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LIGHT-CONE DOMINANCE OF VERTEX FUNCTIONS FOR COMPOSITE FIELDS
AND ASYMPTOTIC BEHAVIOUR OF ELASTIC FORM FACTORS +)

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A B S T R A C T

It is shown that off-shell vertex functions for composite fields are dominated by a light-cone singularity. The threshold behaviour of the corresponding scaling function is related to the rate of decrease of the elastic form factor.

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A major problem in the understanding of deep inelastic e-N scattering has been the failure of conventional renormalizable field theory, like e.g., Yukawa interaction at any finite order in the coupling constant, to give an explanation of the observed Bjorken scaling behaviour ^{1),2)}.

A way out of this difficulty has recently been shown by Drell and Lee ³⁾ by treating the target hadron as a relativistic composite system. The compositeness of the nucleon supplies a sufficiently fast decrease in the transverse momentum so as to give finite scaling functions. The same model has also proved useful in giving a qualitative understanding of the asymptotic behaviour of electromagnetic form factors ⁴⁾.

We accept in this note the idea of a hadron described by a composite field ⁵⁾ and we try to analyze in this context the light-cone structure of its vertex functions ⁶⁾. We are interested in two physically different questions:

- 1) given a field $\psi(x)$ interacting with a current $J(0)$, what is the difference, in the light-cone behaviour of the product, between an elementary ψ and a composite one? In particular, is the off-shell form factor light-cone dominated in the Brandt-Preparata ⁷⁾⁻⁹⁾ sense?
- 2) given a composite field $\psi(x) \sim \phi_a(x)\phi_b(x)$, and given the fact that the elastic form factor depends on the product $\phi_a(x)\phi_b(0)$, how does the light-cone behaviour of the latter determine $F(q^2)$ at large q^2 's. In other words, is the form factor a good probe of the interaction between the constituents at small x^2 ?

Let us consider a bound state of mass M of two point-like particles a and b of equal mass m (all spinless), and let $\psi(x) = N\phi_a(x)\phi_b(x)$ be its field, interacting with the scalar current ¹⁰⁾

$$J(y) = \phi_b^+(y) \phi_b(y). \quad (1)$$

The internal a - b interaction, which is assumed to be regular ¹¹⁾, is characterized by an index δ which gives the light-cone singularity of the $M \rightarrow ab$ vertex function

$$\langle 0 | T[j_a(x) j_b(0)] | P \rangle \underset{x^2 \rightarrow 0}{\sim} (x^2 - i\varepsilon)^{-2+\delta} q(x \cdot P), \quad (2)$$

$$j_{a,b}(x) \equiv (\square + m^2) \phi_{a,b}(x), \quad 0 < \delta < 2,$$

where, for instance $\delta = 1$ corresponds to an interaction mediated by a scalar point-like gluon, and $\delta = 0$ would correspond to a (singular renormalizable) $\lambda \phi^4$ theory.

The form factor of particle M is given by the reduction formula

$$F(q^2) \equiv \langle P_2 | J(0) | P_1 \rangle = i N \int d^4 x e^{i P_2 \cdot x} (\square_x + M^2) \langle 0 | T[\phi_a(x) \phi_b(x) J(0)] | P_1 \rangle, \quad N = \langle 0 | \phi_a(0) \phi_b(0) | M \rangle^{-1} \quad (3)$$

This formula also gives the off-shell continuation in the form

$$V(P_2^2, q^2) \equiv F(P_2^2, q^2) / (M^2 - P_2^2) = i N \int d^4 x e^{i P_2 \cdot x} \langle 0 | T[\phi_a(x) \phi_b(x) J(0)] | P_1 \rangle. \quad (4)$$

If $J(0)$ is given by (1), and if q^2 is large, $F(q^2)$ can be given an explicit form in the impulse, or ladder, approximation ^{4), 12)}

$$\langle 0 | T[\phi_a(x) \phi_b(x) J(0)] | P \rangle \simeq \quad (5a)$$

$$\simeq \int d^4 y G(x, x; y, 0) (\square_y + m^2) \langle 0 | T[\phi_a(y) \phi_b(0)] | P \rangle, \quad (5b)$$

where G is the four-point Green's function. From (5b) we get in fact ¹³⁾

$$F(q^2) = i \int d^4 x \langle P_2 | T[\phi_a^\dagger(x) \phi_b^\dagger(0)] | 0 \rangle (\square + m^2) \langle 0 | T[\phi_a(x) \phi_b(0)] | P_1 \rangle. \quad (6)$$

Our discussion is based on Eqs. (2), (4) and (6).

