CM-P00058511

Archives.

Ref.TH.1597-CERN

LARGE TRANSVERSE MOMENTA IN THE MULTIPERIPHERAL MODEL

D. Amati, L. Caneschi and M. Testa

CERN - Geneva

A B S T R A C T

A unified picture of small and large transverse momenta is provided by the multiperipheral model in its original version. The large k_T spectrum is k_T^{-p} times a scaling function. Results are in agreement with ISR data and testable predictions are made.

Ref.TH.1597-CERN

14 December 1972

Recently, ISR experiments ¹⁾ have explored values of the transverse momentum k_T much larger (ten times or more) than $\langle k_T \rangle$, but still much smaller than p_{inc} , i.e., far from the phase space boundary.

The main features of the large k_T data can be summarized as follows :

- 1) the observed rate is much larger than an exponential extrapolation of low k_T data would suggest ;
- 2) whereas for low k_T the spectrum is energy independent, the large k_T data increase substantially with energy ;
- 3) the sharp peak at small k_T and the shallow large k_T region are smoothly related : there is no evidence for a break or a similar structure ;
- 4) the particle composition of the large k_T events is radically different from the small k_T one, in that it is not dominated by pion production.

In recent times, it seems a general wide-spread idea that the low k_T events are determined by a soft structure, whereas high k_T are expected from a hard or point-like one (partons, quarks, etc.). The evidence for this hard component is provided by the SLAC results. In this picture, the point-like structure should give rise to slowly decreasing k_T distributions and should therefore eventually emerge from the rapidly decreasing soft component ²⁾. We want to show that there is really no need, on the basis of the k_T spectra, for this kind of two-component structure that we find physically unpleasant.

A unique description of the whole k_T spectrum can be achieved by a theoretical model provided that it exhibits Reggeization, hadronic scaling and Bjorken scaling in the appropriate limits. All these requirements are satisfied by the multiperipheral model in its original version ³⁾. For small values of k_T it is well known that the model gives a rapidly decreasing and energy-independent k_T spectrum. The explicit form depends, of course, on the details of the model as, for instance, the masses of the exchanged and created particles : sensible values for them (m_π, m_{ρ_4}) reproduce very satisfactorily the small k_T ($\lesssim 1$ GeV/c) spectrum ⁴⁾. These masses are the only parameters that determine the scale of the model and in particular the rapid decrease in k_T . This implies that there is no fundamental scale beyond the hundred of MeV or, better, if high

"elementary" scales exist, they do not determine the main features of high energy events. We calculate in this paper the multiperipheral single particle spectra for a situation in which k_T is quite larger than these low masses (for the sake of reference, let us say $k_T > 1.5 \text{ GeV}/c$). As expected, in the asymptotic limit ($s \rightarrow \infty$), the spectrum is given by a function of the Feynman parameter x multiplied by an inverse power of k_T in which the scale is provided by the (small) input masses.

Using the notation of Amati, Stanghellini and Fubini (ASF) and the kinematics shown in Fig. 1, the single particle distribution [Eq. (6.15) of ASF] is given by

$$\frac{1}{\sigma} \frac{d\sigma}{d^3k} \equiv N(s, k) = \frac{1}{A(s)} \frac{1}{s} \int du' du'' P(u') P(u'') \int ds' ds'' A(u's') A(u''s'') T(u'' - m_T^2 y_2(1+y_1), u' - m_T^2 y_1(1+y_2), k_T^2) \quad (1)$$

where

$$s_1 = 2k \cdot p_1, \quad s_2 = 2k \cdot p_2, \quad s_1 s_2 = s m_T^2, \quad y_1 = \frac{s'}{s_1}, \quad y_2 = \frac{s''}{s_2} \quad (2)$$

$$T(a, b, c) = \frac{\theta(-a^2 - b^2 - c^2 + 2ab + 2bc + 2ac)}{\sqrt{-a^2 - b^2 - c^2 + 2ab + 2bc + 2ac}}, \quad m_T^2 = m^2 + k_T^2 \quad (3)$$

where m is the mass of the produced particle, $P(u)$ is the propagation function for which we shall assume

$$P(u) \xrightarrow{u \rightarrow \infty} u^{-\lambda} \quad (4)$$

For a single particle scalar theory, $P(u) = 1/(u^2 + \mu^2)$, i.e., $\lambda = 1$.

