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PARTON DISTRIBUTION AND LEVEL STRUCTURE OF DUAL HADRONS

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A B S T R A C T

We study a model of hadrons where a dual parton dynamics for the parton interaction is explicitly assumed. We calculate a parton distribution function and other interesting physical quantities based on the model. Our study is strongly motivated by our interest in investigating a possible connection between the parton model and the dual resonance model of hadrons. The main emphasis will be put on the transverse momentum cut-off and scaling properties of particle distributions which are achieved as a consequence of the assumed level structure and its dual coupling scheme of hadrons. From the study of our model it is concluded that a particle distribution in an n point Veneziano amplitude necessarily obeys the transverse momentum cut-off of $\exp(-\gamma k_{\perp}^2)$ where γ must satisfy the condition $\gamma \geq 2 \ln 2$.

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1. INTRODUCTION

In this paper we shall study a model of hadrons where a dual parton dynamics for the parton interaction is explicitly assumed and try to calculate a parton distribution function and other interesting physical quantities based on the model. The study is motivated by our interest in clarifying a possible connection between the parton and dual resonance approaches, both of which are regarded as describing certain characteristic features of the hadron. The main emphasis will be put on the transverse momentum cut-off and the scaling property of particle distribution functions which are achieved in the present model as a consequence of the assumed level structure and dual coupling scheme of hadrons.

The parton model of Feynman ¹⁾ gives us an intuitive way of understanding the observed ²⁾ scaling property of various high energy scattering processes. The model is based on the physical picture in which a "physical" particle with an extended structure appears to be composed of point-like constituents ("partons") in very high energy scattering. "For example, in quantum electrodynamics the partons are bare electrons and bare photons ³⁾." In the case of hadrons, the partons could be quarks ⁴⁾, or bare hadron states themselves ⁵⁾. In the parton approach, one "tries to get results which are more general and characteristic of a wide class of theories and which can be stated in a way independent of the field theory which served as a logical crutch for their discovery" ⁶⁾. However, if one wants to construct a dynamical theory of the parton model where various physical quantities are calculable (and this must be the next step to be studied), one has to consider a more explicit theoretical framework where the parton interaction inside the hadron is known.

On the other hand, from the extensive theoretical and experimental study of hadron scattering, it seems that the hadron has a characteristic property of duality. Veneziano ⁷⁾ proposed a scattering representation where duality is nicely realized within the framework of the narrow resonance model. Through the study of factorization ⁸⁾ of the Veneziano amplitude, it was shown that the model has a highly degenerate hadron spectrum which can be systematically represented by introducing an infinite set of four-dimensional harmonic oscillators ⁹⁾. This level structure and its coupling scheme seems to be a characteristic consequence of the duality requirement that the infinite sum of direct channel poles gives rise to crossed channel poles due to its divergence as the crossed channel energy varies.

Both the parton model and the dual resonance model are supposed to represent certain, probably true, aspects of hadrons, respectively. "If so, how can we combine both approaches into one consistent picture of the hadron?"

We could be less ambitious and ask "how is duality relevant in the Feynman parton model?" or "how can we make parton interpretation and obtain its important consequences in the dual resonance model?" We believe that these questions are very important and relevant to the future study of hadron dynamics. In this paper, we shall study the model where an attempt is made to understand the common and/or complementary aspects of the two models mentioned above. The present work probably answers best the third question above and possibly the second one as well, but is admittedly not sufficient for providing a convincing answer for the first question.

There have been attempts ¹⁰⁾ of making parton interpretation of the dual resonance hadron. It seems, however, that these previous investigations are particularly unsatisfactory in at least two respects. The identification of partons is based on the infinite number of internal degrees of freedom given by an infinite set of four-dimensional harmonic oscillators with increasing frequencies. This approach suffers from a difficulty that the partons are infinitely bound and cannot be regarded as approximately free in any situation due to the persistent effect of harmonic potentials with infinitely increasing strengths corresponding to infinitely high modes. Secondly, the way of introducing the parton concept in this approach is essentially based on the classical treatment of the internal degrees of freedom which will never describe quantum aspects of partons, i.e., their emission and absorption. From our point of view these features are already in disagreement with Feynman's parton picture.

A simple remedy for these difficulties is to introduce a quantum field $\phi(y)$ and assume a proper parton dynamics for the field. Bare particles of the field are identified as partons. (Actually, to us this seems to be the only way of curing the above-mentioned difficulties.) The asymptotic field of $\phi(y)$ creates physical particles. The model to be pursued here follows this line and the assumed dual parton dynamics for the field $\phi(y)$ allows us to compute the virtual parton distribution inside the hadron, the inclusive distribution of an emitted particle (a "dressed" parton) and other interesting quantities.

Let us consider a tree graph of an n point Veneziano amplitude. By specifying a certain channel, denoted here by $(m,1)$, the tree graph can be drawn as in Fig. 1a. In the internal line an infinite tower of narrow resonances is exchanged. As a result of the dense level structure of the resonances and the scheme of its coupling to the external particles, the amplitude corresponding to the graph has a hidden symmetry which gives rise to the dual nature of the amplitude ¹¹⁾. It was pointed out ¹²⁾ some time ago that one could also visualize the scattering process represented by the tree graph Fig. 1a in terms of the conventional picture of potential scattering. This is graphically shown in Fig. 1b.

A ground state hadron (1) enters into a potential field, scatters (n-2) times making transitions from a certain excited state $\{N_i\}$ to another $\{N_{i+1}\}$, then goes out of the field as another ground state hadron (m). The n point Veneziano amplitude is nothing but the transition amplitude for this process. An unsatisfactory feature of the picture may be that the external lines (1) and (m) corresponding to the scattered hadron and the others (2),(3),..., (m-1),(m+1),..., (n) to the potential field are not treated on the same footing, although by making an appropriate adjustment of the relevant parameters one can restore the symmetry in the final answer for the amplitude ¹²⁾. Nevertheless, this intuitive picture is very suggestive if one wants to understand the hadron structure embedded in the dual resonance model.

In this paper, in some sense, we generalize the basic ingredients behind the above-mentioned picture to its extremum. Rather than considering scattering of the hadron in the external potential field, we explicitly introduce the quantum field $\phi(y)$ and regard the dual resonance states of the hadron as acting as a source of emission and absorption of particles ϕ . Therefore we have a closed system of hadrons in which the complete set of hadron states is given by the bare dual resonance states plus bare ϕ multiparticle states or, equivalently, by the renormalized physical ground state of the source plus physical ϕ multiparticle states. The excited states of the source become unstable after the interaction of ϕ emission and absorption is switched on, therefore they are not anymore linearly independent states.

To get an intuitive picture of the model, it may be helpful to recall the familiar static model ¹³⁾ of the (πN) system in which the nucleon is considered as a source of pion emission or absorption. Our model is much more extensive in its dynamical content and is formulated relativistically so that energy momentum is conserved rigorously.

We remark that it is somewhat unnatural to treat the ϕ field and the dual resonance states on a different basis from the point of view of hadron bootstrap. It would be much more appealing if the physical particle of ϕ turned out to be nothing but the renormalized ground state of the source where the source function itself was composed of the ϕ field. In this case a complete bootstrap is working. The model to be presented may not be very close to this complete solution, but seems to represent certain aspects of hadron dynamics, particularly on the topics which will be discussed in this paper.

In Section 2, we present a model for dual parton dynamics. The physical picture behind the model is stressed.

In Section 3, we calculate a parton distribution function based on the model. In particular, the transverse momentum cut-off is studied in detail.

In Section 4, the structure function of inelastic e-p scattering is calculated. It is explicitly shown that in the Bjorken limit ¹⁴⁾ the structure function is simply related to the parton distribution function obtained in Section 3.

In the final Section, a summary and discussions are given. It is concluded that the transverse momentum cut-off of the form $\exp(-\gamma k_{\perp}^2)$ with $\gamma \geq 2 \ln 2$ is a universal feature for any particle production of n point Veneziano amplitudes.

In the Appendix we give mathematical details of some formulae in the text.

2. MODEL FOR DUAL PARTON DYNAMICS

Our model is based on the crucial assumption that the dual resonance states found in the Veneziano amplitude represent compound systems of strongly bound constituents which are regarded as the effective source of emission and absorption of partons. The equation of motion for this parton dynamics is given by

$$(\square - \mu^2) \phi(y) = -g \rho(y) \quad , \quad (2.1)$$

where $\rho(y)$ is the parton source which makes transitions between various dual resonance states (N_i) of the source :

$$\rho_{N_i N_j}(y) = \langle N_i | \rho(y) | N_j \rangle \quad . \quad (2.2)$$

The field $\phi(y)$ also has a transition matrix in this space :

$$\phi_{N_i N_j}(y) = \langle N_i | \phi(y) | N_j \rangle \quad . \quad (2.3)$$

Here the parton dynamics is completely specified by the level structure and coupling scheme of the source. Therefore the results to be obtained will reflect the characteristic features of them in an essential way.