The large q^2 behaviour of the off-shell vertex function (4) can be investigated either in the Bjorken limit ($\nu \equiv 2P_1 \cdot q \rightarrow \infty$, $\omega \equiv -q^2/\nu$ fixed, $\omega \neq 1$) or in the B.P. limit ($\nu \rightarrow \infty$, P_2^2 fixed). The light-cone dominance means that the latter limit can be smoothly reached from the former as $\omega \rightarrow 1$. If γ is elementary, the perturbation theory shows there is a pole in the scaling function at $\omega = 1$; but Brandt and Preparata ⁷⁾ suggest that it may disappear in the composite case.

In order to see what happens in our model, let us separate those contributions to (5a) which contain a disconnected b line from the connected ones (Fig. 1). Graph 1a corresponds to the leading light-cone singularity, namely

$$\langle 0 | T [\phi_a(x) \phi_b(x) J(0)] | P \rangle \simeq \Delta_F(x, m^2) \chi(x^2, x \cdot P), \quad (7)$$

$$\chi(x^2, x \cdot P) \equiv \langle 0 | T [\phi_a(x) \phi_b(0)] | P \rangle,$$

where by Eq. (2) $\chi(0, \tau)$ is finite ¹⁴⁾ for $\delta > 0$, and we also have ¹⁵⁾

$$\chi(x^2, \tau) \underset{\tau \rightarrow \infty}{\simeq} c h(x^2) \tau^{-\delta-1} (1 + \text{oscillations}) \quad (8)$$

$$0 < \delta < 2, \quad h(0) < \infty.$$

The light-cone contribution (7) shows canonical scaling limit, with a scaling function determined by the wave function $\chi(0, \tau)$ on the light-cone:

$$V_{LC}(P_1^2, q^2) \simeq \frac{1}{\nu} \tilde{V}(\omega), \quad (9)$$

$$\tilde{V}(\omega) = N \int_0^\infty d\tau e^{i(1-\omega)\tau} \chi(0, \tau).$$

It is easily seen that Eq. (9) at $\omega = 1$ gives also the B.P. limit of (7), because for regular interaction ($\delta > 0$) $\chi(0, \tau)$ is convergent enough at $\tau = \infty$ ¹⁶⁾, [Eq. (8)], so as to give $\tilde{V}(1) < \infty$.

This is to be contrasted with the elementary particle case, for which the light-cone weight function is constant at $\tau = \infty$, and, therefore, a pole at $\omega = 1$ appears in Eq. (9).

It remains to be seen that the connected graph 1b vanishes more rapidly than ν^{-1} in the B.P. limit. Consider in fact the integration region for which all hadron masses appearing in graph 1c are finite. In the rest frame of P_1 this is achieved by the parameterization

$$q = (q, 0, q+m), \quad p_1 = (\alpha_1 + \frac{\beta_1}{q}, p_{11}, \alpha_1 - m),$$

$$p_2 = [x_2(q+m) + \frac{\beta_2}{q}, p_{21}, x_2(q+m)]. \quad (10)$$

Changing variables to $\alpha_1, \beta_1, x_2, \beta_2$ gives a Jacobian $|v|^{-1}$, due to the fact that P_2^2 is kept fixed, Ref. 17). However, the α_1 integration, and, therefore, the coefficient of $|v|^{-1}$, vanish in the large q^2 limit because α_1 appears (linearly) only in the mass p_1^2 . The actual behaviour of graph 1c is model dependent. In the ladder approximation one has $v^{-1-\delta} \ln v$ 15), 18), the reason being that either p_1^2 , or $(p_1+q)^2$ and an internal gluon mass have to be large in this limit.

We conclude that behaviour (9) holds for the full vertex function in both the Bjorken and the B.P. limits. In other words, the free part of the constituents' Green's function dominates at large momentum transfer, (graph 1a). The scattering part (graph 1b, e.g., in the ladder approximation) decreases as the elastic form factor and is non-leading for regular interaction ($\delta > 0$).

