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PION-PION PHASE SHIFTS FROM COVARIANT PERTURBATION THEORY
FOR A CHIRAL INVARIANT FIELD THEORETIC MODEL

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A B S T R A C T

The Lehmann model for pion-pion scattering consists in a low energy expansion of the scattering amplitude to fourth order in the momenta on the basis of the chiral invariant pion-nucleon Lagrangian. Because of the non-renormalizability of the pion Lagrangian two parameters remain undetermined in the pion-loop contribution. Using covariant perturbation theory and superpropagator methods, we calculate these two parameters independently of the choice of the pion field co-ordinates. The resulting phase shifts are shown to be in very good agreement with the data.

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1. INTRODUCTION

On the basis of a chiral invariant Lagrangian field theory, Lehmann^{1),2)} has calculated the phase shifts for pion-pion scattering at low energies in the effective range approximation. The amplitudes are computed up to fourth order in the inverse pion decay constant F_π^{-1} which implies that one-loop diagrams are added to the Born term. Since the non-linear chiral invariant pion model is non-renormalizable, two free subtraction constants remain at the one-loop level which cannot be fixed by renormalization conditions. In order to determine these two constants, Lehmann and Trute³⁾ use the superpropagator formalism together with standard perturbation theory. It then turns out that the pion forces alone cannot account for the observed low energy phase shifts. Therefore, Lehmann²⁾ includes the nucleon contributions as given by the chiral invariant pion-nucleon Lagrangian with pseudoscalar coupling. The resulting phase shifts are in qualitative agreement with the experimental values.

The motivation of the present work is to check the consequences of an alternative superpropagator formalism as described in Ref. 4) (hereafter referred to as I). The method is based on a covariant perturbation expansion where the covariance is with respect to transformations of the pion field co-ordinates in terms of which the Lagrangian is defined. In contrast to the standard approach, the covariant formalism leads to subtraction constants in the one-loop amplitude which are independent of the choice of the pion field co-ordinates. The numerical values for the two parameters differ from those obtained by Lehmann and Trute. As a consequence, we obtain somewhat different phase shifts which are in very good agreement with the experimental data.

In Section 2, we briefly summarize the work of Lehmann and repeat some of his formulae which we need in the following. Section 3 contains our contribution to this programme, namely the calculation of the subtraction constants for the pion loop diagram in the framework of covariant perturbation theory. In Section 4, we compare our results with the observed s and p wave phase shifts.

2. THE LEHMANN MODEL

The pion-pion scattering amplitude is written in the standard notation

$$T_{i_1 i_2 i_3 i_4}(s, t, u) = A(s, t, u) \delta_{i_1 i_2} \delta_{i_3 i_4} + A(t, s, u) \delta_{i_1 i_3} \delta_{i_2 i_4} + A(u, t, s) \delta_{i_1 i_4} \delta_{i_2 i_3} \quad (2.1)$$

with $s+t+u=0$ ($m_\pi = 0$), $A(s, t, u) = A(s, u, t)$.

From chiral invariance, Lehmann¹⁾ derived the following low energy expansion for $A(s, t, u)$ to fourth order in the momenta

$$A(s, t, u) = \frac{s}{F_\pi^2} - (96\pi^2 F_\pi^4)^{-1} \left\{ 3s^2 \log[\alpha_1(-s-i\epsilon)] + t(t-u) \log[\alpha_2(-t-i\epsilon)] + u(u-t) \log[\alpha_2(-u-i\epsilon)] \right\} + \dots \quad (2.2)$$

F_π is the pion decay constant and we take $F_\pi = 95$ MeV. α_1 and α_2 are two free parameters which cannot be determined from invariance arguments alone.

The first term in (2.2) is given by the tree diagram. For the pion loop both ordinary and covariant perturbation theory give the same expression identical to the second term in (2.2), with undetermined subtraction constants α_1 and α_2 . The evaluation of these constants by means of the superpropagator formalism will constitute the main part of Section 3.

