CM-P00058851

Ref.TH.1506-CERN

ISOSPIN INVARIANCE CONSTRAINTS, FACTORIZATION AND EXCHANGE
DEGENERACY IN INCLUSIVE PROCESSES

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A B S T R A C T

Isospin constraints are combined with the criterion for rapid scaling in inclusive reactions ($AB\bar{C}$ exotic), factorization and exchange degeneracy. With these assumptions discrepancies arise for reactions like antibaryon + baryon $\rightarrow$ meson + X. Relations are derived which could distinguish, when experimentally tested, between existing criteria for rapid scaling.

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1. RELATIONS DUE TO ISOSPIN INVARIANCE

Isospin invariance has recently been applied by several authors ¹⁾⁻⁴⁾, and in different ways, in order to obtain relations for single and multi-particle inclusive cross-sections. The approach and constraints derived by Llewellyn Smith and Pais ³⁾ seem to be more appropriate to our purpose.

Let us assume, at first, that for the reaction

the initial state has definite isospin I^{AB} with third component I_3^{AB} . In (1) X stands for the whole set of undetected particles. If C is a member of a multiplet of isospin I^C , then isospin invariance for reaction (1) allows only certain isospin values I^X for X . These values are determined by the triangular inequalities like

$$|I^C - I^X| \leq I^{AB} \leq I^C + I^X \quad (2)$$

If C has definite third component I_3^C , there will be as many amplitudes for (1) as there are allowed values for I^X from the inequalities (2). Each of these amplitudes will be equal to a reduced amplitude depending on I^{AB} , I^C and I^X multiplied by the corresponding Clebsch-Gordan coefficient. Summing up the squares of these amplitudes one obtains the inclusive cross-section of (1). Repeating the same analysis for all C belonging to the I^C multiplet, one ends up with the constraints connecting the $2I^C + 1$ cross-sections.

As an example consider

Here I^X can only be 0, 1 or 2, so we can define three reduced amplitudes. Denoting by $d\sigma_{K^+}(\pi^+)$ the cross-section for production of π^+ in (3) we are led finally to the inequalities ^{1),3)}:

$$6 d\sigma_{K^+}(\pi^+) + 2 d\sigma_{K^+}(\pi^-) \gg 6 d\sigma_{K^+}(\pi^0) \gg 3 d\sigma_{K^+}(\pi^-) \quad (4)$$

If, however, the initial state has more than one isospin component, the above analysis has to be modified in order to account for the interference terms. In the particular case of

$$K^0 + p \rightarrow \pi + X \quad (5)$$

interference terms, of equal magnitude but opposite sign, appear only for $I^X = 1$ and in π^+ and π^- production. Since K^+ and K^0 belong to the same isodoublet, one obtains a relation ³⁾ for the cross-sections (3) and (5)

$$\begin{aligned} d\sigma_{K^+}(\pi^+) + d\sigma_{K^+}(\pi^-) - 2d\sigma_{K^+}(\pi^0) &= \\ &= -d\sigma_{K^0}(\pi^+) - d\sigma_{K^0}(\pi^-) + 2d\sigma_{K^0}(\pi^0) \end{aligned} \quad (6)$$

Relations (4) and (6) are model-independent constraints manifesting isospin invariance for inclusive reactions. They will prove very useful in testing different assumptions that are currently proposed in order to understand some features of the inclusive spectra. We should stress that relations (4) and (6) can be transcribed to any other pair of isodoublets. In particular, Eq. (6) is symmetric in interchanging $d\sigma_{K^+}(\pi^+, \pi^0, \pi^-) \leftrightarrow d\sigma_{K^0}(\pi^+, \pi^0, \pi^-)$ and applies to the pairs $(\bar{K}^0, K^-)$, (p, n) and $(\bar{n}, \bar{p})$ in (3) and (5) respectively. Inequality (4) for K^+ applies also for $\bar{K}^0$, p and $\bar{n}$ in (3). We also note that (4) and (6) do not only hold for cross-sections but, being internal symmetry relations, apply equally to distribution functions at a given energy and for any fixed value of particle C momentum.

