space given by the Fock space of the creation operators (1.2) is achieved by multiplication of m_N with the additional projection operator

$$\mathcal{T}(k) = \oint \frac{dy}{2\pi i} y^{L_0-H-1} \quad (2.5)$$

where

$$L_0 = \sum_{n=1}^{\infty} \sum_{i=1}^{d-2} A_n^{i+}(k) A_n^i(k) \quad (2.6)$$

and we have now exhibited the dependence of the operators A on a light-like momentum k which must satisfy the condition

$$k \cdot p_N = 1 \quad (2.7)$$

Goddard and Thorn ²⁾ proved that the "full" Fock space generated by the operators $a_n^{\mu\dagger}$ in (1.1) was equivalent to a sort of Fock space generated by the transverse $A_n^{i\dagger}$ in (1.2) and $K_n^\dagger$ and $L_n^\dagger$ where

$$\begin{aligned} K_n &= \langle k \cdot P(z) \rangle_n \\ L_n &= \langle -\frac{1}{2} : P^2(z) : \rangle_n \end{aligned} \quad (2.8)$$

where

$$\langle X(z) \rangle_n = \oint \frac{dz}{2\pi i z} z^n X(z) \quad (2.9)$$

and

$$P^\mu(z) = p^\mu + \sum_{n=1}^{\infty} \sqrt{n} (a_n^{\mu\dagger} z^n + a_n^\mu z^{-n}) \quad (2.10)$$

Since

$$[K_n, A_m^i(k)] = [L_n, A_m^i(k)] = 0 \quad (2.11)$$

and since H sums the mode numbers of $A_n^{i\dagger}$, $L_n^\dagger$ and $K_n^\dagger$ whereas $\mathcal{L}_0$ sums the mode numbers of just $A_n^{i\dagger}$, we see that $H - \mathcal{L}_0$ is a Hermitian operator such that :

- i) its eigenfunctions span the full Fock space;
- ii) its eigenvalues are integers ≥ 0 ;
- iii) its eigenvalues are zero only on the transverse subspace.

It follows that (2.5) is a Hermitian projection operator acting onto the transverse subspace at any level N .

We now wish to show the promised completeness of transverse states in a tree by considering the operator expression for a tree diagram in the form

$$\langle \Psi | D(p) | \phi \rangle$$

where $|\phi\rangle$ and $|\Psi\rangle$ represent the tree states constructed with the usual vertices and propagators. Then if p satisfies the mass shell condition, (2.1), the result we need is that

$$\langle \psi | m_N (\mathcal{T}-1) | \phi \rangle = 0 \quad (2.12)$$

The proof of (2.12) is facilitated by the following identity, which is our main result. It is valid only if $d=26$ and states that the total mode number operator H equals the "transverse" mode number operator $\mathcal{L}_0$ minus a "spurious" part E :

$$\mathcal{L}_0 - H = E \quad (2.13)$$

where

$$E \equiv (D_0 - 1)(\mathcal{L}_0 - 1) + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n) \quad (2.14)$$

and

$$L_n = \langle -\frac{1}{2} : P^2(z) : \rangle_n \quad D_n = \langle [k \cdot P(z)]^{-1} \rangle_n \quad (2.15)$$

[see (2.9)]. These Fourier components satisfy the algebra :

$$\left. \begin{aligned} [L_n, L_m] &= (n-m) L_{n+m} + \frac{d}{12} (n^3 - n) \delta_{n+m,0} \\ [L_n, D_m] &= -(2n+m) D_{n+m} \\ [D_n, D_m] &= 0 \end{aligned} \right\} \quad (2.16)$$

We shall defer until the next section the direct proof of (2.13), but instead verify the result by noting that the two sides of the equation have identical commutators with the quantities A_n^i , L_n and K_n as well as identical vacuum expectation values. Since A_n^i , K_n and D_n are all mutually commuting the only non-trivial commutator is the L_n one, and we find, using (2.16)

$$[L_n, \mathcal{L}_0 - H] = -n L_n \quad (2.17)$$

$$[L_n, E] = -n L_n + \frac{d-26}{12} (n^3 - n) D_n \quad (2.18)$$

Thus (2.13) follows if $d=26$ (which we shall henceforth suppose).