As mentioned in the Introduction, the pion self-interaction alone does not lead to a satisfactory description of the pion-pion scattering amplitude at low energies. Therefore, Lehmann includes the nucleons. The relevant part of the chiral invariant pion-nucleon Lagrangian with pseudoscalar coupling for the F_π^{-4} approximation in pion-pion scattering is given by :

$$L_I = -i \frac{M_N}{F_\pi} : \bar{\psi} \gamma_5 \vec{\tau} \psi \vec{\pi} : + \frac{M_N}{2F_\pi^2} : \bar{\psi} \psi \vec{\pi}^2 : + \dots \quad (2.3)$$

There are three one-nucleon loop diagrams contributing in this order of approximation which are shown in Fig. 1. The corresponding expressions for $A(s, t, u)$ expanded to fourth order in the momenta are²⁾

$$\begin{aligned} H_{(a)} &= \frac{s^2}{40\pi^2 F_\pi^4} \quad , \quad H_{(b)} = -\frac{s^2}{120\pi^2 F_\pi^4} \quad , \\ H_{(c)} &= -\frac{1}{240\pi^2 F_\pi^4} (9s^2 - 5t^2 - 5u^2) \end{aligned} \quad (2.4)$$

The coupling constants in (2.3) which have been used for the amplitudes (2.4) have yet to be renormalized. For the three-vertex this renormalization is determined by the Goldberger-Treiman theorem ⁵⁾ and amounts to replacing F_π^{-1} by $g_A F_\pi^{-1}$, where g_A is the axial vector coupling constant. The same replacement is made in the coupling constant of the four-vertex in (2.3) which implies that a momentum dependent term is neglected. For a more extensive discussion of the renormalization of the pion-nucleon vertices, we refer to the original paper of Lehmann ²⁾.

With these vertex corrections, the total one-nucleon loop contribution to pion-pion scattering is given by

$$H_N(s, t, u) = - \frac{g_A^4}{48\pi^2 F_\pi^4} (s^2 - t^2 - u^2) \quad (2.5)$$

or, alternatively,

$$H_N(s, t, u) = - \frac{G^4}{48\pi^2 M_N^4} (s^2 - t^2 - u^2) \quad (2.6)$$

where G is the renormalized pion-nucleon coupling constant and the Goldberger-Treiman relation has been used to go from (2.5) to (2.6). Since the Goldberger-Treiman relation is only fulfilled up to approximately 10%, the two amplitudes (2.5) and (2.6) give slightly different phase shifts. In comparing the calculated phase shifts with the experimental values, we will consider both possibilities.

From (2.5) or (2.6), one immediately sees that the nucleon loop contributions to the parameters α_1, α_2 are well-defined, finite quantities,

3. COVARIANT CALCULATION OF THE SUBTRACTION CONSTANTS FOR THE PION LOOP

In this section, we want to study the superpropagator approximation to pion-pion scattering within the framework of covariant perturbation theory set up in I.

The functional $W(J)$ of connected Green functions, which contains the superpropagator diagrams, has the form

$$W(J) = W_2(J) + \sum_{n=1}^{\infty} W_{2n+2}(J) \quad (3.1a)$$

where

$$W_2(J) = \frac{i}{2} \log \text{Det} (S_{a_1 a_2} G_0) \quad (3.1b)$$

$$W_{2n+2}(J) = \frac{i^{2n+1}}{2[(2n+2)!]^2} S_{a_1 \dots a_{2n+2}} \left(\prod_{p(b_i)} G_{a_1 b_1} \dots G_{a_{2n+2} b_{2n+2}} \right) S_{b_1 \dots b_{2n+2}} \quad (3.1c)$$

$W_2(J)$ represents the one-loop functional ⁶⁾ (Fig. 2a) and $W_{2n+2}(J)$ contains the graphs with $2n+2$ internal pion lines (Fig. 2b). The notation is explained in I, but for convenience we repeat some definitions and equations.