In this model a probing beam will never detect directly the source but rather interacts with a parton virtually emitted out of the source which can be regarded as a sort of infinite reservoir of partons. We could visualize our picture in another way. In the strictly narrow resonance world, the dual resonance states are the tightly bound systems of partons. In a more realistic unitarized world, the higher resonance states are actually scattering states of physical particles and unstable. This feature is incorporated by allowing virtual parton emission and absorption from the tightly bound source which results in a surrounding cloud of partons, giving rise to an unstable state of loosely bound partons for the source. The unstableness is put in here by allowing bare particles (regarded here as partons) to be produced as physical particles ¹⁵⁾. The ground state source is stable but gets renormalized, consequently being surrounded by the cloud of loosely bound partons.

The underlying picture of our virtual dissociation is illustrated in Fig. 2 for the case of the ground state hadron. Diagram i) is the familiar parton picture of the stable hadron. Diagram ii) shows a detailed dynamical mechanism of parton emission and absorption of the present model. In the ultimate stage of this dissociation, we assume we have systems of only multi-partons and an "extinct" ground state bare of partons. The systems may be regarded as compounded of Feynman partons loosely bound together. If we regard the parton dissociation in an "inclusive" way as

$$\begin{aligned} \text{"hadron"} &\rightarrow (\text{parton})_1 + (\text{parton})_2 + \dots \\ &\simeq (\text{parton})_1 + X \end{aligned} \quad ,$$

we are then led to diagram iii) where X represents scattering states of bare particles and the "extinct" ground state bare source. In diagram iv) the state X is replaced by the dual resonance states. The last step is a working hypothesis we shall make in this paper for actual calculations.

For the case of a fixed source, it is well known that the problem with Eq. (2.1) can be solved and the solution for the S matrix is given by ¹³⁾

$$S = \exp \left[i \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{\rho_A^*(\omega)}{\sqrt{2\omega}} A_{in,A}^\dagger + \frac{\rho_A(\omega)}{\sqrt{2\omega}} A_{in,A} \right\} \right] \quad , \quad (2.4)$$

where

$$\rho_{\underline{k}}(k_0) = \int d^4y \rho(y) e^{-i\underline{k}y} \quad (2.5)$$

and

$$\omega = \sqrt{\underline{k}^2 + \mu^2} \quad (2.6)$$

The S matrix in terms of the out-going field is obtained just by replacing A_{in} and $A_{in}^\dagger$ by A_{out} and $A_{out}^\dagger$ in the above expression.

In our case of interest it is more complicated. Firstly, we allow recoil of the source so that energy momentum is conserved in each process. Secondly, we allow the source to make transitions from a certain state to another. Because of these essential generalizations, the problem can only be treated in a perturbative way. However, we wish to point out that due to the very dense level structure and dual coupling scheme of the source even the results obtained in lowest order reflect non-trivial features of the assumed dynamics, as is also the case for the tree approximation of the Veneziano amplitude. Figure 2 was meant to illustrate this point intuitively. The dual resonance states are considered to reflect certain multiparticle effects of physical particles.

The transition level of the parton source $\rho(y)$ is specified by the Hamiltonian ¹⁶⁾

$$H_0 = P^2 + \sum_{n=1}^{\infty} \sum_{\mu=0}^3 n a_{n,\mu}^\dagger a_{n,\mu} \quad (2.7)$$

where P_μ is the total momentum of the source and $a_{n,\mu}$ and $a_{n,\mu}^\dagger$ describe its internal excitation. The level structure is obtained by imposing the condition

$$[H_0 - \alpha(0)] |\{N\}, P\rangle = (P^2 + \sum_n \sum_\mu n N_{n,\mu} - \alpha(0)) |\{N\}, P\rangle = 0 \quad (2.8)$$

which picks up a class of eigenvalue solutions with an eigenvalue $\alpha(0)$. For later convenience, we introduce here a centre-of-mass co-ordinate x_μ conjugate to total momentum P_μ and the internal variables $x_{n,\mu}$ and $p_{n,\mu}$ which are defined by

$$x_{n,\mu} = \frac{1}{\sqrt{2n}} (a_{n,\mu} + a_{n,\mu}^\dagger) \quad , \quad (2.9)$$

$$p_{n,\mu} = -i\sqrt{\frac{n}{2}} (a_{n,\mu} - a_{n,\mu}^\dagger) \quad .$$

Introducing a time parameter τ for the system which is conjugate to the Hamiltonian H_0 , the dynamical variables just introduced can be shown to obey, for the free source (i.e., for the source without any radiation of $\emptyset$) the following equations of motion

$$\begin{aligned} \frac{\partial^2}{\partial \tau^2} x_\mu &= 0 \quad , \\ \frac{\partial^2}{\partial \tau^2} x_{n,\mu} &= -n^2 x_{n,\mu} \quad . \end{aligned} \quad (2.10)$$

The following equal time commutation relations are assumed to hold for the system :

$$\begin{aligned} [a_{m,\mu}, a_{n,\nu}^\dagger] &= \delta_{mn} \delta_{\mu\nu} \quad , \\ [x_\mu, p_\nu] &= i\delta_{\mu\nu} \quad , \\ [x_{m,\mu}, p_{n,\nu}] &= i\delta_{mn} \delta_{\mu\nu} \quad . \end{aligned} \quad (2.11)$$

Now the source function $\rho(y)$ is assumed to be given by the expression

$$\rho(y) = \rho_+(y) + \rho_-(y) = \int d\tau \rho_+(y, \tau) + \int d\tau \rho_-(y, \tau) \quad (2.12)$$

and

$$\rho_\pm(y, \tau) = : \delta^4(y - x(\tau) - X_\pm(\tau)) : \quad , \quad (2.13)$$

where $X_{\pm}(\tau)$ is specified by

$$X_{\pm}(\tau) = \sum_{n=1}^{\infty} 2(\pm)^n x_n(\tau) \quad (2.14)$$

and the normal ordering is to be taken with respect to a_n and $a_n^{\dagger}$. The solid justification for assuming this form of $\rho(y)$ is hard to make here, but there is an obvious motivation for this expression. If one evaluates on-shell emission of particle ϕ with this vertex, one reproduces the dual resonance vertex of the Veneziano amplitude ¹⁷⁾. The form assumed ¹⁸⁾ is a simplest generalization of the on-shell form to virtual emission of ϕ . We realize that our vertex might introduce some unsatisfactory features for the virtual case which could be avoided for the on-shell case, but our point of view is that these points should be clarified with a complete unitarization programme which is beyond the scope of the present paper ¹⁹⁾.

The equation of motion, Eq. (2.1), can be solved using the Green's function technique ¹³⁾. We obtain

$$\begin{aligned} \phi(y) &= \phi_{in}(y) + g \int d^4 y' \Delta_{ret}(y-y') \rho(y') \\ &= \phi_{out}(y) + g \int d^4 y' \Delta_{adv}(y-y') \rho(y') \end{aligned} \quad (2.15)$$

where the retarded and advanced Green's functions are defined by

$$\Delta_{ret}(y) = \int \frac{d^3 k}{(2\pi)^3} e^{i k y} \frac{\sin \omega y_0}{\omega} \theta(y_0) \quad , \quad (2.16)$$

$$\Delta_{adv}(y) \equiv \Delta_{ret}(y, -y_0) \quad . \quad (2.17)$$

Introducing a function

$$\Delta(y) \equiv \Delta_{ret}(y) - \Delta_{adv}(y) = \int \frac{d^3 k}{(2\pi)^3} e^{i k y} \frac{\sin \omega y_0}{\omega} \quad , \quad (2.18)$$

we also obtain

$$\phi_{out}(y) = \phi_{in}(y) + g \int d^3y' \Delta(y-y') \rho(y') \quad (2.19)$$