Notice that the quantities L_n , K_n and D_n all satisfy

$$X_n E = (E - n) X_n \quad (2.19)$$

Hence

$$X_n \mathcal{T}(k) = \oint \frac{dy}{2\pi i y} y^{E-n} X_n \quad (2.20)$$

Since, as we have seen, $E \leq 0$, we see that for $n \geq 1$, the integral on the right-hand side is zero and so

$$X_n \mathcal{T}(k) = 0 \quad n \geq 1 \quad (2.21)$$

where X_n may be any one of the quantities L_n , K_n or D_n . Also, since by Fourier's theorem, and the fact that by (2.7) $K_0 = 1$,

$$D_0 - 1 = - \sum_{n=1}^{\infty} [D_{-n} K_n + K_{-n} D_n] \quad (2.22)$$

we also see that

$$(D_0 - 1) \mathcal{T}(k) = 0 \quad (2.23)$$

It is now obvious that $\mathcal{T}$ is idempotent, i.e.,

$$\mathcal{T}^2 = \mathcal{T} \quad (2.24)$$

Armed with these results, we can now return to the proof of "completeness", Eq. (2.12). Actually, what we shall prove is the following unintegrated version of (2.12)

$$\langle \psi | m_n (y^E - 1) | \phi \rangle = 0 \quad (2.25)$$

where, of course, $|\phi\rangle$ and $|\psi\rangle$ satisfy the usual Ward identities

$$L_n |\phi\rangle = (L_0 + n - 1) |\phi\rangle \quad (2.26)$$

Since

$$y^E - 1 = E \int_1^y dz z^{E-1} \quad (2.27)$$

we can use (2.26) and the relevant commutators (2.16) and (2.19) to eliminate all the L_n 's and L_{-n} 's from the E occurring linearly in (2.27) in favour of L_0 's. These are in turn eliminated by the mass shell condition

$$(L_0 - 1) m_N = 0 \quad (2.28)$$

and (2.25), and hence (2.12), are then found to follow.

We now wish to consider the commutation of the projection operator $\mathcal{T}(k)$ with a vertex which we shall take to emit a tachyon with momentum q and to join two internal lines with momentum p_1 and $p_2 = p_1 + q$. Then in order to satisfy (2.7) for both legs

$$k \cdot p_1 = k \cdot p_2 = 1 \quad (2.29)$$

we must also have $k \cdot q = 0$. This is kinematically possible but, of course, means we are working in a special frame of reference. In this frame, the D_n 's all commute with the vertex operator $V_0(q)$ while the L_n commutators are given by the standard result

$$[L_n, V_0(q)] = [L_0, V_0(q)] + n V_0(q) \quad (2.30)$$

Then, using these commutators, the mass shell condition (2.28) and the algebra (2.16) we find

$$m_{N_1} [E, V_0(q)] m_{N_2} = 0 \quad (2.31)$$

from which it follows, since E and m_N commute, that

$$m_{N_1} [\mathcal{T}(k), V_0(q)] m_{N_2} = 0 \quad (2.32)$$

Thus, in the special frame already mentioned, the projection operator and the vertex commute, provided the two legs are on the mass shell.

We should like to eliminate the necessity for the mass shell here and in previous results. We can note that the integral giving $\mathcal{T}(k)$ is well defined off mass shell and works equally well at any mass shell level in the previous analysis. To check that it is a good off mass shell projection operator, we should like to prove, instead of Eq. (2.12)

$$\langle \psi | D(p) (\mathcal{T}(k) - 1) | \phi \rangle = 0 \quad (2.33)$$

The previous result (2.12) shows that this is analytic in p^2 and so gives rise to a contact term (which may not be Lorentz covariant because of the k dependence). We have tried to prove (2.33) and believe that it is probably true but have not yet constructed a satisfactory proof. In the sequel to this paper in which we calculate the planar loop, we shall see that it is sufficient to have the projection operator on the mass shell.

3. DIRECT EVALUATION OF $\mathcal{L}_0$ -H

This section is devoted to the direct systematic derivation of our fundamental relation (2.13) and (2.14) which expresses $\mathcal{L}_0$ -H in terms of the gauge operators. The analysis is fairly complicated and we shall therefore just present it in outline emphasizing the general procedure which, as the next section illustrates, can be useful elsewhere in dual theory.

