THE SPIN PREDICTIONS OF THE RELATIVISTIC QUARK MODEL
FOR BARYON DECUPLET PRODUCTION

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A B S T R A C T

Single quark scattering contributions are considered in the case of ground state decuplet baryon production. It is shown within the framework of an explicitly covariant approach that the spin consequences of the quark additivity assumption hold in the t channel helicity frame independently of much of the details of the model.

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1. - INTRODUCTION

The predictions of the quark model for the spin dependence of high energy scattering processes generally agree quite well with experiment showing that the quark spin structure may be an important symmetry. [For a summary of the quark model predictions, see 1),2).] These predictions follow from the assumption that the hadron scattering at high energy proceeds in the way as depicted on Fig. 1. This is the additivity assumption ³⁾⁻⁵⁾. Considering the hadrons as bound states of quarks, the additivity assumption states that during the interaction of hadrons the binding forces acting between the quarks are negligible.

The quark additivity works well in particular for the spin dependence of the decuplet production. From the quark model point of view there is a class of processes which is especially simple, namely (restricting ourselves to the case of proton target) :

$$K^+ p \rightarrow K^0 \Delta^{++} \quad (1a)$$

$$K^0 p \rightarrow K^+ \Delta^0 \quad (1b)$$

$$K^- p \rightarrow \pi^- \chi_*^+ \quad (1c)$$

$$K^- p \rightarrow \pi^0 \chi_*^0 \quad (1d)$$

$$\bar{K}^0 p \rightarrow \pi^+ \chi_*^0 \quad (1e)$$

$$\bar{K}^0 p \rightarrow \pi^0 \chi_*^+ \quad (1f)$$

$$K^- p \rightarrow \bar{K}^0 \Delta^0 \quad (1g)$$

$$\bar{K}^0 p \rightarrow K^- \Delta^{++} \quad (1h)$$

$$\pi^+ p \rightarrow K^+ \chi_*^+ \quad (1i)$$

$$\pi^- p \rightarrow K^0 \chi_*^0 \quad (1j)$$

In these cases the initial and final quark configurations are such that only a single quark process can contribute. In (1a-f) one quark from the meson must go over to the baryon and vice versa. (In the small t region of the hadron process this means a nearly backward elastic scattering of the "active" quarks.) In (1g-j) one quark-antiquark pair in the initial state must annihilate and create a new quark pair. The first two processes (1a-b) have the further merit of being exotic in the s channel, hence no s channel resonances appear and the asymptotic regime begins lower ^{*}). For definiteness in what follows we shall consider the quark backward scattering process, but this makes a difference only in $SU(3)$ indices and the spin structure is the same also for annihilation processes.

As it is known ^{1),2)} the prediction of the quark model for the processes (1a-j) is that

$$\begin{aligned} H_{\frac{1}{2}\frac{1}{2}} &= H_{-\frac{3}{2}\frac{1}{2}} = 0, \\ H_{\frac{3}{2}\frac{1}{2}} &= \sqrt{3} H_{-\frac{1}{2}\frac{1}{2}}, \end{aligned} \quad (2)$$

where $H_{\lambda_D \lambda_B}$ denotes the helicity amplitudes, λ_D and λ_B being the helicity of the decuplet particle and of the nucleon, respectively. The relations in Eq. (2) are, however, not rotation invariant, and therefore they can be true only in one frame (the so-called "additivity frame"). This arbitrariness which is due to the different possible relativistic definition of spin made always the spin predictions of the quark model somewhat suspicious. [For a recent discussion on this, see Ref. 6) where also detailed references are given to the earlier literature.] This motivates the investigation of the problem in an explicitly covariant approach where the possible effects of the model dependent assumptions are easy to study.

^{*}) In the resonance region the binding forces play clearly an important rôle also during the interaction, therefore additivity cannot be expected to work.