Making a Fourier decomposition of $\phi(y)$ as

$$\phi(y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left[e^{i\vec{k}\vec{y}} A_{\vec{k}}(y_0) + e^{-i\vec{k}\vec{y}} A_{\vec{k}}^\dagger(y_0) \right] \quad (2.20)$$

and similarly for $\phi_{in}(y)$ and $\phi_{out}(y)$ with

$$\phi_{in}(y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left[e^{i\vec{k}\vec{y} - i\omega y_0} A_{in,\vec{k}} + e^{-i\vec{k}\vec{y} + i\omega y_0} A_{in,\vec{k}}^\dagger \right] \quad (2.21)$$

and

$$\phi_{out}(y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left[e^{i\vec{k}\vec{y} - i\omega y_0} A_{out,\vec{k}} + e^{-i\vec{k}\vec{y} + i\omega y_0} A_{out,\vec{k}}^\dagger \right] \quad (2.22)$$

we get the Fourier transformed expressions of Eqs. (2.15) and (2.19)

$$\begin{aligned} A_{\vec{k}}(y_0) &= A_{in,\vec{k}} e^{-i\omega y_0} + g \int \frac{dk_0}{2\pi} \frac{\rho_{\vec{k}}(k_0)}{\sqrt{2\omega}} \frac{e^{-ik_0 y_0}}{\omega - k_0 - i\epsilon} \\ &= A_{out,\vec{k}} e^{-i\omega y_0} + g \int \frac{dk_0}{2\pi} \frac{\rho_{\vec{k}}(k_0)}{\sqrt{2\omega}} \frac{e^{-ik_0 y_0}}{\omega - k_0 + i\epsilon} \end{aligned} \quad (2.23)$$

$$\begin{aligned} A_{\vec{k}}^\dagger(y_0) &= A_{in,\vec{k}}^\dagger e^{i\omega y_0} + g \int \frac{dk_0}{2\pi} \frac{\rho_{\vec{k}}^\dagger(k_0)}{\sqrt{2\omega}} \frac{e^{ik_0 y_0}}{\omega - k_0 + i\epsilon} \\ &= A_{out,\vec{k}}^\dagger e^{i\omega y_0} + g \int \frac{dk_0}{2\pi} \frac{\rho_{\vec{k}}^\dagger(k_0)}{\sqrt{2\omega}} \frac{e^{ik_0 y_0}}{\omega - k_0 - i\epsilon} \end{aligned} \quad (2.24)$$

and

$$\begin{aligned} A_{out, \underline{k}} &= A_{in, \underline{k}} + ig \frac{\rho_{\underline{k}}(\omega)}{\sqrt{2\omega}} , \\ A_{out, \underline{k}}^\dagger &= A_{in, \underline{k}}^\dagger - ig \frac{\rho_{\underline{k}}^\dagger(\omega)}{\sqrt{2\omega}} , \end{aligned} \quad (2.25)$$

where $\rho_{\underline{k}}(k_0)$ is defined by Eq. (2.5).

We note that $\rho_{\underline{k}}(k_0)$ is an operator which in general is not a commutable quantity. Let us define

$$\rho_{\underline{k}}(\underline{k}_0, \tau) = \int d^4y \rho(y, \tau) e^{-iky} , \quad (2.26)$$

then

$$\rho_{\underline{k}}(\underline{k}_0) = \int d\tau \rho_{\underline{k}}(\underline{k}_0, \tau) . \quad (2.27)$$

The crucial features of our dynamics are specified by $\rho_{\underline{k}}(\underline{k}_0, \tau)$ whose equation of motion is given by

$$\frac{\partial}{\partial \tau} \rho_{\underline{k}}(\underline{k}_0, \tau) = i [H, \rho_{\underline{k}}(\underline{k}_0, \tau)] , \quad (2.28)$$

where the total Hamiltonian H for the internal system of the source in interaction is given by

$$H = H_0 + H_I \quad (2.29)$$

with the interaction Hamiltonian specified by

$$H_I = -g \int d^4y \rho(y, \tau) \phi(y) . \quad (2.30)$$

H_0 is given by Eq. (2.7).

It was mentioned above that in the case of a fixed source the S matrix is given by Eq. (2.4). We can generalize the expression to the present problem where $\rho(y, \tau)$ is an operator. Let us introduce a unitary matrix in the interaction representation

$$U(\tau_2, \tau_1) = T_\tau \exp \left[i \int_{\tau_1}^{\tau_2} d\tau \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{\rho_k^{\circ \dagger}(\omega, \tau)}{\sqrt{2\omega}} A_{in, k} + \frac{\rho_k^{\circ}(\omega, \tau)}{\sqrt{2\omega}} A_{in, k}^{\dagger} \right\} \right] \quad (2.31)$$

where we have the commutativity

$$[\rho_k^{\circ}(\omega, \tau), A_{in, k}] = 0, \quad [\rho_k^{\circ}(\omega, \tau), A_{in, k}^{\dagger}] = 0, \quad \text{etc.} \quad (2.32)$$

and the time ordering with respect to τ should be taken. Furthermore, we note the relation

$$\rho_k^{\circ}(\omega, \tau) = e^{iH_0\tau} \rho_k^{\circ}(\omega, \tau=0) e^{-iH_0\tau}, \quad \text{etc.} \quad (2.33)$$

Then the S matrix in the present case takes the form

$$\begin{aligned} S &= U(\infty, -\infty) \\ &= T_\tau \exp \left[i \int d\tau \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{\rho_k^{\circ \dagger}(\omega, \tau)}{\sqrt{2\omega}} A_{in, k} + \frac{\rho_k^{\circ}(\omega, \tau)}{\sqrt{2\omega}} A_{in, k}^{\dagger} \right\} \right] \end{aligned} \quad (2.34)$$

We also note the familiar relation which connects a Heisenberg operator with the corresponding operator in the interaction representation such as

$$\rho_k^{\circ}(\omega, \tau) = U^{-1}(\tau, -\infty) \rho_k^{\circ}(\omega, \tau) U(\tau, -\infty) \quad (2.35)$$

With this whole set-up of the model, we shall calculate the distribution function of bare particles (the parton distribution function) in the next section.

3. PARTON DISTRIBUTION FUNCTION

The renormalized ground state of the source with momentum $\underline{P}$ is defined by

$$A_{in, \underline{k}} |\{0\}, \underline{P}\rangle = 0 \quad (3.1)$$

with the normalization

$$\langle\langle \{0\}, \underline{P}' | \{0\}, \underline{P} \rangle\rangle = 2P_0 (2\pi)^3 \delta^3(\underline{P} - \underline{P}') \quad (3.2)$$

where

$$P_0 = \sqrt{\underline{P}^2 + m^2} \quad (3.3)$$

with mass m ($m^2 = -\alpha(0)$) of the ground state hadron. The state is stable, so we also have

$$A_{out, \underline{k}} |\{0\}, \underline{P}\rangle = 0 \quad (3.4)$$

There is no production of physical particles out of this stable state, but it has a surrounding cloud of virtual particles $\emptyset$.

The parton distribution function $F(\underline{k})$ is defined as the probability of finding a parton with momentum $\underline{k}$ in the physical ground state of the hadron. The probability of finding a parton with a given momentum $\underline{k}$ is clearly proportional to the number $n_{\underline{k}}$ of partons with the given momentum in the virtual cloud of the physical ground state, which is given by

$$n_{\underline{k}} = \frac{1}{2P_0} \langle\langle \{0\}, \underline{P} | A_{\underline{k}}^\dagger A_{\underline{k}} | \{0\}, \underline{P} \rangle\rangle \left[(2\pi)^3 \delta^3(\underline{0}) 2\pi \delta_\tau(0) \right]^{-1} \quad (3.5)$$

where $A_{\underline{k}} \equiv A_{\underline{k}}(y_0=0)$ and $A_{\underline{k}}^\dagger \equiv A_{\underline{k}}^\dagger(y_0=0)$ and the expression is divided by the volume factor $(2\pi)^3 \delta^3(\underline{0})$ and the proper time factor $2\pi \delta_\tau(0)$. Therefore the distribution function $F(\underline{k})$ can be given by

$$F(\underline{k}) \frac{d^3 \underline{k}}{(2\pi)^3 2\omega} = \frac{n_{\underline{k}}}{N_A} \frac{d^3 \underline{k}}{(2\pi)^3} \quad (3.6)$$

where the constant N_A is the total number of partons in the virtual cloud :

$$N_A = \int \frac{d^3 \underline{k}}{(2\pi)^3} n_{\underline{k}} \quad (3.7)$$

The distribution function is normalized as

$$\int \frac{d^3 \underline{k}}{(2\pi)^3 2\omega} F(\underline{k}) = 1 \quad (3.8)$$

Substituting Eqs. (2.23) and (2.24) into Eq. (3.5) and using the defining property Eq. (3.1) of the state $|\{0\}, \underline{P}\rangle$, we obtain for the distribution function

$$F(\underline{k}) = \frac{g^2}{N_A} \frac{1}{2P_0} \langle \{0\}, \underline{P} | \int \frac{d\underline{k}_0'}{2\pi} \int d\tau' \frac{\rho_{\underline{k}}^{\dagger}(\underline{k}_0', \tau')}{\omega - \underline{k}_0' + i\varepsilon} \quad (3.9)$$

$$\cdot \int \frac{d\underline{k}_0}{2\pi} \int d\tau \frac{\rho_{\underline{k}}(\underline{k}_0, \tau)}{\omega - \underline{k}_0 - i\varepsilon} | \{0\}, \underline{P} \rangle \rangle \left[(2\pi)^3 \delta^3(\underline{0}) 2\pi \delta_{\tau}(0) \right]^{-1}$$

So far no approximation has been made and the expression (3.9) is exact.