We start with $\mathcal{L}_0$ as given by Eq. (2.6), but with the explicit formula for the transverse A_n^i (4) :

$$A_n^{i+} = A_{-n}^i = \oint \frac{dx}{2\pi i x} \epsilon^i \cdot p(x) e^{-ink \cdot Q(x)} \quad (3.1)$$

where ϵ^i , the polarization vector, is orthogonal to both k and p , the integral encloses the origin, and

$$Q^\mu(z) = q^\mu - i p^\mu \ln z - i \sum_{m=1}^{\infty} \frac{1}{\sqrt{m}} [a_m^{\mu+} z^m - a_m^\mu z^{-m}] \quad (3.2)$$

so, according to (2.10)

$$p^\mu(z) = i z \frac{dQ^\mu(z)}{dz} \quad (3.3)$$

We find

$$\mathcal{L}_0 = \sum_{n=1}^{\infty} \oint \frac{dx}{2\pi i x} \oint \frac{dy}{2\pi i y} \left[: P_{\mu}(x) \epsilon^{\mu\nu} P_{\nu}(y) : + \frac{(d-2)xy}{(x-y)^2} \right] \quad (3.4)$$

$$\cdot e^{-ik \cdot Q(x)} e^{ik \cdot Q(y)}$$

We have explicitly normal ordered the product $P(x)P(y)$ and thus have found the condition $|y| < |x|$. We are now allowing space-time to have arbitrary dimension d . The polarization tensor is

$$\epsilon_{\mu\nu} = \sum_{i=1}^{d-2} \epsilon_{\mu}^i \epsilon_{\nu}^i = -g_{\mu\nu} + k_{\mu} p_{\nu} + p_{\mu} k_{\nu} - p^2 k_{\mu} k_{\nu} \quad (3.5)$$

as can be deduced from the conditions

$$k^{\mu} \epsilon_{\mu\nu} = p^{\mu} \epsilon_{\mu\nu} = 0 \quad (3.6)$$

Because k is light-like, the sum over mode number n in (3.4) is a geometric series which can be summed to give

$$\mathcal{L}_0 = \oint \frac{dx}{2\pi i x} \oint \frac{dy}{2\pi i y} \left[: P_{\mu}(x) \epsilon^{\mu\nu} P_{\nu}(y) : + \frac{(d-2)xy}{(x-y)^2} \right] \quad (3.7)$$

$$\cdot \frac{e^{-ik \cdot Q(x)}}{e^{-ik \cdot Q(y)} - e^{-ik \cdot Q(x)}}$$

Notice that the integrand has an isolated singularity at $x=y$. If we distort the contour past the singularity, we obtain the contribution of the residue of the pole there, together with an integral like the original but with the reversed inequality: $|y| > |x|$. But by an interchange of the integration variables x and y , together with a trivial integration, we find that this integral is just the negative of the original (3.7) which must therefore equal one-half of the pole contribution:

$$\mathcal{L}_0 = \frac{1}{2} \oint \frac{dy}{2\pi i y} \oint_y \frac{dx}{2\pi i x} \left[: P_\mu(x) \epsilon^{\mu\nu} P_\nu(y) : + \frac{(d-2)xy}{(x-y)^2} \right] \cdot \frac{e^{-ik \cdot Q(x)}}{e^{-ik \cdot Q(y)} - e^{-ik \cdot Q(x)}} \quad (3.8)$$

The x contour just encircles the point y and so can be evaluated by the residue theorem. The first term gives

$$\frac{1}{2} \oint \frac{dy}{2\pi i y} : P_\mu(y) \epsilon^{\mu\nu} P_\nu(y) : / k \cdot P(y) \quad (3.9)$$

Because it is itself already normal ordered and because it commutes with the quantity inside the normal ordering [because of (3.6)], the $[k \cdot P(y)]^{-1}$ can be moved inside the normal ordering. If we substitute expression (3.5) for $\epsilon^{\mu\nu}$ we can carry out some of the divisions, obtaining quantities which can be integrated trivially. The result is

$$\frac{P^2}{2} - \frac{1}{2} \oint \frac{dy}{2\pi i y} : \frac{P^2(y)}{k \cdot P(y)} : \quad (3.10)$$

The second, dimension dependent term in (3.8), has a triple pole at $x=y$. Careful evaluation gives

$$- \left(\frac{d-2}{24} \right) \left(-1 + \oint \frac{dy}{2\pi i y} \frac{1}{k \cdot P(y)} \left[1 + \left[y \frac{d}{dy} \ln k \cdot P(y) \right]^2 \right] \right) \quad (3.11)$$

So $\mathcal{L}_0$ is given as the sum of the expressions (3.10) and (3.11). Most of the structure appears in the second term of (3.10). If the Fourier components of $-\frac{1}{2} : P^2(y) :$ and $[k \cdot P(y)]^{-1}$, which are L_n and D_n [see Eqs. (2.15)], were c number quantities, this second term would simply be

$$\sum_{n=-\infty}^{\infty} D_{-n} L_n \quad (3.12)$$

These Fourier components are not c number quantities [since they obey the algebra (2.16)] and we would rather obtain the "normal ordered" expression :

$$D_0 L_0 + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n) \quad (3.13)$$