The inverse propagator $S_{a_1 a_2}$ has the form

$$S_{a_1 a_2} = -(\mathbb{D}_\mu)_{a_1 b} (\mathbb{D}^\mu)_{b a_2} - C_{a_1 a_2} \quad (3.2)$$

where

$$\begin{aligned} C_{ab} &= F_\pi^{-2} (g_{ik} \partial_\mu \pi^i \partial^\mu \pi^k \delta_{ab} - e_{ai} e_{bj} \partial_\mu \pi^i \partial^\mu \pi^j) \delta(x_a, x_b) = \\ &= F_\pi^{-2} ((\partial_\mu \vec{\pi})^2 \delta_{ab} - \partial_\mu \pi^a \partial^\mu \pi^b) \delta(x_a, x_b) + O(\pi^4) \end{aligned} \quad (3.3)$$

$$(\mathbb{D}_\mu)_{ab} = \delta_{ab} \partial_\mu^{(x_a)} \delta(x_a, x_b) + g_{\mu, ab} \delta(x_a, x_b) \quad (3.4)$$

$$\begin{aligned} g_{\mu, ab} &= e_a^k e_b^l \Gamma_{k, i\ell} \partial_\mu \pi^i + g_{k\ell} e_a^k \partial_\mu e_b^\ell = \\ &= \frac{1}{2F_\pi^2} (\pi^a \partial_\mu \pi^b - \pi^b \partial_\mu \pi^a) + O(\pi^4) \end{aligned} \quad (3.5)$$

The $\Gamma_{k, i\ell}$ are the Christoffel symbols for the metric g_{ij} and the $e_{ai}(\pi)$ are dreibein fields with $e_{ai} e_{aj} = g_{ij}$. In this section the pion fields always stand for the classical pion fields which are solutions of the equations of motion

$$\frac{\delta S}{\delta \pi^i(x)} + J_i(x) = 0 \quad (3.6)$$

and are thus functionals of the external sources $J_i(x)$. S is the action functional $\frac{1}{2} \int dx g_{ij}(\pi) \partial_\mu \pi^i \partial^\mu \pi^j$.

Note that the terms in C_{ab} and $\rho_{\mu,ab}$ which are bilinear in the pion fields are independent of the choice of the pion field co-ordinates. Since only these bilinear terms will be relevant for the pion-pion scattering amplitude, the calculations can obviously be done in a co-ordinate independent way.

Furthermore,

$$S_{a_1 \dots a_{2n+2}} = S_{2n+2}^{(1)} + S_{2n+2}^{(2)} \quad (3.7a)$$

with

$$S_{2n+2}^{(1)} = (4F_\pi^{-2})^n (-1)^{n+1} C_{a_1 a_2} \delta_{a_3 a_4} \dots \delta_{a_{2n+1} a_{2n+2}} \quad (3.7b)$$

and

$$S_{2n+2}^{(2)} = (4F_\pi^{-2})^n (-1)^n (D_\mu)_{ba_1} (\delta_{bc} \delta_{a_2 a_3} - \delta_{ba_2} \delta_{ca_3}) \cdot \delta_{a_4 a_5} \dots \delta_{a_{2n} a_{2n+1}} (D^\mu)_{ca_{2n+2}} \quad (3.7c)$$

The vertex functionals (3.7) are to be considered as completely symmetrized with respect to the $a_1, \dots, a_{2n+2}$ and δ functions like $\delta(x_{a_3}, x_{a_4})$ are absorbed into the Kronecker symbols $\delta_{a_3 a_4}$. Finally, for the superpropagator approximation, we may set the full propagator $G_{ab}(x)$ equal to $\delta_{ab} G_0(x)$, where $G_0(x) = i/(4\pi^2(x^2 - i\epsilon))$.

The contribution from the one-loop functional (3.1b) to pion-pion scattering has the following form

$$i W_2(J) = \frac{1}{4} \int dx dx' \left\{ C_{ab}(x) C_{ab}(x') G_0^2(x-x') + 2 \rho_{\mu,ab}(x) \rho_{\nu,ab}(x') [G_\mu(x-x') G_\nu(x-x') - G_{\mu\nu}(x-x') G_0(x-x')] \right\} \quad (3.8)$$

where

$$G_\mu(x) = \partial_\mu G_0(x)$$

$$G_{\mu\nu}(x) = \partial_\mu \partial_\nu G_0(x)$$

The calculation of the relevant contributions from $W_{2n+2}^{(J)}$ ($n \geq 1$) proceeds by inserting equations (3.7) into (3.1c) and performing the combinatorics.