Based on the argument presented in the last section, we shall approximate the right-hand side of Eq. (3.9) by

$$F(\underline{k}) \approx \frac{g^2}{N_A} \frac{1}{2P_0} \langle \{0\}, \underline{P} | \int \frac{d\underline{k}_0'}{2\pi} \int d\tau' \frac{\rho_{\underline{k}}^{\dagger}(\underline{k}_0', \tau')}{\omega - \underline{k}_0' + i\varepsilon} \quad (3.10)$$

$$\cdot \int \frac{d\underline{k}_0}{2\pi} \int d\tau \frac{\rho_{\underline{k}}^0(\underline{k}_0, \tau)}{\omega - \underline{k}_0 - i\varepsilon} | \{0\}, \underline{P} \rangle \rangle \left[(2\pi)^3 \delta^3(\underline{0}) 2\pi \delta_{\tau}(0) \right]^{-1}$$

This expression can be calculated exactly. Using the relation given by Eq. (2.33) and performing the τ integration, we obtain

$$F(\underline{k}) \approx \frac{g^2}{N_A} \frac{1}{2P_0} \langle \{0\}, \underline{P} | \int \frac{d\underline{k}_0'}{2\pi} \frac{\rho_{\underline{k}}^{\dagger}(\underline{k}_0', 0)}{\omega - \underline{k}_0' + i\varepsilon} 2\pi \delta(H_0 - \alpha(0))$$

$$\cdot \int \frac{d\underline{k}_0}{2\pi} \frac{\rho_{\underline{k}}^0(\underline{k}_0, 0)}{\omega - \underline{k}_0 - i\varepsilon} | \{0\}, \underline{P} \rangle \rangle \left[(2\pi)^3 \delta^3(\underline{0}) \right]^{-1} \quad (3.11)$$

The Fourier transformed form of the source $\rho_{\underline{k}}^0(k_0, 0)$ is explicitly given by

$$\rho_{\underline{k}}^0(k_0, 0) = e^{-i\underline{k}x(0)} \sigma_{\underline{k}}^0(k_0) = e^{-i\underline{k}x(0)} \left\{ \sigma_{+, \underline{k}}^0(k_0) + \sigma_{-, \underline{k}}^0(k_0) \right\} \quad (3.12)$$

and

$$\sigma_{\pm, \underline{k}}^0(k_0) = : e^{-i\underline{k}X_{\pm}(0)} : \quad (3.13)$$

In the $x(0)$ representation, where we have

$$\langle x(0) | \{0\}, \underline{P} \rangle = e^{i\underline{P}x(0)} | \{0\} \rangle \quad (3.14)$$

we can perform the $x(0)$ integration to obtain

$$F(\underline{k}) = \frac{1}{2P_0} \int d k_0 \frac{A(\underline{P} \cdot \underline{k}, k^2)}{(\omega - k_0)^2} \quad (3.15)$$

where

$$A(\underline{P} \cdot \underline{k}, k^2) = \frac{g^2}{N_A} \langle \{0\} | \sigma_{\underline{k}}^{0\dagger}(k_0) \delta((\underline{P} - \underline{k})^2 + m^2 + \sum_n n a_n^\dagger a_n) \sigma_{\underline{k}}^0(k_0) | \{0\} \rangle \quad (3.16)$$

The k_0' integration was done in the above. Substituting the explicit form of $\rho_{\underline{k}}^0(k_0)$ into the above equation, we get four contributions. Two of them are from crossed terms corresponding to $(\sigma_+^{0\dagger} \sigma_-^0)$ and $(\sigma_-^{0\dagger} \sigma_+^0)$, while the other two are from diagonal terms of $(\sigma_+^{0\dagger} \sigma_+^0)$ and $(\sigma_-^{0\dagger} \sigma_-^0)$. Thanks to the coherent expression of $\sigma_{\underline{k}}^0(k_0)$ the right-hand side of Eq. (3.16) can be further calculated to be

$$A(\underline{P} \cdot \underline{k}, k^2) = \frac{2g^2}{N_A} \sum_{N=0}^{\infty} \frac{T(N+2k^2)}{T(N+1)T(2k^2)} [(-)^{N+1}] \delta(k^2 - 2\underline{P} \cdot \underline{k} + N) \quad (3.17)$$

The first term in the bracket corresponds to the contribution from the crossed terms while the second term corresponds to that from the diagonal ones.

Let us define an on-shell vector

$$k_\omega = (\underline{k}, \omega = \sqrt{\underline{k}^2 + \mu^2}) \quad (3.18)$$

Then we can transform Eq. (3.15) into a Lorentz covariant form as

$$F(\underline{k}) = \frac{1}{2} \int d(Pk) \frac{A(Pk, k^2)}{(Pk - Pk_\omega)^2} \quad (3.19)$$

Now we can go to the infinite momentum frame by taking a limit $|P| \rightarrow \infty$ in order to get a parton distribution function in that frame as was originally considered by Feynman¹⁾. For this purpose we shall choose the momenta as

$$\begin{aligned} P &= (0, 0, P, P + \frac{m^2}{2P}) \\ k &= (\underline{k}_\perp, k_\parallel, \pm (|k_\parallel| + \frac{k_\perp^2 - v}{2|k_\parallel|})) \\ k_\omega &= (\underline{k}_\perp, k_\parallel, |k_\parallel| + \frac{k_\perp^2 + \mu^2}{2|k_\parallel|}) \end{aligned} \quad (3.20)$$

where

$$v = k^2 \quad (3.21)$$

and the $+$ ($-$) sign in k should be chosen for $k_0 > 0$ (< 0). Following Feynman, let us define

$$x = \frac{k_\parallel}{P} \quad (3.22)$$

Then we can immediately see that only when we choose a positive k_0 under the condition

$$0 < x \leq 1 \quad (3.23)$$

the distribution function under the limit $P \rightarrow \infty$ becomes finite and we obtain

$$\lim_{P \rightarrow \infty} F(\underline{k}) = F(x, k_\perp^2) \quad (3.24)$$

Otherwise we get the results

$$\lim_{P \rightarrow 0} F(\underline{k}) = O\left(\frac{1}{P^2}\right) \quad \text{for } x \notin [0, 1] \quad (3.25)$$

With the choice of the momenta specified above, we finally obtain

$$F(x, k_{\perp}^2) = \int dv \frac{x}{(v + \mu^2)^2} A(x, k_{\perp}^2, v) \quad (3.26)$$

where

$$A(x, k_{\perp}^2, v) = \frac{2g^2}{N_A} \sum_{N=0}^{\infty} \frac{\Gamma(N+2v)}{\Gamma(N+1)\Gamma(2v)} [(-)^{N+1}] \delta\left(N + \frac{k_{\perp}^2}{x} + m^2x - \frac{1-x}{x}v\right) \quad (3.27)$$

and x is limited to the region given by Eq. (3.23).

For the subsequent discussion, it is more convenient to introduce a variable V by

$$V = \frac{v}{x} - \frac{k_{\perp}^2}{x} - m^2x \quad (3.28)$$

In terms of this variable, Eqs. (3.26) and (3.27) can be written as

$$F(x, k_{\perp}^2) = \int_0^1 dV \frac{A(x, k_{\perp}^2, V)}{\left(V + \frac{k_{\perp}^2}{x} + m^2x + \frac{\mu^2}{x}\right)^2} \quad (3.29)$$

where

$$A(x, k_{\perp}^2, V) = \frac{2g^2}{N_A} \sum_{N=0}^{\infty} \frac{\Gamma(N + 2k_{\perp}^2 + 2m^2x^2 + 2xV)}{\Gamma(N+1)\Gamma(2k_{\perp}^2 + 2m^2x^2 + 2xV)} \cdot [(-)^{N+1}] \delta(N + k_{\perp}^2 + m^2x^2 - (1-x)V) \quad (3.30)$$

The integration region of V conveniently chosen above is sufficient since it covers the whole non-vanishing region of $A(x, k_{\perp}^2, V)$.