$A(u, s)$ is the off-shell amplitude [$\bar{A}(s) \equiv A(-\mu^2, s)$ the on-shell one] which is the eigenfunction of the multiperipheral integral equation. General properties of $A(u, s)$ are ^{*)}, 3) :

*) All these general properties are determined by the damping in u given by $P(u)$. Our results will depend on the validity of Eqs. (5) and (6) and not specifically on the scalar character ($\lambda = 1$) of the theory. Due to this, our attitude will be to consider λ as a parameter.

$$A(u, s) \xrightarrow[\substack{s \rightarrow \infty \\ u \text{ finite}}]{ } s^\alpha \varphi(u) \quad (5)$$

where α , the Regge intercept, is related to the eigenvalue problem of the multiperipheral integral equation. Another property of $A(u, s)$ is the scaling in the Bjorken limit, i.e.,

$$A(u, s) \xrightarrow[\substack{u \rightarrow \infty \\ \omega \text{ fixed}}]{ } \frac{1}{u} \psi(\omega) \quad (6)$$

where $\omega = (s/u) + 1$. Besides, Regge and Bjorken limits can be interchanged in the multiperipheral model, i.e.,

$$\varphi(u) \xrightarrow{u \rightarrow \infty} \frac{C}{u^{1+\alpha}} \quad \psi(\omega) \xrightarrow{\omega \rightarrow \infty} C \omega^\alpha \quad (7)$$

The triangle function of Eq. (3) forces at least one of u' or u'' to be of the order of k_T^2 . A simple analysis shows indeed that the dominant contribution for large k_T comes when one of the sides of the triangle [let us say $u'' = m_T^2 y_2 (1 + y_1)$] is of the order of k_T^2 , while the other remains of the order of the masses. Using Eqs. (4) and (6), one gets for large k_T

$$N \propto \frac{1}{s^{1+\alpha} (k_T^2)^{2\alpha}} \int d\omega \int du' P^2(u') \int ds' A(u', s') \left(1 - \omega \frac{k_T^2}{s_1}\right)^{2\alpha-1} \psi(\omega) + (8) \\ + (s_1 \leftrightarrow s_2)$$

Let us now express the scaling function $\psi(\omega)$ in terms of an integral over A . This can easily be obtained from the defining integral equation [Eq. (4.3') of ASF], i.e. :

$$\psi(\omega) = \frac{1}{\omega} \int du' \int ds' P^2(u') A(u', s') \quad (9)$$

Introducing (9) into (8), we obtain :

$$N \propto \frac{1}{s^{1+\alpha} (k_T^2)^{2\alpha}} \frac{s_1}{k_T^2} \int d\omega \left(1 - \omega \frac{k_T^2}{s_2}\right)^{2\alpha} \psi(\omega) \psi\left[\frac{s_1}{k_T^2} \left(1 - \omega \frac{k_T^2}{s_2}\right)\right] + (s_1 \leftrightarrow s_2) \quad (10)$$

Remembering the asymptotic behaviour (7) of $\psi(\omega)$, it is better to introduce the structure function

$$\chi(\omega) \equiv \frac{\omega^{-\alpha}}{c} \psi(\omega) \quad \lim_{\omega \rightarrow \infty} \chi(\omega) = 1. \quad (11)$$

Let us introduce the dimensionless parameters x and y by

$$x = \frac{s_2}{s} = \frac{k_T^2}{s_1}, \quad y = \frac{k_T^2}{xs} = \frac{k_T^2}{s_2}. \quad (12)$$

We note that x (or y) reduces to the Feynman parameter for small k_T .
Then

$$N \propto \frac{1}{(k_T^2)^{2\lambda+\alpha+1}} G(x, y), \quad G(x, y) = \frac{1}{2} (g(x, y) + g(y, x)) \quad (13)$$

where

$$g(x, y) \equiv c(\alpha, \lambda) y^{\alpha+1} \int_0^{1/y} d\omega (1-\omega y)^{2\lambda+\alpha} \omega^\alpha \chi(\omega) \chi\left(\frac{1-\omega y}{x}\right); c(\alpha, \lambda) \equiv \beta^{-1}(2\lambda+\alpha+1, \alpha) \quad (14)$$

The function G is normalized to 1 in the limit

$$\frac{1}{x} = \frac{s_1}{k_T^2} \rightarrow \infty, \quad \frac{1}{y} = \frac{s_2}{k_T^2} \rightarrow \infty.$$