This can be evaluated in a way analogous to that which led from (2.6) to (3.10) and (3.11) and the result is

$$-\frac{1}{2} \oint \frac{dy}{2\pi i y} : \frac{p^2(y)}{k \cdot p(y)} : - \oint \frac{dy}{2\pi i y} \frac{1}{k \cdot p(y)} \left[y \frac{d}{dy} \ln k \cdot p(y) \right]^2 \quad (3.14)$$

Putting together all these results, we have

$$\begin{aligned} L_0 - H = & (D_0 - 1) \left(L_0 - \frac{d-2}{24} \right) + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n) + \\ & + \frac{26-d}{24} \oint \frac{dy}{2\pi i y} \frac{1}{k \cdot p(y)} \left[y \frac{d}{dy} \ln (k \cdot p(y)) \right]^2 \end{aligned} \quad (3.15)$$

which reduces to the desired result if $d=26$. It is interesting that the last term which enters if $d \neq 26$ has a structure reminiscent of the Brower longitudinal operators. This suggests that for $d \neq 26$ there exists an equation similar to (2.13) and (2.14) but with an extra contribution from a longitudinal mode counting operator.

4. THE PROJECTION OPERATOR FOR THE NEVEU-SCHWARZ MODEL

The Neveu-Schwarz model ⁸⁾ has a "full" Fock space generated by the operators in (1.1) and in addition anticommuting operators :

$$\{b_m^\mu, b_n^{\nu\dagger}\} = -g^{\mu\nu} \delta_{nm} \quad \begin{aligned} n, m &= 0, 1, 2, \dots \\ \mu, \nu &= 0, 1, \dots, d-1 \end{aligned} \quad (4.1)$$

The "transverse" Fock space [in what is called the $\mathcal{F}_2$ space; see Ref. 9) for the explanation] is generated by "transverse" operators $A_n^{i\dagger}$ and $B_n^{i\dagger}$ which have been explicitly constructed in terms of the a_n^μ and b_n^μ by several authors ¹⁰⁾. If H and $\mathcal{L}_0$ are the mode number operators for the full and transverse Fock spaces corresponding to those in Section 2, but appropriately extended to incorporate the new states, it follows by a similar argument that the "transverse space" projection operator is

$$\mathcal{T}_{NS}(k) = \oint \frac{dy}{2\pi i y} y^{2(\mathcal{L}_0 - H)} \quad (4.2)$$

The extra factor two in the exponent is necessary since $\mathcal{L}_0 - H$ may now have integral or half-integral eigenvalues. Using the expressions ¹⁰⁾ for A_n^i and B_n^i and repeating the arguments of the previous section, we have found that, in ten dimensions, $\mathcal{L}_0 - H$ can be expressed as a linear combination of the gauges which are appropriate to the $\mathcal{F}_2$ space, namely L_n, G_r and the mass shell operator $L_0 - \frac{1}{2}$:

$$\mathcal{L}_0 - H = E_{NS} \quad (4.3)$$

where

$$E_{NS} = (D_0 - 1)(L_0 - \frac{1}{2}) + \sum_{n=1}^{\infty} (D_{-n} L_n + L_{-n} D_n) + \sum_{r=\frac{1}{2}}^{\infty} (E_{-r} G_r - G_{-r} E_r) \quad (4.4)$$

and now

$$\left. \begin{aligned} L_n &= \langle -\frac{1}{2} : P^2(z) : + \frac{1}{2} : z H(z) \frac{dH(z)}{dz} : \rangle_n \\ D_n &= \langle [k \cdot P(z)]^{-1} (1 - 2k \cdot H(z) \frac{d}{dz} (k \cdot H(z)) (k \cdot P(z))^{-2} \rangle_n \\ G_r &= \langle P(z) \cdot H(z) \rangle_r \\ E_r &= \langle z \cdot [k \cdot P(z)]^{-3/2} \frac{d}{dz} \{ k \cdot H(z) (k \cdot P(z))^{-1/2} \} \rangle_r \end{aligned} \right\} \quad (4.5)$$

It is understood that the labels n and m are integers and r and s half-integers. $H^\mu(z)$ is, as usual,

$$H^\mu(z) = \sum_{n=0}^{\infty} [b_n^\mu z^{n+1/2} + b_n^\mu z^{-n-1/2}] \quad (4.6)$$