Let us now remark that by known spectral properties 19) we have

$$\chi(0, \tau) = \int_0^1 d\omega' v(\omega') e^{-i(1-\omega')\tau}, \quad (11)$$

so that Eq. (9) becomes

$$\tilde{V}(\omega) = \int_0^1 d\omega' \frac{v(\omega')}{\omega' - \omega + i\varepsilon} \quad (12)$$

and by Eqs. (8) and (12), we therefore get

$$\text{Disc. } \tilde{V}(\omega) = v(\omega) \underset{\omega \rightarrow 1}{\simeq} \frac{Nc}{4\delta} (1-\omega)^\delta. \quad (13)$$

If we substitute this behaviour in a finite mass sum rule, by assuming scaling for $|q^2|$, $|M'^2| > M_1^2$, we get

$$\int_{M^2}^{M_1^2} dM'^2 \text{Disc. } V(M'^2, q^2) \simeq \int_{1-M_1^2/|q^2|}^1 d\omega \nu(\omega) \simeq (14)$$

$$\simeq (M_1^2/|q^2|)^{\delta+1}, \quad (|q^2| \gg M_1^2).$$

This means that the non-scaling piece of the absorptive part of V behaves like $|q^2|^{-\delta-1}$, and suggests that the elastic form factor has a similar behaviour.

An explicit calculation can be performed on Eq. (6) by using the asymptotic behaviour (8). The relative space-time picture which arises is the following ¹⁵⁾. In the Breit frame ($\underline{P}_1 = -\underline{P}_2 = \underline{q}/2$) it is seen that at large q^2 $|\underline{x} \cdot \underline{P}_1|$ and $|\underline{x} \cdot \underline{P}_2|$ have to be large, unless $x_0 \simeq x_3 \simeq 1/|q|$, in which case $|\underline{x}^2|$ is small ²⁰⁾. There are, therefore, two competing contributions: the region of fixed $\underline{x}^2$ and large $|\underline{x} \cdot \underline{P}|$, and the light-cone region of small $|\underline{x}^2|$. Since no damping oscillations are present in Eq. (8) and since the power fall-off is slow, it turns out that the former region dominates. Equation (6) is therefore a convolution of the form

$$F(q^2) \simeq i |C|^2 \int d^4x (m^2 h^2 - 4 \underline{x}^2 h'^2) \cdot |\underline{x} \cdot \underline{P}_2|^{-\delta-1} |\underline{x} \cdot \underline{P}_1|^{-\delta-1} \quad (15)$$

where the constant C is to be determined by the condition $F(0) = 1$ ¹²⁾. Since each power $(\underline{P} \cdot \underline{x})^{-\delta-1}$ corresponds to a pole at $J = -\delta-1$ in its Mellin transform, the resultant behaviour of $F(q^2)$ is the one obtained from a double pole at $J = -\delta-1$, namely ¹⁵⁾

$$F(q^2) \simeq \frac{1}{2} \pi^2 (4m^2)^\delta \frac{|C|^2}{S} |q^2|^{-\delta-1} \ln |q^2|. \quad (16)$$

We therefore find the following results.

- 1) The off-shell vertex function is light-cone dominated for composite fields with regular interaction between constituents. The scaling function has a finite $\omega \rightarrow 1$ limit, and the vanishing of its discontinuity [Eq. (13)] is connected with the fall-off of the elastic form factor [Eq. (16)] by a relation of the same nature as Drell-Yan-West relation²⁰⁾.
- 2) The fixed x^2 , large $x \cdot P$ region dominates over the light-cone region of $\chi(x^2, x \cdot P)$ in determining the large q^2 fall-off of $F(q^2)$. In this respect, one has a rather confusing situation. On one hand, since Eqs. (8) and (2) are related through the index δ , the asymptotic behaviour of $F(q^2)$ is dependent on the interaction at small distances. On the other hand, a light-cone expansion of $\chi(x^2, x \cdot P)$ would not tell the whole truth, because the large $|x \cdot P|$ region is the dominant one.

If the coefficient of the leading behaviour Eq. (8) happens to vanish, the small $|x^2|$ behaviour may become dominant in Eq. (6). Since we have¹⁴⁾

$$\chi(x^2, p \cdot x) \underset{x^2 \rightarrow 0}{\simeq} f_0(p \cdot x) + (x^2)^\delta f_1(p \cdot x) + \dots, \quad (17)$$

$(\delta \neq 1),$

an easy computation shows that in this case

$$F(q^2) \simeq |q^2|^{-1-2\delta}, \quad (18)$$

a behaviour different from Eq. (16). The relation between (13) and (16), as well as the Drell-Yan-West relation would no longer hold in the present form.

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a)

b)

c)

a) Disconnected vs. b) Connected contributions to the matrix element (5a). Graph c) is the momentum space counterpart of graph b).

FIG.1