2. - THE SINGLE QUARK SCATTERING CONTRIBUTION

Let us begin by considering the different ingredients of the diagram in Fig. 1, which we shall consider as a Feynman diagram. The momenta of the internal quark lines depend on the relative momenta of the quarks inside the hadrons : q_M and q_L for the mesons and q_B, q respectively q_D, q for the nucleon and the decuplet particle. These are defined in terms of the quark momenta, say, p_{Mj} ($j = 1, 2$) and p_{Bj} ($j = 1, 2, 3$) as follows

$$\begin{aligned} q_M &= \frac{1}{2}(p_{M1} - p_{M2}), \\ q_B &= \frac{1}{3}(p_{B1} + p_{B2} - 2p_{B3}), \quad q = \frac{1}{2}(p_{B1} - p_{B2}), \\ p_M &= p_{M1} + p_{M2}, \quad p_B = p_{B1} + p_{B2} + p_{B3} \end{aligned} \quad (3)$$

and analogously for q_L, q_D (q is the same in D and B).

The hadron quark vertices depend on the relative momenta of quarks and, besides, on spin and $SU(3)$ indices. The meson quark vertex $p_M \rightarrow p_{M1} + p_{M2}$ is given for instance by

$$F(q_M^2) \left(\frac{\not{p}_{M1} + m_q}{2m_q} \cdot \frac{\not{p}_M + m_M}{2m_M} \right)_{a_1 b_M} \left(\frac{\not{p}_M - m_M}{2m_M} \cdot \frac{\not{p}_{M2} - m_q}{2m_q} \right)_{a_M a_2} \quad (4)$$

Here $m_M(m_q)$ is the meson (quark) mass and $a \equiv A \alpha$ in general denotes an $SU(6)$ index, A being the $SU(3)$ part and α a Dirac index. The indices a_M, b_M belong to the meson and a_1, a_2 to the quark-antiquark pair. The function F stands for the internal wave function of the meson. The relativistic $SU(6)$ symmetry reduces in the case of collinear processes to $SU(6)_w$, hence Eq. (4) could be defined also as the $SU(6)_w$ symmetric vertex. For the baryon quark vertex, we use the simplest generalization of Eq. (4), namely (in similar notations as above) :

$$\begin{aligned} F(q_V^2, q_B^2) \delta_{\{abc\}}^{\{a'b'c'\}} & \left(\frac{\not{p}_{B1} + m_q}{2m_q} \cdot \frac{\not{p}_B + m_B}{2m_B} \right)_{a_1 a'} \\ & \left(\frac{\not{p}_{B2} + m_q}{2m_q} \cdot \frac{\not{p}_B + m_B}{2m_B} \right)_{a_2 b'} \left(\frac{\not{p}_{B3} + m_q}{2m_q} \cdot \frac{\not{p}_B + m_B}{2m_B} \right)_{a_3 c'} \end{aligned} \quad (5)$$

Here

$$\delta_{\{abc\}}^{\{a'b'c'\}} = \frac{1}{3!} \sum_{\text{perm.}} \delta_{p(a)}^{a'} \delta_{p(b)}^{b'} \delta_{p(c)}^{c'} \quad (6)$$

is the symmetrizer of three indices. The relation of Eqs. (4), (5) to vertices for definite spin states (e.g., helicity states) is evident if one notices

$$\frac{\not{p}+m}{2m} = \sum_{\sigma} u(p, \sigma) \bar{u}(p, \sigma),$$

$$\frac{\not{p}-m}{2m} = \sum_{\sigma} v(p, \sigma) \bar{v}(p, \sigma), \quad (7)$$

where u and v denote the usual Dirac spinors.

The effect of the internal motion of quarks for the spin structure can be seen simply considering the projection operators appearing for quarks in the vertices (and also as the numerators of quark propagators). The quark line with momentum $\frac{1}{2}p_M - q_M$ in Fig. 1, for instance, has the projection operator

$$\frac{1/2 \not{p}_M + m_q}{2m_q} - \frac{\not{q}_M}{2m_q}. \quad (8)$$

If we choose the quark mass to be one half of the meson mass m_M , then the first part is identical to the meson projection operator and therefore has no effect (it "conserves" the spin orientation). Nevertheless, the second term, which is due to the internal motion of quarks does not conserve the spin.

Hence one can expect some spin prediction at all only in the case when the second term is negligible. It is natural, however, to characterize the relative momenta of the quarks inside the hadrons roughly by the inverse slope α^{-1} of the observed Regge trajectories (which is of order 1 GeV^2)^{*)}.