2. RAPID SCALING AND FACTORIZATION

The invariant single-particle distribution for reaction (1) at incoming energy s is

$$f(s, \vec{p}_C) = E_C \frac{d^3\sigma}{d^3p_C} \quad (7)$$

where $\vec{p}_C$ and E_C are the three-momentum and energy of C. In the fragmentation region of B (see Fig. 1a), f can be expressed as a sum of two terms: an energy independent Pomeron exchange term ($\mathbb{P}$), and an energy dependent one due to secondary Regge exchanges (collectively denoted by R). According to Chan et al. ⁵⁾ rapid scaling occurs (e.g., the R contribution vanishes) when $AB\bar{C}$ is exotic. (This criterion has been supplemented ⁶⁾ by the condition that $B\bar{C}$ should not be exotic.) Factorization of the

Pomeron in the fragmentation region has been tested in several reactions, Refs. 7), 8), and has been reasonably verified. Factorization of the secondary trajectories has recently been investigated ^{9),10)}. A successful description of several hadronic inclusive reactions ⁹⁾ and inclusive photo-production ¹⁰⁾ data has been obtained. In this description several assumptions have been made. It seems therefore reasonable to pursue further the implications arising from these assumptions, and combine them with the isospin invariance constraints.

We consider reactions where pions are produced as fragments of proton targets ($B=p$, $C=\pi$). We denote the Pomeron-beam (A) and Regge-beam vertices by γ^A and g_R^A respectively. Accordingly, we denote the Pomeron-fragmentation ($B\bar{C}$) and Regge-fragmentation vertices by $\gamma(\bar{C})$ and $g_R(\bar{C})$ respectively (see Fig. 1a). Duality, absence of exotic exchanges in exclusive two-body scattering combined with the Chan et al. ⁵⁾ criterion and factorization have been exploited ^{9),10)} to derive a set of relations between vertex functions. For $A=\pi^+$, π^- , K^+ , K^0 , K^- and $\bar{K}^0$ and $C=\pi^+$, π^- and π^0 while B being a proton the relations are:

$$g_p^K = g_\omega^K = g_{A_2}^K = g_f^K \equiv g^K \quad (8a)$$

$$g_p^\pi = g_f^\pi \equiv g^\pi \quad (8b)$$

$$g_p^N = g_{A_2}^N \quad ; \quad g_f^N = g_\omega^N \quad ; \quad (g_p^N \neq g_f^N) \quad (8c)$$

$$g_p(\pi^+) = g_\omega(\pi^+) = g_{A_2}(\pi^+) = g_f(\pi^+) \equiv g(\pi^+) \quad (9a)$$

$$g_p(\pi^-) = g_{A_2}(\pi^-) \quad ; \quad g_f(\pi^-) = g_\omega(\pi^-) \quad (9b)$$

$$g_p(\pi^0) = g_{A_2}(\pi^0) \quad ; \quad g_f(\pi^0) = g_\omega(\pi^0) \quad (9c)$$

At this point we note that the assumptions used to obtain (8) and (9), are leading to inconsistencies if A is nucleon or antinucleon. For $A=p$, n , $\bar{p}$ and $\bar{n}$, $B=p$ and $C=\pi^-$ the Regge term of $f(s, \vec{p}_C)$ (which we call f^R) will be:

$$f^R_{\pi^+}(pp \rightarrow \pi^- X) = S^{\frac{1}{2}} \left[-g_p^N g_p(\pi^+) - g_w^N g_w(\pi^+) + g_{A_2}^N g_{A_2}(\pi^+) + g_f^N g_f(\pi^+) \right] \quad (10a)$$

$$f^R_{\pi^+}(np \rightarrow \pi^- X) = S^{\frac{1}{2}} \left[+g_p^N g_p(\pi^+) - g_w^N g_w(\pi^+) - g_{A_2}^N g_{A_2}(\pi^+) + g_f^N g_f(\pi^+) \right] \quad (10b)$$

$$f^R_{\pi^+}(\bar{p}p \rightarrow \pi^- X) = S^{\frac{1}{2}} \left[+g_p^N g_p(\pi^+) + g_w^N g_w(\pi^+) + g_{A_2}^N g_{A_2}(\pi^+) + g_f^N g_f(\pi^+) \right] \quad (10c)$$

$$f^R_{\pi^+}(\bar{n}p \rightarrow \pi^- X) = S^{\frac{1}{2}} \left[-g_p^N g_p(\pi^+) + g_w^N g_w(\pi^+) - g_{A_2}^N g_{A_2}(\pi^+) + g_f^N g_f(\pi^+) \right] \quad (10d)$$

$AB\bar{C}$ is exotic in (10a), (10b) and (10d) while $B\bar{C}$ is non-exotic in all four reactions. Therefore, we expect f^R to vanish in (a), (b) and (d). This implies $g_p^N = g_f^N$ which is inconsistent with (8) and far away from the experimental value $g_p^N \approx 5g_f^N$ (11). The situation is reminiscent of the failure to accommodate factorization with duality and absence of exotics for baryon-antibaryon scattering (12). As in that case, we cannot draw an inclusive dual diagram with no exotics for reaction (10c) (see Fig. 1b).