We obtain

$$S_{2n+2}^{(1)}(\Pi G_0) S_{2n+2}^{(1)} = \frac{2}{15} (4F_\pi^{-2})^{2n} \int dx dx' [G_0(x-x')]^{2n+2} \cdot$$

$$\cdot \left\{ (2n+5)(2n+3)(2n+1)! C_{ab}(x) C_{ab}(x') + n(2n+3)(2n+1)! C_{aa}(x) C_{bb}(x') \right\} \quad (3.9a)$$

$$S_{2n+2}^{(1)}(\Pi G_0) S_{2n+2}^{(2)} = 0 \quad (3.9b)$$

$$S_{2n+2}^{(2)}(\Pi G_0) S_{2n+2}^{(2)} = \frac{4}{3} (4F_\pi^{-2})^{2n} (2n+3)(2n+1)! \cdot$$

$$\cdot \int dx dx' \rho_{\mu,ab}(x) \rho_{\nu,ab}(x') [G_\mu(x-x') G_\nu(x-x') - G_{\mu\nu}(x-x') G_0(x-x')] G_0^{2n}(x-x') \quad (3.9c)$$

We now insert (3.8) and (3.9) into (3.1) and take for C_{ab} and $\rho_{\mu,ab}$ only the terms bilinear in the pion fields as given by (3.3) and (3.5). After performing some partial integrations, one finally arrives at the following expression for the connected functional for pion-pion scattering in the superpropagator approximation :

$$W(J) = \int dx dx' \partial_\mu \pi^a(x) \partial^\mu \pi^b(x) \partial_\nu \pi^a(x') \partial^\nu \pi^b(x') S_1(x-x') +$$

$$+ \int dx dx' (\partial_\mu \vec{\pi}(x))^2 (\partial_\nu \vec{\pi}(x'))^2 S_2(x-x') +$$

$$+ \frac{1}{2} \int dx dx' \pi^a(x) \partial_\mu \pi^b(x) (\pi^a(x') \partial^\mu \pi^b(x') - \pi^b(x') \partial^\mu \pi^a(x')) S_3(x-x') \quad (3.10)$$

with

$$i S_1(x) = -\frac{1}{15 F_\pi^4} \sum_{n=0}^{\infty} (4 F_\pi^{-2})^{2n} \frac{(2n+5)(2n+3)}{(2n+2)(2n+2)!} (i G_0(x))^{2n+2} \quad (3.11a)$$

$$i S_2(x) = -\frac{1}{15 F_\pi^4} \sum_{n=0}^{\infty} (4 F_\pi^{-2})^{2n} \frac{(6n+5)(2n+3)}{(2n+2)(2n+2)!} (i G_0(x))^{2n+2} \quad (3.11b)$$

$$i S_3(x) = \frac{16 \pi^2}{3 F_\pi^4} \sum_{n=0}^{\infty} (4 F_\pi^{-2})^{2n} \frac{1}{(2n+2)!} (i G_0(x))^{2n+3} \quad (3.11c)$$

Applying Fourier transformation and functional differentiation, we obtain from (3.10) the pion-pion scattering amplitude

$$\begin{aligned} \bar{H}_\pi(s, t, u) = & t^2 \tilde{S}_1(t) + u^2 \tilde{S}_1(u) + 2s^2 \tilde{S}_2(s) - \\ & - (2s+t) \tilde{S}_3(t) - (2s+u) \tilde{S}_3(u) \end{aligned} \quad (3.12)$$

The quantities $\tilde{S}_i(q^2) = \int dx e^{iqx} S_i(x)$ ($i=1,2,3$) are defined by the superpropagator method. This definition is not unique due to the ambiguity of time ordering for products of propagators. In the following we will use the same definition of time ordering as Lehmann and Truete and refer to their paper ³⁾ for a justification of this choice.