Therefore, we obtain, as previously stated, an asymptotic power dependence in k_T . Let us analyze our results ^{*}, Eqs. (13), (14), for different kinematical situations. In the pionization region, i.e., $x = y = (k_T/\sqrt{s})$:

$$G(x, y) = F\left(\frac{\sqrt{s}}{k_T}\right) = c(\alpha, \lambda) \left(\frac{k_T}{\sqrt{s}}\right)^{1+\alpha} \int_0^{\sqrt{s}/k_T} (1-\omega \frac{k_T}{\sqrt{s}})^{2\lambda+\alpha} \omega^\alpha \chi(\omega) \chi\left(\frac{1-\omega \frac{\sqrt{s}}{k_T}}{k_T/\sqrt{s}}\right) d\omega \quad (15)$$

and, as said before, F approaches 1 for $(\sqrt{s}/k_T) \rightarrow \infty$. For $s \rightarrow \infty$ at fixed x (i.e., $(1/y) = (xs/k_T^2) \rightarrow \infty$) (asymptotic fragmentation region) :

$$\lim_{\substack{s \rightarrow \infty \\ x \text{ fixed}}} G(x, y) \equiv f(x) = \frac{1}{2} \chi\left(\frac{1}{x}\right) + \frac{c(\alpha, \lambda)}{2} x^{1+\alpha} \int_0^{1/x} (1-\omega x)^{2\lambda+\alpha} \omega^\alpha \chi(\omega) d\omega \quad (16)$$

^{*}) These results are of course similar to those of Ref. 2). Moreover, every field theoretical model would give asymptotic behaviour of this kind, purely on dimensional grounds.

From Eqs. (13), (14), we see that the distribution in the three variables s , x and k_T^2 (or s , x and y) is completely determined by the structure function $X(\omega)$. This function is of course determined by the eigenvalue problem together with the Regge intercept α . Both, however, are dependent on the details of the model. We will choose $\alpha = 1$, and for what regards $X(\omega)$ we expect a behaviour analogous to the structure functions measured at SLAC. It is true that at SLAC the measure concerns matrix elements of the electromagnetic current, while here we are concerned with the currents which are sources of our particles; however, the multiperipheral model gives both structure functions on similar grounds and therefore one expects an analogous behaviour for them.

To start the comparison with experiments, let us test whether the scaling law of Eqs. (13), (15) in the pionization region (and $k_T > 1.5$ GeV/c) is compatible with ISR data.

We can have first an indication on the power of k_T in Eq. (13) by comparing data at fixed $(\sqrt{s}/k_T)$. The data suggest a k_T^{-6} behaviour, i.e., $\lambda = \frac{1}{2}$ in Eq. (13).

We notice that such a behaviour gives a reasonable extension to lower k_T . As an indication we have verified that a function of the form $(k_T^2 + \mu^2/\mu^2)^{-3}$ with $\mu \sim 300$ MeV gives a rough description of the spectrum, normalized to 1 for $k_T = 0$, from $k_T = 0$ up to $k_T \sim 1.5$ GeV/c. To test the scaling we display in Fig. 2 the experimentally measured values of $f = (k_T/\mu)^6 [E(d\sigma/d^3k)/E(d\sigma/d^3k)|_{k_T=0}]$ as function of $(\sqrt{s}/k_T)$. The fact that the points that refer to the three different available energies follow the same pattern is an indication of the validity of the proposed scaling law.

The solid curve shown in Fig. 2 represents the scaling function F computed from Eq. (15) using the function $X(\omega)$ represented by the dashed line in the same Figure. We can thus reproduce the data with a function $X(\omega)$ that has the typical features expected for a structure function. As compared with the SLAC structure function, we see that our $X(\omega)$ is systematically smaller than $(\nu W_2(\omega))/(\nu W_2(\omega))$ for $\omega < 10$ *).

*) Differences of this kind are not surprising. νp structure functions, for instance, are also expected to be substantially smaller than $(\nu W_2(\omega))/(\nu W_2(\omega))$ on the basis of the Adler sum rule 5).

At first sight, it looks astonishing to find important non-asymptotic effects for $s \sim 2000 \text{ GeV}^2$. The reason is that even if s is large, for $k_T \sim 3 \text{ GeV}/c$ the value for the relevant variable is $(\sqrt{s}/k_T) \sim 15$. Furthermore, the function F (as seen in Fig. 2) has a very late asymptotism. Indeed in order for F to reach its asymptotic regime, both χ in the integrand of Eq. (15) must have reached their asymptotic limit.

The model has many distinctive features that are experimentally testable.