We mention further that when the number of baryons (or anti-baryons) involved in $AB\bar{C}$ increases, one should be cautious about any criterion for early scaling. For instance, in the fragmentation region in pp reactions, it is found that scaling is reached for π^+ already at an incident energy of about 12 GeV/c, while for π^- scaling starts at higher energy (13). But if one considers $pp \rightarrow \bar{p}X$ a rapid scaling should be expected, according to all existing criteria (14). In contrast, a strong energy dependence of the cross-section for $\bar{p}$ production persists up to the ISR data (13).

3. ISOSPIN CONSTRAINTS ON FRAGMENTATION VERTICES

We are in a position now to apply the isospin relations (4) and (6) to the distribution function $f(S, \vec{p}_C)$. For reaction (1), and taking B always a proton, we denote this function by $f_A(C)$.

Employing the vertex relations (8) and (9) we obtain, by letting A be the isodoublet (K^0, K^+)

$$f_{K^+}(\pi^+) = f_{K^0}(\pi^+) = \gamma^K \gamma(\pi^-) \quad (11a)$$

$$f_{K^+}(\pi^0) = f_{K^0}(\pi^0) = \gamma^K \gamma(\pi^0) \quad (11b)$$

$$f_{K^+}(\pi^-) = f_{K^0}(\pi^-) = \gamma^K \gamma(\pi^+) \quad (11c)$$

From (6) then we derive:

$$\gamma(\pi^+) + \gamma(\pi^-) = 2\gamma(\pi^0) \quad (12)$$

For A being the isodoublet $(K^-, \bar{K}^0)$ besides the Pomeron there will be Regge contributions. Making use of (12) and (6) we obtain the relation for the Regge fragmentation vertices:

$$g_f(\pi^+) + g_f(\pi^-) = 2g_f(\pi^0) \quad (13)$$

Inequality (4) for K^+ , combined with (12), can be sharpened to:

$$3f_{K^+}(\pi^+) \geq f_{K^+}(\pi^-) \quad (14)$$

or

$$3\gamma(\pi^-) \geq \gamma(\pi^+) \quad (14')$$

From the above relations it is clear that π^0 distributions are needed for any further analysis. There are indeed experiments in progress, and, hopefully, such single particle distributions will be soon available. In that case, for the beam A being K^+ , K^- , π^+ or π^- both (12) and (13) could be checked. If this were the case, this would give strong support for the validity of the assumptions made in deriving (8) and (9). We would not expect, however, the secondary trajectory relation (13) to hold for A being p or $\bar{p}$. Therefore, for intermediate energies the relation

$$f_p(\pi^+) + f_p(\pi^-) = 2f_p(\pi^0) \quad (15)$$

should not be verified. At high energies, of course, where the Pomeron contribution prevails, Eq. (15) should hold as a result of (12). (Note, however, the caution for the number of baryons in $AB\bar{C}$.) Relations (12)-(14) can be of further use to distinguish between some of the existing criteria for weak energy dependence of distribution functions in the fragmentation region. Ellis et al.¹⁴⁾ claim that early scaling occurs if $AB\bar{C}$ and AB are exotic, while for Einhorn et al.¹⁴⁾ the condition is AB and $A\bar{C}$ to be exotic. According to Einhorn et al., the relations (11) do not hold, therefore, we would expect (12) not to be valid, and

$$f_{K^+}(\pi^+) + f_{K^+}(\pi^-) \neq 2 f_{K^+}(\pi^0) \quad (16)$$

Consider now the Ellis et al., criterion. Again we would obtain relations (11), (12), (13) and (14). We could, furthermore, accommodate in the scheme factorization for $A=p, \bar{p}, n$ and $\bar{n}$ since for $\bar{n}p \rightarrow \pi^- X$ secondary trajectory contributions do not vanish. Accordingly we expect (15) to be true even at low energies.

The above arguments indicate that π^0 distributions could be of some help in understanding several features of the inclusive processes.

ACKNOWLEDGEMENT

I wish to thank Drs. H-M. Chan, P.H. Frampton, F. Hadjioannou, J. Iliopoulos and H.I. Miettinen for helpful discussions. I am grateful to the CERN Theoretical Study Division for the hospitality extended to me while this work was done.

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- a) Diagram for reaction $A + B \rightarrow C + X$ where C is the fragment of B . In the diagram, Pomeron ($\mathbb{P}$) and Regge (R) exchanges contribute.
- b) It is not possible to construct an inclusive dual diagram for an antibaryon ($\bar{B}$) + baryon (B) $\rightarrow$ meson (M) + X reaction.

FIG.1