One introduces a Sommerfeld-Watson representation for the infinite sums in (3.11), deforms the contour of integration to a region where the Fourier transformation can be done unambiguously ⁷⁾ and then goes back to the original contour.

This procedure leads to

$$\tilde{S}_1(q^2) = -\frac{1}{480 F_\pi^4} \cdot \frac{1}{2\pi i} \int_L ds (4\pi^2 F_\pi^2)^{2s} (-q^2)^{2s} \frac{(2s+5)(2s+3)}{\Gamma(2s+1) [\Gamma(2s+3)]^2 \sin^2 \pi s} \quad (3.13a)$$

$$\tilde{S}_2(q^2) = -\frac{1}{480 F_\pi^4} \cdot \frac{1}{2\pi i} \int_L ds (4\pi^2 F_\pi^2)^{-2s} (-q^2)^{2s} \frac{(6s+5)(2s+3)}{\Gamma(2s+1) [\Gamma(2s+3)]^2 \sin^2 \pi s} \quad (3.13b)$$

$$\tilde{S}_3(q^2) = -\frac{1}{96 F_\pi^4} \cdot \frac{1}{2\pi i} \int_L ds (4\pi^2 F_\pi^2)^{-2s} (-q^2)^{2s+1} \frac{1}{\Gamma(2s+2) [\Gamma(2s+3)]^2 \sin^2 \pi s} \quad (3.13c)$$

where the contour L encircles the positive real axis counter-clockwise, as shown in Fig. 3. For the F_π^{-4} approximation of the scattering amplitude, one has to evaluate the integrals (3.13) around the double pole at $s=0$. Inserting the residues into Eq. (3.12) gives, finally

$$\begin{aligned} H_\pi(s, t, u) = & - (16\pi^2 F_\pi^4)^{-1} \left[\frac{s^2}{2} \log\left(\frac{-s-i\epsilon}{4\pi^2 F_\pi^2}\right) + \frac{t(t-u)}{6} \log\left(\frac{-t-i\epsilon}{4\pi^2 F_\pi^2}\right) + \right. \\ & \left. + \frac{u(u-t)}{6} \log\left(\frac{-u-i\epsilon}{4\pi^2 F_\pi^2}\right) + \gamma(s^2+t^2+u^2) - \frac{1}{60} (22s^2+57t^2+57u^2) \right] \end{aligned} \quad (3.14)$$

where γ is Euler's constant ($\gamma = 0.577\dots$).

Comparing this result with the general expression (2.2) we obtain the pion loop contribution to the parameters α_1 and α_2 :

$$\alpha_1^\pi = (4\pi^2 F_\pi^2)^{-1} \exp\left(3\gamma - \frac{101}{60}\right) \quad (3.15a)$$

$$\alpha_2^\pi = (4\pi^2 F_\pi^2)^{-1} \exp\left(3\gamma - \frac{57}{20}\right) \quad (3.15b)$$

While these values depend in principle on the choice of time ordering, their ratio $\alpha_1^\pi / \alpha_2^\pi = e^{7/6}$ does not. In contrast with the standard approach by Lehmann and Trute, this ratio is not equal to one in the covariant approach. Furthermore, as already emphasized, α_1^π and α_2^π are independent of which co-ordinate system is chosen for the pion field.

4. RESULTS AND DISCUSSION

For the comparison of our results with the observed phase shifts, we again adopt the nomenclature of Ref. 1).