- A) - The asymptotic distribution in the fragmentation region is given by Eq. (16). The x and k_T behaviours are factorized. The x behaviour is given in Fig. 3 for the χ of Fig. 2^{*)}.
- B) - The asymptotic regime is reached faster in the fragmentation region than in the pionization region. As displayed in Eq. (16), the relevant variable which has to be large in the asymptotic limit is (xs/k_T^2) instead of $(\sqrt{s}/k_T)$. In Fig. 2 we show $G(x,y)$ for $x = 0.2$ as a function of $(\sqrt{s}/k_T)$ for our choice of $\chi(\omega)$.
- C) - Nevertheless non-asymptotic effects are observable also for $x \neq 0$ as exemplified in Fig. 3 where the lack of factorization in x and k_T^2 is displayed for $s = 2000 \text{ GeV}^2$.
- D) - Exclusive events with a large transverse momentum are expected to be similar to the usual ones (constant density in rapidity) except that a slice of length proportional to $\log k_T$ is replaced by two low mass systems with opposite k_T and with an arbitrary separation inside this gap. Due to the fact that a slice has in average no quantum numbers, we expect the two particles with large k_T to compensate their quantum numbers (e.g., baryon number and strangeness). The jet-like structure has been also suggested by the parton-like calculations²⁾, however, in these models the quantum numbers of the two large k_T jets are not expected to be correlated.
- E) - Let us discuss the nature of the particles with large transverse momentum. In the multiperipheral model there is a dynamical suppression of high mass particles for small values of k_T . This is due to the fact that the natural variable which controls the spectrum is $m_T^2 = p_T^2 + m^2$ and therefore a mass increase suppresses the spectrum as strongly as a k_T increase. This effect

*) Actually, the x dependence shown here is much more sensible to the choice of $\chi(\omega)$ than the function $F(\sqrt{s}/k_T)$ from which our χ has been tentatively determined. Therefore we attach only a qualitative meaning to the predictions shown in Fig. 3.

is washed out for large k_T and therefore we expect particles that are equally coupled to be equally emitted. Typically this should be the case for members of the same $SU(3)$ multiplet (as π and K). Low lying baryons, being strongly coupled ⁶⁾, are also expected to be abundant at large k_T . Keeping in mind also the kinematical fact that in the decay of a large k_T small Q resonance the heavier particles get the dominant share of the momentum, the observed scarcity of pions at large k_T appears quite natural.

F) - Our results for large k_T can be extrapolated to lower energies. The relevant contribution to Eq. (14) comes then from small values of ω , where χ is quite small : hence we expect a strong depressing factor. This is, in the framework of our model, the reason of the very small yield observed at PS energies. The depression factor turns out to be of the order of 10^{-3} and the deviation from the asymptotic curve is so large that the spectrum falls even more rapidly than an exponential.

G) - We have frequently stressed that our results are valid for large k_T .

For smaller values of the transverse momentum, it is more convenient to share it between the two adjacent propagators rather than to absorb it in only one. Therefore we expect that when $k_T/2$ is not much larger than $\langle k_T \rangle$, this will be the dominant contribution and consequently asymptotism will be reached much faster than for large k_T , so we understand quite naturally why there is no strong energy dependence for values of $k_T \lesssim 4 \langle k_T \rangle$.

ACKNOWLEDGEMENTS

We thank Marcello Ciafaloni for useful conversations.

REFERENCES

- 1) CERN - Columbia - Rockefeller Collaboration, and Saclay - Strasbourg Collaboration, as reported by M. Jacob - XVI International Conference on High Energy Physics, Batavia (1972), CERN Preprint TH. 1570 (1972).
- 2) S.M. Berman, J.D. Bjorken, and J.B. Kogut - Phys.Rev. D4, 3388 (1971) ;
J.F. Gunion, S.J. Brodsky and R. Blankenbecler - Phys.Rev. D6, 2652 (1972) ;
P.V. Landshoff and J.C. Polkinghorne - Cambridge DAMTP 72/43 (1972).
- 3) D. Amati, A. Stanghellini and S. Fubini - Nuovo Cimento 26, 896 (1962).
- 4) B.R. Webber - Phys.Rev.Letters 27, 448 (1971).
- 5) J.D. Bjorken and S.F. Tuan - Comments on Nuclear and Particle Physics V, 71 (1972).
- 6) L. Caneschi and A. Pignotti - Phys.Rev.Letters 22, 1219 (1969).

FIGURE CAPTIONS

Figure 1 The multiperipheral diagram considered.

Figure 2 Data for $\varphi(k_T) / \varphi(0)$
with $\varphi(k_T) = \left(\frac{.09 + k_T^2}{.09} \right)^3 \frac{E}{k^2} \frac{d^2 \sigma}{dk d\Omega}$

from Ref. 1), plotted as function of $(\sqrt{s}/k_T)$. The solid curve represents our results computed with the structure function $X(\omega)$ represented by the dashed curve as function of ω . The dotted curve is the predicted distribution as function of $(\sqrt{s}/k_T)$ for fixed $x = 0.2$.

Figure 3 Expected x dependence of the spectrum for $s = 2000 \text{ GeV}^2$ (dashed curves) and $s \rightarrow \infty$ (full curve).

Fig. 1

Fig. 2

- π^0 et $s = 950 \text{ GeV}^2$
- × $\pi^\pm$ et $s = 2000 \text{ GeV}^2$
- π^0 et $s = 2850 \text{ GeV}^2$

Fig. 3