In order to make the summation in Eq. (3.30), we first note the equality

$$\delta(N + k_{\perp}^2 + m^2 x^2 - (1-x)V) = \frac{1}{\pi} \operatorname{Im}_{(1-x)+i\epsilon} \int_0^1 dz z^{N + k_{\perp}^2 + m^2 x^2 - (1-x)V - 1} \quad (3.31)$$

which holds for positive V . In the above the notation $\operatorname{Im}_{(1-x)+i\epsilon}$ was introduced to specify that the imaginary part should be taken with respect to the variable $(1-x)$. Substituting this relation into Eq. (3.30) and performing the summation, we obtain the following expression

$$A(x, k_{\perp}^2, V) = \frac{1}{\pi} \operatorname{Im}_{(1-x)+i\epsilon} B(x, k_{\perp}^2, V) \quad (3.32)$$

where $B(x, k_{\perp}^2, V)$ is defined by

$$B(x, k_{\perp}^2, V) = \frac{2g^2}{N_A} \int_0^1 dz z^{k_{\perp}^2 + m^2 x^2 - (1-x)V - 1} \cdot \left[(1+z)^{-2k_{\perp}^2 - 2m^2 x^2 - 2xV} + (1-z)^{-2k_{\perp}^2 - 2m^2 x^2 - 2xV} \right] \quad (3.33)$$

Let us first consider the contribution corresponding to the first term in the bracket, which we denote by $B_1(x, k_{\perp}^2, V)$. Making a change of variable

$$z = e^{-y} \quad (3.34)$$

we easily obtain

$$B_1(x, k_{\perp}^2, V) = \frac{2g^2}{N_A} \int_0^A dy (e^{y/2} + e^{-y/2})^{-2(k_{\perp}^2 + m^2 x^2)} e^{-h_1(x, y)V} \quad (3.35)$$

where the function $h_1(x, y)$ is defined by

$$h_1(x, y) = (x-1)y + 2x \ln(1 + e^{-y}) \quad (3.36)$$

Let us now consider a function $G_1(x, k_{\perp}^2)$ given by

$$G_1(x, k_{\perp}^2) = \int_0^A dV \frac{B_1(x, k_{\perp}^2, V)}{\left(V + \frac{k_{\perp}^2}{x} + m^2 x + \frac{\mu^2}{x}\right)^2} \quad (3.37)$$

Using the following formula for the V integration

$$\int_0^A dx \frac{e^{-bx}}{(x+a)^2} = e^{ab} \int_0^A dt \frac{t e^{-a(b+t)}}{b+t}, \quad (3.38)$$

we arrive at the expression

$$G_1(x, k_\perp^2) = \frac{2g^2}{N_A} \int_0^A dy (e^{y/2} + e^{-y/2})^{-2(k_\perp^2 + m^2 x^2)} e^{D h_1(x, y)} \cdot \int_0^A dt \frac{t e^{-D(h_1(x, y) + t)}}{t + h_1(x, y)}, \quad (3.39)$$

where

$$D \equiv \frac{k_\perp^2}{x} + m^2 x + \frac{\mu^2}{x} \quad (3.40)$$

In performing the V integration we have kept x in the unphysical region, i.e., bigger than one, so that $h_1(x, y) > 0$. By making analytic continuation into the physical region $0 \leq x \leq 1$, we see explicitly from Eq. (3.39) that $G_1(x, k_\perp^2)$ starts to have an imaginary contribution due to the vanishing of the denominator under the t integration. Using the defining relation

$$F_1(x, k_\perp^2) = \frac{1}{\pi} \text{Im}_{(1-x)+i\epsilon} G_1(x, k_\perp^2) \quad (3.41)$$

and performing the t integration, we get

$$F_1(x, k_\perp^2) = \frac{2g^2}{N_A} \int_0^A dy (e^{y/2} + e^{-y/2})^{-2(k_\perp^2 + m^2 x^2)} e^{D h_1(x, y)} \cdot [-h_1(x, y)] \theta[-h_1(x, y)]. \quad (3.42)$$

It is easy to show that for $0 \leq x \leq 1$, $h_1(x, y)$ is a monotonically decreasing function in y which crosses zero at $y = y_1(x)$ where $y_1(x)$ is given by the equation

$$h_1(x, y_1(x)) = (x-1) y_1(x) + 2x \ln(1 + e^{-y_1(x)}) \equiv 0, \quad (3.43)$$

and

$$y_1(x) > 0 \quad \text{for } 0 < x < 1 \quad (3.44)$$

Therefore, substituting the explicit form of D into Eq. (3.44) we finally get the answer for $F_1(x, k_\perp^2)$:

$$F_1(x, k_\perp^2) = \frac{2g^2}{N_A} \int_{y_1(x)}^1 dy \, e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} h_1(x, y)} [-h_1(x, y)] \quad (3.45)$$

The other contribution corresponding to the second term in the bracket of Eq. (3.33) can be evaluated similarly. But it must be treated more carefully. The form given in Eq. (3.33) contains additional unwanted singular contributions due to the divergence of the integral at $z=1$ which must be excluded appropriately in the correct answer. The details will be given in the Appendix. Here we only present the result :

$$F_2(x, k_\perp^2) = \frac{2g^2}{N_A} \int_{y_2(x)}^1 dy \, e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} [-\bar{h}_2(x, y)] \quad (3.46)$$

where $0 < x < 1$ and

$$\bar{h}_2(x, y) = (x-1)y + 2x \ln(1 - e^{-y}) \quad (3.47)$$

$y_2(x)$ is the solution of the equation

$$h_2(x, y) \equiv (x-1)y - 2x \ln(1 - e^{-y}) = 0 \quad (3.48)$$

with respect to y and for a fixed x with $0 < x < 1$. We have the identity

$$h_2(x, y_2(x)) = (x-1)y_2(x) - 2x \ln(1 - e^{-y_2(x)}) = 0 \quad (3.49)$$

Note that

$$y_2(x) > 0 \quad \text{for } 0 < x < 1 \quad (3.50)$$

The final answer of the total contribution for $F(x, k_\perp^2)$ is obtained as

$$F(x, k_{\perp}^2) = F_1(x, k_{\perp}^2) + F_2(x, k_{\perp}^2) \quad (3.51)$$

A. Transverse Momentum Cut-off

One of the characteristic properties of our $F(x, k_{\perp}^2)$ is its transverse momentum cut-off. From the integral representations Eqs. (3.45) and (3.46), we immediately see that $F_1(x, k_{\perp}^2)$ and $F_2(x, k_{\perp}^2)$ are respectively bounded by

$$F_1(x, k_{\perp}^2) \leq g_1(x) e^{-\frac{y_1(x)}{x} k_{\perp}^2} \quad (3.52)$$

$$F_2(x, k_{\perp}^2) \leq g_2(x) e^{-\frac{y_2(x)}{x} k_{\perp}^2}$$

where $g_1(x)$ and $g_2(x)$ are functions of x only. Furthermore, the functions in the exponents are bounded from below for $0 \leq x \leq 1$:

$$\frac{y_1(x)}{x} \geq 2 \ln 2 \quad (3.53)$$

$$\frac{y_2(x)}{x} \geq 3 \ln 3 - 2 \ln 2$$

These results are proved in the Appendix. Noting that $2 \ln 2 < (3 \ln 3 - 2 \ln 2)$, we can therefore conclude that

$$F(x, k_{\perp}^2) \leq g(x) e^{-(2 \ln 2) k_{\perp}^2} \quad (3.54)$$

where $g(x) = g_1(x) + g_2(x)$. It should be noted that this bound is correct for any finite value of $k_{\perp}^2$. (The limit $k_{\perp}^2 \rightarrow \infty$ need not be taken.)

We would like to point out that this remarkable result of transverse momentum cut-off is a consequence of the conspiring effect of the assumed very dense structure of transition levels of the source and their dual coupling scheme. In fact, we observe that a contribution from each level does not share the property at all. The transverse momentum cut-off manifests itself only when the whole contribution from each level is summed up.