In the effective range approximation, Lehmann derived the following s and p waves from (2.2)

$$\cot \delta_0^s(q^2) = \frac{4\pi F_\pi^2}{q^2} + \frac{25}{18\pi} \log\left(\frac{q^2}{4\pi^2 F_\pi^2}\right) - \frac{\eta}{\pi} \quad (4.1)$$

$$\cot \delta_0^2(q^2) = -\frac{8\pi F_\pi^2}{q^2} + \frac{20}{9\pi} \log\left(\frac{q^2}{4\pi^2 F_\pi^2}\right) - \frac{8}{5\pi} (\xi + \eta) - \frac{7}{18\pi} \quad (4.2)$$

$$\cot \delta_1'(q^2) = \frac{24\pi F_\pi^2}{q^2} - \frac{6\xi}{\pi} \quad (4.3)$$

q is the centre-of-mass momentum and the pion mass is introduced in the equations above to convert momenta into energies by $q^2 = \frac{1}{4}(s - 4m_\pi^2)$. ξ and η are defined by

$$\xi = \frac{1}{2}(\beta_2 - \beta_1 - \frac{1}{9}) \quad (4.4a)$$

$$\eta = \frac{1}{36} (33\beta_1 + 17\beta_2 + \frac{11}{3}) \quad (4.4b)$$

where $\beta_i = -\log(16\pi^2 F_\pi^2 \alpha_i)$ ($i=1,2$).

With the Lehmann values for α_i^{nucl} and our new values (3.15) for α_i^π we get ξ bigger and η smaller than the corresponding quantities calculated by Lehmann²⁾. This modification moves the p wave up and the I=0 s wave down and thus increases the agreement with experiment, while the I=2 s wave is not much changed. In Figs. 4, 5 and 6 we exhibit our results together with the data. Curve 1) is obtained by taking the nucleon contribution in the form (2.5) with $g_A = 1.25$ while for curve 2) we have used (2.6) with $G^2/4\pi = 14.6$. The p wave passes through 90° at 860 MeV for curve 1) and 760 MeV for curve 2).

Once the definition of time ordering in the superpropagator formalism is accepted, one has a parameter-free description of low energy pion-pion scattering that seems to work surprisingly well. In principle, the equations (4.1), (4.2)

and (4.3) can only be trusted in the region of elastic unitarity. However, the measured inelasticities seem to be rather small up to almost 1 GeV for the $I=2$ s wave and the $I=1$ p wave, while this is not the case for the $I=0$ s wave⁸⁾. That the predicted $I=2$ s wave and $I=1$ p wave agree with the data up to higher energies compared to the $I=0$ s wave, is therefore not unexpected. However, this is not necessarily a very convincing explanation, especially because the range of validity of the F_π^{-4} approximation is unknown. The corrections due to the non-vanishing pion mass have been estimated¹⁾ in the tree approximation to be of the order of a few degrees.

In our opinion, the Lehmann model provides a very satisfactory description of pion-pion scattering on the basis of a chiral invariant field theory. Moreover, we consider it appealing that the requirement of pion co-ordinate independence of the scattering amplitude in the determination of the parameters α_1 and α_2 leads to phase shifts that are in excellent agreement with the present data.

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FIGURE CAPTIONS

Figure 1 : Nucleon loop contributions to pion-pion scattering from the Lagrangian (2.3).

Pion lines : - - - - -

Nucleon lines : ————

Figure 2 : a) One-pion loop diagram for pion-pion scattering.
b) Superpropagator diagrams for pion-pion scattering.

Figure 3 : Contour of integration in the complex s plane for Eqs. (3.13).

Figure 4 : $I=0$ s wave phase shift δ_0^0 . Curve 1) : $g_A = 1.25$;
Curve 2) : $g^2/4\pi = 14.6$.

Data points :

■ Baton et al. 9), ■ Protopopescu et al. 10),
□ Baillon et al. 11), □ Sonderegger et al. 12),
▲ Bensinger et al. 13), ▲ Zylbersztejn et al. 14).

Figure 5 : $I=2$ s wave phase shift δ_0^2 . Curves 1) and 2) as in Fig. 4.

Data points :

■ Baton et al. 9), ■ Colton et al. 15),
□ Katz et al. 16), □ Walker et al. 17).

Figure 6 : $I=1$ p wave phase shift δ_1^1 . Curves 1) and 2) as in Fig. 4.

Data points :

■ Baton et al. 9), ● Protopopescu et al. 10).

a)

b)

c)

Fig. 1

a)

b)

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6