Therefore the condition for spin predictions is $m_q^2 \gg \alpha^{-1}$. In that case the whole projection operator is approximated by $\frac{1}{2}$ simply and the spins are conserved along the quark lines. This hypothesis is in contrast with the usual formulations of the quark additivity⁶⁾, where the quark mass is generally taken as a fraction of hadron masses ($\frac{1}{2}$ of mesons and $1/3$ of baryons)^{**) .}

A consequence of the large quark mass is that the very much off-mass shell quarks in Fig. 1 cannot really propagate. In the case of $m_q^2 \rightarrow \infty$ the range of propagation shrinks to zero.

*) This is definitely true in oscillator models like for instant that of Ref. 7).

**) Note that it was shown by Feynman, Kislinger and Ravndal⁷⁾ that the argument of the nucleon magnetic moments in favour of the "light" quarks is not necessarily valid in relativistic quark models. In the oscillator model of Ref. 7) the magnetic moments came out right without any assumption about the quark mass.

Besides the propagators and vertices the further ingredient in the graph in Fig. 1 is the off-mass shell quark-quark scattering amplitude. For our present purposes when the mesons are pseudoscalar and the question is the spin state of the produced decuplet particle, it is in fact enough to consider a more simple situation which is depicted on Fig. 2. As the upper part of the quark-quark scattering diagram is not interesting it is possible to integrate over the upper loop of Fig. 1 and then the off-mass shell meson quark scattering amplitude appears. We assume that this latter is given by

$$a_q + b_q \left(\frac{\not{p}_M}{m_M} + \frac{\not{p}_L}{m_L} \right) \quad (9)$$

with some invariant functions a_q, b_q *).

Writing down the amplitude corresponding to the diagram on Fig. 2 in the limit $m_q \rightarrow \infty$ the spin structure simplifies considerably. After integration over the two independent loop momenta (say, q and q_D) one obtains the following expression :

$$T = \delta_{A_L}^{A_M} \delta_{\{A_D B_D C_D\}}^{\{E_D F_D G_D\}} (C \gamma_\mu)_{\beta_D \alpha_D} \bar{u}^\mu(p_D, \sigma_D)_{\gamma_D} \delta_{\{A'_D \alpha'_D B'_D \beta'_D C'_D \gamma'_D\}}^{\{A_D \alpha_D B_D \beta_D C_D \gamma_D\}} \cdot \left(\frac{\not{p}_D + m_D}{2m_D} \cdot \frac{\not{p}_B + m_B}{2m_B} \right)_{\alpha'_D \alpha'_B} \left(\frac{\not{p}_D + m_D}{2m_D} \cdot \frac{\not{p}_B + m_B}{2m_B} \right)_{\beta'_D \beta'_B} \left\{ \frac{\not{p}_D + m_D}{2m_D} \left[A_q + \right. \right. \\ \left. \left. + B_q \left(\frac{\not{p}_M}{m_M} + \frac{\not{p}_L}{m_L} \right) \right] \frac{\not{p}_B + m_B}{2m_B} \right\}_{\gamma'_D \gamma'_B} \delta_{\{A_B \alpha_B B_B \beta_B C_B \gamma_B\}}^{\{A'_B \alpha'_B B'_B \beta'_B C'_B \gamma'_B\}} \varepsilon^{A_B B_B D_B} (\gamma_5 C)_{\beta_B \alpha_B} u(p_B, \sigma_B)_{\gamma_B} \quad (10)$$

*) It can be shown that all of our conclusions remain valid if we take a more general meson-quark amplitude containing also terms like $c_q (\not{p}_M - \not{p}_L)$ and $d_q (\frac{1}{3} \not{p}_D - \not{p}_D)$.

Here the amplitude is already projected out for the process when the initial baryon is in the octet (u is the Dirac spinor of it, C_B and D_B are its octet indices) and the final baryon is in the decuplet ($\tilde{u}^\mu$ is its Rarita-Schwinger spinor, $\{E_D F_D G_D\}$ are its $SU(3)$ indices). The $SU(3)$ indices A_M, B_M and A_L, B_L belong to the pair of pseudoscalar mesons. [For the details of the relativistic $SU(6)$ formalism we refer to 8).] The invariant functions A_q, B_q depending on $s = (p_M + p_B)^2$ and $t = (p_B - p_D)^2$ result from the quark amplitudes a_q, b_q after the loop integrations are performed. The structure of the diagram is such that the integration is at fixed t , therefore a_q and b_q are smeared out by the bound state wave functions F over a range of s (and the off-mass-shell momentum squared of the quarks).