B. Longitudinal Distribution

The distribution function in the longitudinal direction which will be denoted here by $f(x)$ can be obtained by integrating $F(x, k_{\perp}^2)$ over the transverse momentum $k_{\perp}$. The phase space factor is given by

$$\frac{d^3k}{(2\pi)^3 2\omega} \xrightarrow{P \rightarrow v} \frac{1}{16\pi^2} \frac{dx}{x} dk_{\perp}^2 \quad (3.55)$$

Let us define

$$\begin{aligned} F_1(x) &= \frac{1}{16\pi^2} \int_0^{\infty} dk_{\perp}^2 F_1(x, k_{\perp}^2) \\ &= \frac{g^2}{8\pi^2 N_A} \int_{y_1(x)}^{\infty} dy \left(\frac{x}{y} \right) e^{-m^2 xy} e^{\frac{\mu^2}{x} h_1(x, y)} [-h_1(x, y)] \end{aligned} \quad (3.56)$$

$$\begin{aligned} F_2(x) &= \frac{1}{16\pi^2} \int_0^{\infty} dk_{\perp}^2 F_2(x, k_{\perp}^2) \\ &= \frac{g^2}{8\pi^2 N_A} \int_{y_2(x)}^{\infty} dy \left(\frac{x}{y} \right) e^{-m^2 xy} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} [-\bar{h}_2(x, y)] \end{aligned} \quad (3.57)$$

Then the longitudinal distribution function $f(x)$ is given by

$$f(x) dx = F(x) \frac{dx}{x} \equiv [F_1(x) + F_2(x)] \frac{dx}{x} \quad (3.58)$$

From the expressions, Eqs. (3.56) and (3.57), we can study the limiting behaviour of $f(x)$ at the end points $x=0$ and $x=1$. The exact evaluation is difficult to make, but we can investigate the limiting behaviour up to log factors. The results are

$$F_{1,2}(x) \underset{x \rightarrow 1}{\sim} O((1-x)^{1+m^2}) \quad (3.59)$$

whereas

$$\begin{aligned} F_1(x) &\underset{x \rightarrow 0}{\sim} O(x^2) \\ F_2(x) &\underset{x \rightarrow 0}{\sim} O(x^2) + O(x^{2+2\mu^2}) \end{aligned} \quad (3.60)$$

Therefore we see that $f(x)$ vanishes at $x=1$ as it should from the threshold condition when the distribution function was applied to inelastic $e-p$ scattering (see next section). We also notice that in Eq. (3.60) the contribution of $O(x^2)$ corresponds to a fixed singularity in the angular momentum plane and that of $O(x^{2+2\mu^2})$ to the Regge pole exchange of $O(x^{1-\beta(0)})$ with $\beta(t) = t-1-2\mu^2$ [see the footnote in Ref. 17].

The above results are consistent with the normalization condition Eq. (3.8) and apparently suggest that the normalization constant N_A is finite, i.e., the total number N_A of partons in the virtual cloud is finite in our model. In connection with this observation, as the limiting behaviour Eq. (3.60) shows, the present calculation in lowest order of approximation does not generate Pomeron contribution yet which is supposed to be represented by the exchange of an infinite number of "wee" partons ¹⁾ in the parton model.

4. INELASTIC $e-p$ SCATTERING

It is known from the work of Bjorken and Paschos ⁴⁾ that the parton distribution function can be related to the structure function of deep inelastic $e-p$ scattering in the Bjorken limit ¹⁴⁾. In this section we explicitly show that this is indeed the case in the present model.

To simplify the argument, we assume that the field $\phi(y)$ couples to a scalar photon field as

$$e J(y) A(y) \quad (4.1)$$

where the source current $J(y)$ is given by

$$J(y) = : \phi(y) \phi(y) : \quad (4.2)$$

Then the structure function is defined by

$$W(P, q, q^2) = \frac{1}{4\pi} \int d^4y e^{-iqy} \ll \{0\}, P | [J(y), J(0)] | \{0\}, P \gg [2\pi \delta_\tau(0)]^{-1}. \quad (4.3)$$

Now we calculate the commutator in the above expression in lowest order in g . Then we obtain for the connected part of the commutator

$$[J(y), J(0)]_C \cong 2i\Delta(y) : \phi(y)\phi(0) : \quad (4.4)$$

where

$$\Delta(y) = -\frac{i}{(2\pi)^3} \int d^4k e^{ik \cdot y} \varepsilon(k_0) \delta(k^2 + \mu^2) \quad (4.5)$$

Substituting these into Eq. (4.3) and performing the y integration, we get

$$W(P, q, q^2) = \int d^4k \varepsilon(q_0 + k_0) \delta((q+k)^2 + \mu^2) \cdot \langle\langle \{0\}, \underline{P} | : \phi(-k) \phi(0) : | \{0\}, \underline{P} \rangle\rangle [2\pi \delta_\tau(0)]^{-1} \quad (4.6)$$

where the matrix element in the integrand is defined by

$$\langle\langle \{0\}, \underline{P} | : \phi(y) \phi(0) : | \{0\}, \underline{P} \rangle\rangle = \int d^4k e^{-ik \cdot y} \langle\langle \{0\}, \underline{P} | : \phi(-k) \phi(0) : | \{0\}, \underline{P} \rangle\rangle \quad (4.7)$$

and the explicit expression is given by

$$\langle\langle \{0\}, \underline{P} | : \phi(-k) \phi(0) : | \{0\}, \underline{P} \rangle\rangle = 2\pi \delta_\tau(0)$$

$$\frac{g^2}{(2\pi)^3 (2\omega)^2} \langle\{0\} | \sigma_{\underline{k}}^0(k_0) \delta((P-k)^2 + m^2 + \sum_n n a_n^\dagger a_n) \quad (4.8)$$

$$\sigma_{\underline{k}}^0(k_0) | \{0\} \rangle \left[\frac{1}{(\omega - k_0)^2 + \varepsilon^2} + \frac{1}{(\omega - k_0 - i\varepsilon)(\omega + k_0 - i\varepsilon)} \right] + (k_0 \rightarrow -k_0)^* .$$

In order to obtain the parametrization of momenta in the infinite momentum limit, let us choose the centre-of-mass frame of the (e-p) system where the electron and proton ²⁰⁾ carry the centre-of-mass momenta $-\underline{P}$ and $\underline{P}$ respectively. Then the four momentum q_μ of the exchanged photon is specified as ⁵⁾

$$q = \left(q_\perp, q_3 = -\frac{2m\nu + q^2}{4P}, q_0 = \frac{2m\nu - q^2}{4P} \right) \quad (4.9)$$

and

$$q_{\perp}^2 = q^2 \left(1 - \frac{m\nu}{p^2} \right) \quad (4.10)$$

where

$$\nu \equiv - \frac{q \cdot P}{m} \quad (4.11)$$

The target proton momentum and the parton momentum inside the proton are defined in a similar way to those given by Eq. (3.20). Then the infinite momentum frame is defined by taking the limit $|\vec{P}| \rightarrow \infty$. In this limit only the first and the third term in Eq. (4.8) survive, while the other two give a vanishing contribution of order $(1/P^2)$. Note that the appropriate expressions for the parton momenta k_{μ} in the first and the third terms are

$$k = \left(\underline{k}_{\perp}, xP, xP + \frac{k_{\perp}^2 - \nu}{2xP} \right) \quad (4.12)$$

and

$$k = \left(\underline{k}_{\perp}, -xP, -xP - \frac{k_{\perp}^2 - \nu}{2xP} \right) \quad (4.13)$$

respectively, under the condition $0 < x \leq 1$. Thus we finally obtain the expression

$$W(Pq, q^2) = N_A \int \frac{d^2 \underline{k}_{\perp}}{16\pi^2} \int \frac{dx}{x} \int d\nu \frac{A(x, k_{\perp}^2, \nu)}{(\nu + \mu^2)^2} \quad (4.14)$$

$$\cdot \left[\delta(q^2 - 2m\nu x - \underline{k}_{\perp} \underline{q}_{\perp} + \nu + \mu^2) - \delta(q^2 + 2m\nu x + \underline{k}_{\perp} \underline{q}_{\perp} + \nu + \mu^2) \right]$$

The Bjorken limit of the above expression can be obtained by making the following expansion under the integration :

$$\delta(q^2 \pm 2m\nu \pm \underline{k}_{\perp} \underline{q}_{\perp} + \nu + \mu^2) = \frac{1}{q^2} \delta\left(1 \pm \frac{2m\nu}{q^2} x\right) + \quad (4.15)$$

$$\frac{\nu + \mu^2 \pm \underline{k}_{\perp} \underline{q}_{\perp}}{q^4} \delta\left(1 \pm \frac{2m\nu}{q^2} x\right) + \frac{(\nu + \mu^2 \pm \underline{k}_{\perp} \underline{q}_{\perp})^2}{2q^6} \delta\left(1 \pm \frac{2m\nu}{q^2} x\right) + \dots$$