These Fourier components satisfy the algebra in d space-time dimensions :

$$\left. \begin{aligned} [L_n, L_m] &= (n-m) L_{n+m} + \frac{d}{8} n(n^2-1) \delta_{n+m,0} \\ [L_n, G_r] &= \left(\frac{n}{2} - r\right) G_{n+r} \\ \{G_r, G_s\} &= 2 L_{r+s} + \frac{d}{2} (r^2 - 1/4) \delta_{r+s,0} \\ [G_r, D_n] &= 2 E_{n+r} \\ \{G_r, E_s\} &= - \frac{3r+s}{2} D_{r+s} \\ [L_n, D_m] &= - (2n+m) D_{n+m} \\ [L_n, E_r] &= - \left(\frac{3}{2} n + r\right) E_{n+r} \\ [D_n, D_m] &= \{E_r, E_s\} = [D_n, E_r] = 0 \end{aligned} \right\} \quad (4.7)$$

Notice that the L, D algebra is preserved. As a check, we can calculate from (4.7), for general d

$$[G_r, L_0 - H] = -r G_r \quad (4.8)$$

$$[G_r, E_{Ns}] = -r G_r + \frac{d-10}{2} (r^2 - 1/4) E_r \quad (4.9)$$

In ten dimensions, these commutators will be equal, as will also be the L commutators. Now we can prove, just as before

$$L_n \tilde{T}_{Ns} = 0 \quad n \geq 1 \quad (4.10)$$

$$G_r \tilde{T}_{Ns} = 0 \quad r \geq \frac{1}{2} \quad (4.11)$$

as well as other similar relations, if we now assume ten dimensions.

We can now prove a completeness relation similar to (2.12) to show that the transverse states span the physical set of states in the Fock space. As for the proof of (2.12), we prove the unintegrated version

$$\langle \psi | m_{N, N_3} (y^{2E_{N_3}} - 1) | \phi \rangle = 0 \quad (4.12)$$

In this case we define

$$m_{N, N_3} \equiv \oint \frac{dx}{2\pi i x} x^{2(L_0 - 1/2) - 1} \quad (4.13)$$

The states $|\psi\rangle$ and $|\phi\rangle$ satisfy the Ward identities

$$L_n |\phi\rangle = (L_0 + n - 1/2) |\phi\rangle \quad (4.14)$$

$$G_r |\phi\rangle = (L_0 + r - 1/2) |\phi_0\rangle \quad (4.15)$$

where $|\phi_0\rangle$ differs from $|\phi\rangle$ in that the first vertex is the ordinary Veneziano model vertex instead of a Neveu-Schwarz model vertex. We write

$$y^{2E_{N_3}} - 1 = 2E_{N_3} \int_1^y dz z^{2E_{N_3} - 1} \quad (4.16)$$

We can now use the commutator relations (4.7) together with (4.14) and (4.15) to eliminate all L_n, L_{-n}, G_r and G_{-r} from the E occurring linearly in (4.16) in favour of L_0 's. Equation (4.12) is then seen to follow when using the mass shell condition

$$(L_0 - 1/2) m_{N, N_3} = 0 \quad (4.17)$$

and hence the completeness of the transverse space in ten dimensions is proved.

5. CONCLUSIONS

In both extant dual models we have constructed the projection operator onto "transverse states" and recast it into a form involving gauges. In this form it can be transformed away in tree diagrams if the line concerned is on the mass shell, thereby showing the completeness of the transverse states in the critical dimension. As we show in the sequel ⁷⁾ the latter form of the projection operator is also the useful one in loop calculations.

We should like to emphasize that since the spectrum of the operator $\mathcal{L}_0 - H$ is always integral (or half-integral in the Neveu-Schwarz model), irrespective of whether or not the mass shell condition is satisfied, we believe it may also be a good projection operator off mass shell, since it is certainly well defined off mass shell and many of its properties still hold, for example, Eqs. (2.20) and (2.24).

It is possible that the Neveu-Schwarz projection operator may play a rôle in the construction of fermion-fermion amplitudes, since up to now there has been a problem in eliminating spurious states on meson lines joining fermion lines ¹¹⁾.

Finally, we should mention that it should be possible to derive analogous results for non-critical dimensions by including extra contributions for longitudinal physical states, but that we have not done this yet.

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R E F E R E N C E S

- 1) An account with references appears in :
V. Alessandrini, D. Amati, M. Le Bellac and D. Olive, Phys.Reports 1C, 269 (1971).
We follow the notation and conventions of this article, except that we no longer restrict d to be 4. Thus $g^{\mu\nu} = \text{diag}(1, -1, -1, \dots, -1)$.
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