EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

SOLUTION OF THE EQUATIONS OF MOTION OF A CHARGED
PARTICLE IN A STATIONARY MAGNETIC FIELD

M. Regler^{*)}

CERN, Geneva, Switzerland

Geneva - May 1969

(Submitted to J. Comput. Phys.)

^{*)} Visiting scientist from the Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften.

SOLUTION OF THE EQUATIONS OF MOTION OF A CHARGED
PARTICLE IN A STATIONARY MAGNETIC FIELD

M. Regler^{*)}

Institut für Hochenergiephysik der Österreichischen
Akademie der Wissenschaften

ABSTRACT

The equation $\ddot{\underline{r}}(t) = \text{const} \cdot [\dot{\underline{r}}(t) \times \underline{B}(\underline{r}(t))]$ ($\dot{}$ denotes d/dt) is solved by comparison of coefficients done by means of a FORTRAN program. $\underline{B}$ must be given analytically with non-negative powers in the Cartesian coordinates x_α ($\alpha = 1, 2, 3$) in the considered region, and must not depend explicitly on the parameter t . For a magnetic field, the parameter could be the time, the path length, the momentum, path length/momentum, the proper time, etc. For Monte Carlo calculations, a suitable choice of the parameter allows the representation of a group of particles by one set of constants, for example to calculate the decay point and the direction of a particle after a generated lifetime.

*) At present visiting scientist at CERN, Geneva, Switzerland.

It is often necessary in high-energy physics experiments to be able to make fast calculations of the paths of charged particles in a magnetic field, for both Monte Carlo and reconstruction programs. If it is possible to represent the field analytically in a Cartesian coordinate system by non-negative powers of x_1, x_2, x_3 (see Ref. 1), then the differential equation may be solved by comparison of coefficients.

The equation of motion then has the form

$$x''_{\alpha} = \text{const.} \sum_{\beta=1}^3 \sum_{\gamma=1}^3 \sum_{\ell_{\gamma}=1}^{N_{\gamma}} \epsilon_{\alpha\beta\gamma} x'_{\beta} C_{\gamma}(\ell_{\gamma}) x_1^{i_{\gamma}(\ell_{\gamma})} x_2^{j_{\gamma}(\ell_{\gamma})} x_3^{k_{\gamma}(\ell_{\gamma})}$$

$$B_{\gamma} = \sum_{\ell_{\gamma}=1}^{N_{\gamma}} C_{\gamma} x_1^{i_{\gamma}(\ell_{\gamma})} x_2^{j_{\gamma}(\ell_{\gamma})} x_3^{k_{\gamma}(\ell_{\gamma})}$$

1, when α, β, γ is an even permutation of 1, 2, 3

$\epsilon_{\alpha\beta\gamma} = 0$, when at least two indices are equal

-1, when α, β, γ is an odd permutation of 1, 2, 3

' = d/dt.

This equation can be solved by an infinite series of the following type:

$$x_{\alpha} = \sum_{\lambda_{\alpha}=1}^{\infty} a_{\alpha}(\lambda_{\alpha}) t^{\lambda_{\alpha}-1}.$$

If $a_\alpha(n_\alpha + 1)$ is the first non-zero coefficient, the series may be re-written as:

$$x_\alpha = t^{n_\alpha} \cdot \sum_{\lambda_\alpha=1}^{\infty} \bar{a}_\alpha(\lambda_\alpha) t^{\lambda_\alpha-1}$$

$$\bar{a}_\alpha(\lambda_\alpha) = a_\alpha(\lambda_\alpha + n_\alpha).$$

Now the coefficients of the series may be calculated (see Ref. 2)

$$x_\alpha^{m_\alpha} = t^{n_\alpha m_\alpha} \cdot \sum_{\mathcal{K}_\alpha=1}^{\infty} c_\alpha(\mathcal{K}_\alpha) t^{\mathcal{K}_\alpha-1} = \sum_{q=1}^{\infty} d_{\alpha, m_\alpha}(q) t^{q-1}$$

$$= 0 \text{ for } q \leq n_\alpha m_\alpha$$

$$d_{\alpha, m_\alpha}(q) = \bar{a}_\alpha(1)^{m_\alpha} \text{ for } q = n_\alpha m_\alpha + 1$$

$$= \frac{1}{\bar{q} \bar{a}_\alpha(1)} \cdot \sum_{p=1}^{\bar{q}} (p m_\alpha - \bar{q} + p) \bar{a}_\alpha(p+1) d_{\alpha, m_\alpha}(n_\alpha m_\alpha - p + \bar{q} + 1)$$

$$\bar{q} = q - n_\alpha m_\alpha - 1 \geq 1.$$

The product of the powers is now formed:

$$x_1^{i_\alpha(\ell_\alpha)} x_2^{j_\alpha(\ell_\alpha)} x_3^{k_\alpha(\ell_\alpha)} = \sum_{\mu=1}^{\infty} K_{\alpha, \ell_\alpha}(\mu) t^{\mu-1}$$

$$K_{\alpha, \ell_{\alpha}}(\mu) = \sum_{\substack{3 \\ \sum_{\eta=1} u_{\eta} = \mu + 2}} d_{1, i_{\alpha}(\ell_{\alpha})}(u_1) d_{2, j_{\alpha}(\ell_{\alpha})}(u_2) d_{3, k_{\alpha}(\ell_{\alpha})}(u_3).$$

B then has the form

$$B_{\alpha} = \sum_{\delta=1}^{\infty} G_{\alpha}(\delta) t^{\delta-1}$$

$$G_{\alpha}(\delta) = \sum_{\ell_{\alpha}=1}^{N_{\alpha}} C_{\alpha}(\ell_{\alpha}) K_{\alpha, \ell_{\alpha}}(\delta).$$

Now the a_{α} may be calculated recursively from the given initial conditions $a_{\alpha}(1), a_{\alpha}(2)^{*}$:

$$a_{\alpha}(\rho+2) = \frac{\text{const}}{\rho+1} \cdot \sum_{\beta=1}^3 \sum_{\gamma=1}^3 \epsilon_{\alpha\beta\gamma} \cdot \sum_{\mu+\gamma=\rho+1} a_{\beta}(\mu+1) G_{\gamma}(\nu)$$

$$a_{\alpha}(\rho+2) = F_{\alpha, \rho+2} (a_1(< \rho+2), a_2(< \rho+2), a_3(< \rho+2))$$

*) The $a_{\alpha}(2)$ can be dependent on one another in certain representations.

The FORTRAN^{*)} program to solve this was written in double precision and tested. By a suitable choice of the parameter (proper time) the program is particularly adapted to Monte Carlo calculations. A listing can be obtained from the author.

Acknowledgements

The author is indebted to Dr. M. Vivargent and Dr. K. Winter for their continuing interest in this work, which was done for the K_{e3} experiment of the CERN-Orsay-Vienna collaboration.

REFERENCE

- 1) H. Wind, "Processing magnetic field data", J. Comput. Phys. 2, No. 3, February 1968.
- 2) I.S. Gradshteyn and I.M. Ryzhik, Table of integrals series and products, Academic Press, New York and London 1965 (0.314).

*) The principal loop exists also in CDC 6000 machine language, coded by M. Metcalf, Data Handling Division, CERN, Geneva, Switzerland.

α = alpha

β = beta

γ = gamma

δ = delta

ϵ = epsilon

η = eta

μ = mu

ν = nu

ρ = rho

λ = lambda

$\sum$ = summation

∞ = infinite

' = prime

$\sim$ = vector