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ON THE SLOPE OF THE SCALAR $K_{\ell 3}$ DECAY FORM FACTOR

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A B S T R A C T

We discuss a mechanism, based on exact threshold theorems in the limit of $SU_2 \times SU_2$ symmetry, which can be used to estimate a certain suppression of the scalar $K_{\ell 3}$ decay form factor at low momentum transfers, while maintaining the validity of the Callan-Treiman relation.

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Considerable experimental and theoretical efforts have been devoted to the understanding of the $K_{\ell 3}$ decay form factors. For these we adopt a standard notation

$$\langle \pi^0(q) | V_\mu^{K^+} | K^+(p) \rangle = \frac{1}{\sqrt{2}} [f_+(t)(p+q)_\mu + f_-(t)(p-q)_\mu] \quad (1)$$

normalized to $f_+(0) = 1$ in the SU_3 symmetry limit, with the Cabibbo angle separated off. The subject of this discussion will be the normalized scalar combination of these form factors

$$\langle \pi^0(q) | \mathcal{V}_\mu^{K^+} | K^+(p) \rangle = \frac{i}{\sqrt{2}} (m_K^2 - m_\pi^2) f_+(0) F_0(t) \quad (2)$$

Apart from the normalization condition $F_0(0) = 1$, one also expects to find $F_0(m_K^2)$ expressed in terms of the $\pi_{\ell 2}$ and $K_{\ell 2}$ decay constants F_π and F_K , through the Callan-Treiman relation¹⁾

$$F_0(m_K^2) \approx \frac{F_K}{F_\pi f_+(0)} \approx 1.28 \quad (3)$$

up to corrections of order $O(m_\pi^2)$ which vanish in the limit of chiral $SU_2 \times SU_2$ symmetry. A linear interpolation between these two points would lead to a slope parameter

$$\lambda_0 = \left[\frac{d}{dt} F_0(t) \right] \Big|_{t=0} \cdot m_\pi^2 \approx 0.022 \quad (4)$$

while a number of experiments tend to favour a smaller and probably even negative value. We quote from a review of Chounet and Gaillard²⁾

$$\lambda_0 = -0.025 \pm 0.020 \quad (5)$$

We note that this value is not yet established³⁾.

In this paper we examine the question to what extent the slope parameter λ_0 may be suppressed at low t , while respecting the Callan-Treiman relation. To make this last condition more precise, we work in the limit of chiral $SU_2 \times SU_2$ symmetry, with $m_\pi^2 = 0$. We find that a suppression

of λ_0 by at least 20%, down to $\lambda_0 \leq 0.018$, is expected to occur quite naturally and a downward shift by about 0.01 is the most we can plausibly accommodate. Any further suppression of λ_0 , possibly even to negative values, should be attributed to violations of $SU_2 \times SU_2$. This result is certainly in accordance with many other estimates, too many to quote them here, and the mechanisms we will discuss can readily be seen to operate in existing models; the main point of this paper is to show their rather general validity. In the limit of $SU_2 \times SU_2$ symmetry, exact threshold theorems can be used to give the above estimates without additional controversial assumptions; we have not yet been able to treat the effects of $SU_2 \times SU_2$ violations in comparable generality.

A suppression of λ_0 at low t can only be made compatible with the Callan-Treiman condition if a certain amount of positive curvature is introduced in $F_0(t)$. We find that the attractive $(K\pi)$ scattering threshold can produce such an effect, and in the limit of chiral $SU_2 \times SU_2$ symmetry, with $m_\pi^2 = 0$ and the threshold being shifted down to m_K^2 , this effect is completely determined to the leading order in $(m_K^2 - t)$. This is seen by considering the low energy contributions to a dispersion relation for the second derivative $F_0''(t)$

$$F_0''(t) = \frac{2}{\pi} \int_{m_K^2}^{\Lambda^2} \frac{\text{Im} F_0(s)}{(s-t)^3} ds + \dots \quad (6)$$

The contribution of the elastic $(K\pi)$ intermediate states dominates at threshold, it is given by

$$\text{Im} F_0(s) \approx \frac{1}{8\pi} F_0(s) \frac{p}{\sqrt{s}} T^*(s) \quad (7)$$

with

$$F_0(s) \approx F_0(m_K^2) = \frac{F_K}{F_\pi f_+(0)} \quad (8)$$

and

$$T(s) \approx (s - m_K^2) / F_\pi^2 \quad (9)$$

where Eq. (9) is determined by Adler's and Weinberg's low energy theorems for the $(K\pi)$ s wave scattering amplitude $T(s)$ in the isospin $(\frac{1}{2})$ channel. Integration of Eq. (6) gives immediately the leading behaviour

$$F_0''(t) = \frac{F_0(m_K^2)}{8\pi^2 F_\pi^2 m_K^2} \log\left(\frac{\Lambda^2}{m_K^2 - t}\right) + (\text{finite terms}) \quad (10)$$

To give a more realistic estimate of the effect we now propose to go back to a dispersion relation for $F_0(t)$, we separate a low energy region from a high energy region by a cut-off parameter Λ^2 , and for the purpose of using it only in $0 \leq t \leq m_K^2$ we replace the high energy contribution by a linear form in t . To give greater weight to the low energy region we use a once subtracted form

$$F_0(t) \approx F_0(m_K^2) + \frac{(t - m_K^2)}{\pi} \int_{m_K^2}^{\Lambda^2} \frac{\gamma_m F_0(s)}{(s - m_K^2)(s - t)} ds + \gamma(t - m_K^2) \quad (11)$$