This expansion is well defined when substituted into the above expression since the v integrations in each term are convergent ²¹⁾. Therefore we obtain an asymptotic expansion of $W(qP, q^2)$ for $q^2 \rightarrow \infty$ and fixed $2m\nu/q^2$. The leading contribution is given by

$$\begin{aligned}
 \lim_{\substack{q^2 \rightarrow \infty, \\ \frac{q^2}{2m\nu} \text{ fixed}}} W(Pq, q^2) &= \frac{N_A}{q^2} \int \frac{dk_{\perp}^2}{16\pi^2} \int \frac{dx}{x} \int dv \frac{A(x, k_{\perp}^2, v)}{(v + \mu^2)^2} \\
 &\quad \cdot \left[\delta\left(1 - \frac{2m\nu}{q^2} x\right) - \delta\left(1 + \frac{2m\nu}{q^2} x\right) \right] \\
 &= \frac{N_A}{q^2} \int \frac{dk_{\perp}^2}{16\pi^2} \int \frac{dx}{x} F(x, k_{\perp}^2) \left[\delta\left(1 - \frac{2m\nu}{q^2} x\right) - \delta\left(1 + \frac{2m\nu}{q^2} x\right) \right] \\
 &= N_A \frac{1}{q^2} \frac{1}{x} F(x) = N_A \frac{1}{q^2} f(x)
 \end{aligned} \tag{4.16}$$

where

$$x = \frac{q^2}{2m\nu} \tag{4.17}$$

This formula explicitly shows that in the Bjorken limit each parton contributes incoherently to the structure function $W(qP, q^2)$; the contribution from a single parton is $f(x)/q^2$, which is multiplied by the number N_A of virtual partons in the cloud.

In a more realistic case of a vector photon coupling, we expect to obtain the following result for the structure function $\nu W_2(qP, q^2)$:

$$\lim_{\substack{q^2 \rightarrow \infty, \\ \frac{q^2}{2m\nu} \text{ fixed}}} \nu W_2(Pq, q^2) = N_A F(x) = N_A x f(x) \tag{4.18}$$

This result is simply based on the following power counting; from the term $\nu(1/q^2)$ we get $1/x$, while the spin factor for double helicity flip contributes a factor x^2 . Therefore the net result is a multiplication factor of $(1/x)x^2 = x$. As is expected, we obtain the Bjorken scaling in this case.

The result Eq. (4.18) can also be obtained by simply applying the argument of Bjorken and Paschos ⁴⁾ to our parton distribution function.

5. SUMMARY AND DISCUSSION

Based on our parton interpretation of narrow resonance levels of dual hadrons, we have studied a model where a dual parton dynamics is explicitly assumed. We have calculated, in particular, the parton distribution function and have shown that it has the remarkable transverse momentum cut-off as a consequence of the assumed dense level structure and its dual coupling scheme of hadrons.

Intuitively the experimentally well established transverse momentum cut-off may be regarded as a reflection of some finite extended size of hadrons. However, our point of view here is that we rather associate the transverse momentum cut-off with a conspiring effect of the very dense quantum states of hadrons which may be saturated at least substantially in terms of the dual resonance states. As we have seen in the present model, each transition into a certain hadron state does not show any cut-off, but only when all the allowed transitions are summed up, does the transverse momentum cut-off start to manifest itself.

Our formalism can be applied to a large class of reactions. An $(n+2)$ point Veneziano amplitude can be obtained by calculating the matrix element

$$\langle\langle \{0\}, P_{n+2}; k_{n+1}, k_n, \dots, k_{m+1}, \text{out} \mid \{0\}, P_{m+1}; k_m, k_{m-1}, \dots, k_2, \text{in} \rangle\rangle \quad (5.1)$$

in lowest order in g , i.e., in the tree approximation. (In order to reproduce a completely symmetric amplitude, trajectory intercepts in each channel must be appropriately adjusted.) We can also write down the formula for inclusive momentum spectrum of the reaction

$$a + b \rightarrow c + X$$

It is given by

$$\frac{\omega_c}{\sigma_{\text{Tot}}} \frac{d\sigma}{d^3k_c} \sim \frac{\omega_c}{(2\pi)^3} \langle\langle \{0\}, P_a; k_b, \text{in} \mid A_{\text{out}, k_c}^\dagger A_{\text{out}, k_c} \mid \{0\}, P_a; k_b, \text{in} \rangle\rangle \quad (5.2)$$

where an appropriate normalization must be taken into account. The state $\langle\langle \{0\}, P_a; k_b, \text{in} \rangle\rangle$ represents the physical incoming scattering state of particle a and particle b . The right-hand side of the above equation is proportional to the number of outgoing physical particles with the given momentum k_c which are emitted out of the incoming state of particle a and particle b . By integrating over the momentum k_c of particle c , we obtain

$$\begin{aligned} \langle n_c \rangle &= \int \frac{d^3 \underline{k}_c}{\omega_c} \frac{\omega_c}{\sigma_{tot}} \frac{d\sigma}{d^3 \underline{k}_c} \\ &\sim \langle \langle \{0\}, \underline{p}_a; \underline{k}_b, in \mid \int \frac{d^3 \underline{k}_c}{(2\pi)^3} A_{out, \underline{k}_c}^\dagger A_{out, \underline{k}_c} \mid \{0\}, \underline{p}_a; \underline{k}_b, in \rangle \rangle^{(5.3)} \end{aligned}$$

The last expression apparently represents the average total number of particle c emitted out of the state $\mid \{0\}, \underline{p}_a; \underline{k}_b, in \rangle$. The calculation of the connected part of the above matrix element Eq. (5.2) in lowest order in g is within our capability. The lowest order contribution is essentially the tree approximation of the quantity where the X states are replaced by the bare dual resonance states $\{N_{n,\mu}\}$. In this approximation the contribution essentially reduces to the problem studied by De Tar, Kang, Tan and Weis²²⁾ and Virasoro²³⁾ where they considered the above inclusive reaction starting from six-point Veneziano amplitudes and first pointed out that one gets a transverse momentum cut-off for the momentum spectrum of emitted particle c .

From our study of the assumed parton dynamics which can represent an n point Veneziano amplitude as a special case, we know that the parton source $\rho(y)$ has a virtual cloud of bare particles whose momentum spectrum is bounded in its transverse direction with respect to the scattering direction we are interested in. Therefore, we naturally expect that any number of outgoing particles emitted out of this virtual cloud will reflect the characteristic momentum spectrum of the cloud around the source, hence they will show a similar transverse momentum cut-off of $\exp(-\gamma k_\perp^2)$. Whether the decreasing rate in the cut-off represented by γ is universal or not has to be studied in detail. One expects that as a consequence of possible particle correlations in higher order processes, the value of γ will change in general. However, the natural guess will be that the value does not become smaller than the minimum value $(2\ln 2)$ we have obtained for the virtual cloud in Section 3.

This is due to the observation that virtual particles with larger $k_\perp$ are trapped more inside the source cloud, and are therefore emitted as physical particles with greater difficulty, whereas the ones with smaller $k_\perp$ are emitted more easily because they are populated more densely around the edge of the virtual cloud. Therefore the $k_\perp$ distribution of emitted physical particles will be more sharply peaked around $k_\perp = 0$ than that of the virtual cloud. Thus we conclude that a particle distribution in an n point Veneziano amplitude will necessarily obey the transverse momentum cut-off of $\exp(-\gamma k_\perp^2)$ with $\gamma \geq 2\ln 2$ ²⁴⁾, since the n point amplitude involves only physically emitted particles.

An obvious application of the present model may be the evaluation of form factors. To derive physical implications from this application in connection with the results obtained in this paper, one needs a deeper (technically as well as physically) understanding of the explicit representations of various quantities obtained above. We hope to discuss it in a later publication.