The evaluation of the expression (10) gives for the $SU(3)$ part of the amplitude

$$T_{SU(3)} = \frac{2}{3} \delta_{A_L}^{A_M} \delta_{\{RC_B C_M\}}^{\{E_D F_D G_D\}} \varepsilon^{R B_L D_B}, \quad (11)$$

whereas the spin part turns out to be

$$T_{spin} = \tilde{u}^\mu(p_D, \sigma_D) u(p_B, \sigma_B) \varepsilon_{\mu\nu\rho\sigma} \frac{p_D^\nu}{m_D} \frac{p_B^\rho}{m_B} \left(\frac{p_M}{m_M} + \frac{p_L}{m_L} \right)^\sigma B_q. \quad (12)$$

The essential point is that the amplitude A_q does not contribute, therefore the entire amplitude T is proportional to an (unknown) complex function of s, t namely B_q . An immediate consequence of this is the prediction that the polarization in the processes (1a-j) is zero.

It is a trivial exercise to calculate from Eq. (12) the helicity amplitudes. In the Jackson frame ⁹⁾, if the momenta are chosen as in Fig. 3, the result for the helicity amplitudes $F_{\sigma_D \sigma_B}$ is :

$$\begin{aligned} F_{\frac{3}{2} \frac{1}{2}} &= \sqrt{3} F_{-\frac{1}{2} \frac{1}{2}}, \quad F_{-\frac{3}{2} \frac{1}{2}} = \sqrt{3} F_{\frac{1}{2} \frac{1}{2}}, \\ F_{\frac{3}{2} \frac{1}{2}} &= \sin(\chi_M + \chi_B) \frac{q_B q_M (m_M + m_L)}{m_B^2 m_M m_L} \sqrt{m_B^2 + q_B^2} B_q(s, t), \\ F_{-\frac{3}{2} \frac{1}{2}} &= 0. \end{aligned} \quad (13)$$

We have therefore the result that Eq. (2) holds in the t channel helicity frame. This result is in agreement with the conjecture by Białas, Kotanski and Zalewski ¹⁰⁾.

To explicitly see how the results depend on the frame, one can, for instance, calculate the helicity amplitudes in the s channel helicity frame $F'_{\sigma_D \sigma_B}$ where the z axis in Fig. 3 is chosen to be parallel to p_L . In that case the first two relations in Eq. (13) remain valid ($F'_{\frac{3}{2} \frac{1}{2}}$ remains, of course, non-zero) but instead of the last relation one has

$$F'_{-\frac{3}{2} \frac{1}{2}} = \tan \frac{\chi_B}{2} \cdot F'_{\frac{3}{2} \frac{1}{2}} \quad (14)$$

For the spin density matrix elements it follows from Eq. (13) that

$$\rho_{33} = \frac{3}{8}, \quad \rho_{31} = 0, \quad \rho_{3-1} = \frac{\sqrt{3}}{8}; \quad (15)$$

which are the well-known Sakurai-Stodolsky relations ¹¹⁾. The measurement of Neuhofer et al. ¹²⁾ in the case of the reaction (1a) indicates that Eq. (15) is in reasonable agreement with the experiment already at the relatively low energy 2.25 GeV/c. The spin density matrix elements, however, are not sensitive to the choice of the additivity frame ²⁾. Therefore, Eq. (14) can be tested only in an experiment with polarized target.

3. - CONCLUSIONS

In conclusion, we should like to emphasize the interest in measuring the spin structure of reactions (1a-j) at high energies and in as wide momentum transfer range as possible. This is because our derivation shows that the spin predictions of the quark model are to a large extent model independent. They do not depend essentially on any of the details of the model (such as the bound state wave functions or the quark scattering amplitudes) and they are not affected by relativistic corrections (recoils, spin boosts, etc.). In the case of a failure one should blame therefore the essential assumptions of the model: the assumption that the numerator of the quark propagators like in Eq. (8) can be replaced by $\frac{1}{2}$, and the idea of single quark scattering.

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FIGURE CAPTIONS

- Figure 1 Meson-baryon scattering in the quark model. Hadrons are the double lines, quarks the single ones.
- Figure 2 Decuplet production by meson-quark scattering.
- Figure 3 The momenta in the Jackson frame.

FIG. 1.

FIG. 2.

FIG. 3.