The parameter γ is determined to match the value $F_0(0) = 1$. It will of course strongly depend on Λ^2 , yet the over-all dependence of $F_0(t)$ on Λ^2 for $0 \leq t \leq m_K^2$ is mild. The cut-off should be placed below $[900 \text{ MeV}]^2$ because otherwise the unitarity bound on $|T(s)|$ would be violated by the extrapolation of Eq. (9). Yet a value as high as $\Lambda^2 = 3m_K^2$ should still be sensible, since the $(K\pi)$ s wave phase shift does actually rise to quite substantial values and any residual over-estimate through the use of Eq. (7) for $|T(s)|$ will be more than compensated by a most likely underestimate implied in the use of an ansatz like Eq. (8) for $|F_0(s)|$. In fact it is quite difficult to estimate the behaviour of $F_0(s)$ in $m_K^2 \leq s \leq \Lambda^2$; a constant extrapolation is most likely an underestimate, since an attractive $(K\pi)$ phase shift tends to enhance $|F_0(s)|$. Yet for a realistic computation we would need more detailed knowledge about effects at $s \geq (1 \text{ GeV})^2$ than we actually have. For the purposes of this paper we will therefore simply introduce an average $\langle F_0 \rangle$, whose precise value is unknown. It is most likely larger than $F_0(m_K^2)$, but it would require quite specific dynamical assumptions to get it more than three times as large, since even an effective pole at $(1 \text{ GeV})^2$ would barely give such an effect. The evaluation of the integral is elementary, and we find for λ_0 (at $t = 0$)

$$\lambda_0 m_K^2 / m_\pi^2 = (F_0(m_K^2) - 1) - m_K^2 \frac{\langle F_0 \rangle}{32\pi^2 F_\pi^2} \left(\frac{\Lambda^2 - m_K^2}{\Lambda^2} \right)^2 \quad (12)$$

where the influence of the various parameters is clearly seen. For $\Lambda^2 = 3m_K^2$ and $\langle F_0 \rangle \approx F_0(m_K^2)$ we find $\lambda_0 \approx 0.017$, for $\langle F_0 \rangle \approx 3F_0(m_K^2)$ we find $\lambda_0 \approx 0.01$; we would need $\langle F_0 \rangle \approx 5F_0(m_K^2)$ to get $\lambda_0 \approx 0$. It is not easy to see how such a large value should be generated through low energy dynamics, nor is it easy to see how high energy structure should influence the curvature properties of $F_0(t)$ at low t so drastically. So we conclude that a negative value of λ_0 can plausibly only arise as by substantial influence of $SU_2 \times SU_2$ violations. We also note the gradient of $F_0(t)$ at $t = m_K^2$ in units of $[1/m_\pi^2]$; the resulting value of 0.035 for $\Lambda^2 = 3m_K^2$ and $\langle F_0 \rangle = F_0(m_K^2)$ is significantly larger than the value of 0.022 given by the linear interpolation in Eq. (4); this is due to the asymmetric positive curvature.

The results of this paper are compatible with the work of Dashen, Li, Pagels and Weinstein⁴⁾, but only in a formal manner. These authors establish a theorem on the derivative $F'_0(t)$ at $t = m_K^2 + m_\pi^2$, which in our case, with $m_\pi^2 = 0$, reads

$$m_K^2 F'_0(m_K^2) = \frac{1}{2} \left[\frac{F_K}{F_\pi} - \frac{F_\pi}{F_K} \right] + O(m_K^2) \quad (13)$$

to be valid including corrections of order $\epsilon \log \epsilon$ in an $SU_3 \times SU_3$ symmetry breaking parameter ϵ , but neglecting correction of order ϵ with $m_K^2 = O(\epsilon)$. This value for $F'_0(m_K^2)$ is reproduced by a linear extrapolation of $F_0(t)$ between $t = 0$ and the Callan-Treiman point according to Eq. (4). The correction terms to the linear extrapolation, as deduced here from Eq. (11)

$$m_K^2 F'_0(m_K^2) = (F_0(m_K^2) - 1) + \frac{\langle F_0 \rangle m_K^2}{16\pi^2 F_\pi^2} \left(\frac{\Lambda^2 - m_K^2}{\Lambda^2} \right) \quad (14)$$

are explicitly proportional to m_K^2 and therefore not explicitly included in Eq. (13), yet actually they need not be negligible as compared with the terms retained.

The results in this paper are also compatible with the bounds of $F_0(t)$ given by Bourrely⁵⁾, which when translated into λ_0 , allow for the range $0.008 \leq \lambda_0 \leq 0.020$. Interestingly enough, Bourrely excludes the linear extrapolation of Eq. (4). It is to be noted, however, that Bourrely's values are based on rather different assumptions, the most prominent of which are an unsubtracted spectral representation for the two-point function of the strangeness changing vector current divergences and the $(3, 3^*) + (3^*, 3)$ model of chiral $SU_3 \times SU_3$ symmetry breaking.

We conclude by repeating that substantial violations of $SU_2 \times SU_2$ symmetry appear necessary to suppress λ_0 to values significantly below 0.01; these effects are hard to estimate in general terms. At first sight one would expect a weakening of the present mechanism, since the scattering

threshold is removed from m_K^2 to $(m_K + m_\pi)^2$, which makes a linear approximation within $0 \leq t \leq m_K^2$ rather more plausible. But this need not be the only effect, the enhancement of the threshold by a positive scattering length of order $O(m_\pi)$ being another one, and above all we do not know in which direction the possible corrections to the Callan-Treiman relation will work. In a model due to one of us, these effects seem to be quite substantial ⁶⁾. In any case, there appears to be still some flexibility about $SU_2 \times SU_2$ violations, and the only firm conclusion of this discussion should be not to regard a rather small (positive) slope of the scalar form factor $F_0(t)$ as indicating a failure of the Callan-Treiman relation, with all the consequences such a failure would imply.

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