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A P P E N D I X

A. In this Appendix we show the steps of getting the result Eq. (3.46) in the text.

Let us consider a quantity

$$B_2(x, k_\perp^2, V) = \frac{2g^2}{N_A} \int_0^1 dz z^{k_\perp^2 + m^2 x - (1-x)V - 1} (1-z)^{-2k_\perp^2 - 2m^2 x^2 - 2xV} \quad (A.1)$$

For $V > 0$, $B_2(x, k_\perp^2, V)$ generates two kinds of singularities in x : one of them is due to the divergence of the integral at $z=0$, while the other to its divergence at $z=1$. We are interested in the first singularity. Making a change of variable

$$z = e^{-y} \quad (A.2)$$

we obtain

$$B_2(x, k_\perp^2, V) = \frac{2g^2}{N_A} \int_0^{\infty} dy e^{-y(k_\perp^2 + m^2 x^2)} (1-e^{-y})^{-2(k_\perp^2 + m^2 x^2)} e^{-\bar{h}_2(x, y)V} \quad (A.3)$$

where $\bar{h}_2(x, y)$ is defined by

$$\bar{h}_2(x, y) = (x-1)y + 2x \ln(1-e^{-y}) \quad (A.4)$$

Now define a function

$$G_2(x, k_\perp^2) = \int_0^{\infty} dV \frac{B_2(x, k_\perp^2, V)}{(V + \frac{k_\perp^2}{x} + m^2 x + \frac{\mu^2}{x})^2} \quad (A.5)$$

then using the formula Eq. (3.38) in the text for the V integration, we obtain

$$G_2(x, k_\perp^2) = \frac{2g^2}{N_A} \int_0^{\infty} dy e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} \cdot \int_0^{\infty} dt \frac{t e^{-D(\bar{h}_2(x, y) + t)}}{t + \bar{h}_2(x, y)}, \quad (A.6)$$

where D is given by Eq. (3.40). In performing the V integration $\bar{h}_2(x, y)$ was kept positive by appropriately choosing the values of x and y . From the expression Eq. (A.6) we see that $G_2(x, k_\perp^2)$ has two branch points at $x=1$ and $x=0$. The corresponding cuts are developed along $x \in (-\infty, 1]$ and $x \in [0, \infty)$. In the region $x \in [0, 1]$ in which we are interested, there is an overlap of the two cuts. The total discontinuity in this region is easily obtained as

$$\begin{aligned} \tilde{F}_2(x, k_\perp^2) &= \frac{1}{\pi} \text{Im}_{0 < x < 1} G_2(x, k_\perp^2) \\ &= \frac{2g^2}{N_A} \int_0^{\infty} dy e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} [-\bar{h}_2(x, y)] \int_0^{\infty} dt \delta(t + \bar{h}_2(x, y)) \quad (A.7) \\ &= \frac{2g^2}{N_A} \int_0^{\infty} dy e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} [-\bar{h}_2(x, y)] \theta[-\bar{h}_2(x, y)]. \end{aligned}$$

In order to separate the total discontinuity into two parts, the one due to the cut along $x \in (-\infty, 1]$ and the other to that along $x \in [0, \infty)$, we introduce the following identity in the above equation :

$$\begin{aligned} \int_0^{\infty} dy \theta[-\bar{h}_2(x, y)] \dots &= \int_0^{\infty} dy \theta[(1-x)y + 2x \ln(1-e^{-y})] \dots \\ &+ \int_0^{\infty} dy \theta[-(1-x)y - 2x \ln(1-e^{-y})] \dots, \quad 0 < x < 1. \end{aligned} \quad (A.8)$$

First let us note $\bar{h}_2(x, y)$ is negative for $0 < x < 1$ and $y > 0$, and reaches its maximum value $\bar{h}_2(x, \bar{y}_2(x)) < 0$ at $y = \bar{y}_2(x) = \ln[(1+x)/(1-x)]$. On the other hand, the function $h_2(x, y)$ which appears in the θ functions of Eq. (A.8) and is defined by

$$h_2(x, y) = (x-1)y - 2x \ln(1 - e^{-y}) \quad (\text{A.9})$$

is always negative (positive) for $y > 0$ and $x < 0$ ($x > 1$), whereas for $0 < x < 1$ $h_2(x, y)$ is a monotonically decreasing function of y in the region $y > 0$ and crosses zero at $y = y_2(x)$ which is defined by

$$h_2(x, y_2(x)) = (x-1)y_2(x) - 2x \ln(1 - e^{-y_2(x)}) = 0 \quad (\text{A.10})$$

and

$$y_2(x) > 0 \quad \text{for} \quad 0 < x < 1. \quad (\text{A.11})$$

The expression Eq. (A.7) for the total discontinuity is well defined only for $0 < x < 1$. The form we obtain by substituting Eq. (A.8) into Eq. (A.7) allows x to vary in the entire region $(-\infty, \infty)$. The first term is non-vanishing only for $x < 1$ and is identically zero for $x \geq 1$. The second term is non-zero only for $x > 0$ and vanishes identically for $x \leq 0$. Furthermore, in the region $0 < x < 1$, the sum of the two terms reproduces the entire contribution given by Eq. (A.7). Therefore we see that the complete separation is done; the first term given by the expression

$$F_2(x, k_\perp^2) = \frac{2g^2}{N_A} \int_0^\infty dy e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{k_\perp^2}{x} \bar{h}_2(x, y)} \cdot [-\bar{h}_2(x, y)] \theta[-h_2(x, y)] \quad (\text{A.12})$$

is precisely the discontinuity across the cut extending along $x \in (-\infty, 1]$ and the second term obtained by replacing the θ function in Eq. (A.12) by $\theta[h_2(x, y)]$ is the discontinuity across the cut along $x \in [0, \infty)$. The former contribution is due to the branch point at $x=1$, whereas the latter to the unwanted singularity at $x=0$. Each discontinuity formula satisfies the required spectral condition that it must vanish identically outside the corresponding cut region of x . From Eq. (A.12) we finally obtain the answer for $F_2(x, k_\perp^2)$

$$F_2(x, k_\perp^2) = \frac{2g^2}{N_A} \int_{y_2(x)}^1 dy e^{-\frac{y}{x}(k_\perp^2 + m^2 x^2)} e^{\frac{\mu^2}{x} \bar{h}_2(x, y)} [-\bar{h}_2(x, y)], \quad (A.13)$$

where $0 < x < 1$.

B. Here we shall prove the results given by Eqs. (3.53). Let us define a function of x as

$$z_1(x) = \frac{y_1(x)}{x} \quad (B.1)$$

Then, from Eq. (3.43) we obtain the defining equation of $z_1(x)$ as

$$(x-1) z_1(x) + 2 \ln(1 + e^{-x z_1(x)}) = 0 \quad (B.2)$$

In order to know the minimum value of $z_1(x)$, we differentiate Eq. (B.2) to obtain

$$z_1'(x) = z_1(x) \frac{1 - e^{-x z_1(x)}}{(1-x) + (1+x) e^{-x z_1(x)}} \quad (B.3)$$

From Eq. (B.1) and Eq. (3.44) [which can also be proved in exactly the same way as we do here, starting with Eq. (3.43)], we know that

$$z_1(x) > 0 \quad \text{for } 0 < x < 1 \quad (B.4)$$

Therefore we have, from Eq. (B.3), that

$$z_1'(x) > 0 \quad (B.5)$$

Thus we conclude that $z_1(x)$ is a monotonically increasing function for $0 < x < 1$ and its minimum value is reached at $x=0$ and is given by

$$z_1(0) = 2 \ln 2 \quad (B.6)$$

from Eq. (B.2).

Similarly, let us define a function

$$z_2(x) = \frac{y_2(x)}{x} \quad (B.7)$$

Then the defining equation of $z_2(x)$ is obtained from Eq. (3.49) as

$$(1-x) z_2(x) + 2 \ln(1 - e^{-x z_2(x)}) = 0. \quad (B.8)$$

Differentiating both sides with respect to x , we get

$$z_2'(x) = z_2(x) \frac{1 - 3 e^{-x z_2(x)}}{(1-x)(1 - e^{-x z_2(x)}) + 2x e^{-x z_2(x)}} \quad (B.9)$$

From Eq. (3.50) we have

$$z_2(x) > 0 \quad \text{for} \quad 0 < x < 1. \quad (B.10)$$

Therefore we see from Eq. (B.9) that $z_2'(x)$ vanishes at the value of x which satisfies the condition

$$x z_2(x) = \ln 3 \quad (B.11)$$

It is straightforward to solve the equations (B.8) and (B.11) and the answer is given by

$$x = x_0 = \frac{\ln 3}{3 \ln 3 - 2 \ln 2} \quad (B.12)$$

and

$$z_2(x_0) = 3 \ln 3 - 2 \ln 2. \quad (B.13)$$

Therefore we conclude that $z_2(x)$ reaches its minimum value given by Eq. (B.13) at the value of $x = x_0$.

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- 15) In this context our parton interpretation is similar to that of Drell et al.⁵⁾ where partons are bare particles themselves in the virtual cloud of a physical hadron.
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- 18) In the string picture of the dual resonance model advocated in Ref. 12), the expression Eq. (2.13) implies that only the end points of the string are active in interaction.
- 19) We only remark here that the model we shall study is formally unitary in a perturbative sense in the particular channel which contains the parton source in it. This will be apparent from our expression of the S matrix, Eq. (2.34), below.
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FIG.1

FIG. 2