QUARK-LEPTON SYMMETRY AND COMPLEMENTARITY

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We argue that the difference between the observed approximate quark-lepton complementarity and the theoretical prediction based on realistic quark-lepton symmetry within the seesaw mechanism may be adjusted by means of a triplet contribution in the seesaw formula.
I. INTRODUCTION

It is well known that the (type I) seesaw mechanism \[1\] is able to explain the smallness of neutrino mass. In fact, for a single fermion generation, the effective neutrino mass \(m_{\nu}\) is given by
\[
m_{\nu} \simeq \left(\frac{m_D}{m_R}\right)m_D,
\]
where the Dirac mass \(m_D\) is of the order of the quark (or charged lepton) mass, and the right-handed Majorana mass \(m_R\) is of the order of the unification or intermediate mass scale. As a result, the effective mass comes out very small with respect to the Dirac mass.

The (type I) seesaw mechanism may as well explain the existence of some large lepton mixings \[2\]. In fact, for three fermion generations, the effective neutrino mass matrix \(M_\nu\) is given by the formula
\[
M_\nu \simeq M_T^D M_R^{-1} M_D, \tag{1}
\]
so that large neutrino mixing can be generated from a nearly diagonal \(M_D\) by means of a strong mass hierarchy or large off-diagonal elements in \(M_R\) \[2, 3, 4, 5\]. A small contribution to lepton mixing from the charged lepton mass matrix \(M_e\) is also expected and could be important to understand the deviation from maximal mixing \[6, 7\].

On the other hand, the triplet contribution to the seesaw formula, leading to the so-called type II seesaw mechanism \[8\], is probably present (see for example \[9\]). The type I seesaw mechanism is based on the introduction, within the standard model, of three heavy right-handed neutrinos. However, small neutrino masses can be generated also by the inclusion of a heavy Higgs triplet \[8, 10\]. In this case, the neutrino mass matrix is given by \(M_\nu = M_L = Y_L v_L\), where \(Y_L\) is a Yukawa matrix and \(v_L\) is the v.e.v. of the triplet, which can be written as
\[
v_L = \gamma v^2/m_T, \tag{2}
\]
with \(v\) the v.e.v. of a standard Higgs doublet, \(m_T\) the triplet mass, and \(\gamma\) a coefficient related to the coupling between the doublet and the triplet. Then, \(v_L\) is small with respect to \(v\), but large mixing in \(M_\nu\) is achieved by hand. Instead, from the type I seesaw formula we get
\[
M_\nu \simeq Y_T^D M_R^{-1} Y_D v^2,
\]
so that large mixing can be generated from the structure of both matrices \(Y_D\) and \(M_R\). Therefore, we can write the type II seesaw formula by adding to the usual type I term the triplet (or type II) term, so that
\[
M_\nu \simeq M_T^D M_R^{-1} M_D + M_L. \tag{2}
\]
The triplet contribution by alone can explain the smallness of neutrino mass but not the existence of large lepton mixings. Nevertheless, it can produce important effects on such mixings within the type II seesaw.

It has also been suggested that the generation of maximal mixings by means of the type I seesaw mechanism could be natural \[11\]. Then, the realistic quark-lepton
symmetry \[12\] is not consistent with the approximate quark-lepton complementarity which is observed in quark and lepton mixings \[13\], that is
\[
\theta_{12} + \vartheta_{12} \simeq \frac{\pi}{4},
\]
(3)
where \(\theta_{12}\) is the 1-2 quark mixing angle (the Cabibbo angle) and \(\vartheta_{12}\) is the 1-2 lepton mixing angle (the solar neutrino mixing angle). The central point of our paper is that the triplet contribution in the type II seesaw formula can indeed correct such a disagreement. We perform an \(SO(10)\) inspired study, where both the realistic quark-lepton symmetry and the relation between \(M_L\) and \(M_R\) are well motivated.

II. FRAMEWORK

Quark mixing and lepton mixing are quite different from each other. The quark mixing matrix \(V\) is called the CKM matrix, the lepton mixing matrix \(U\) is called the MNS matrix. Now, for the quark mixing we have \[14\]
\[
V_{12} = 0.221 - 0.227
\]
\[
V_{23} = 0.039 - 0.044
\]
\[
V_{13} = 0.0029 - 0.0045
\]
and for the lepton mixing \[14\]
\[
U_{12} = 0.48 - 0.62
\]
\[
U_{23} = 0.58 - 0.84
\]
\[
U_{13} < 0.22.
\]
Therefore, the largest quark mixing is smaller or at most similar to the smallest lepton mixing. As we said, the seesaw mechanism could be the origin of this different behaviour.

Quark and lepton mixings come out from the diagonalization of quark and lepton mass matrices. A typical expression for the quark mass matrices is given by \[15, 16\]
\[
M_u \simeq \begin{pmatrix}
0 & \lambda^6 & \lambda^6 \\
\lambda^6 & \lambda^4 & \lambda^4 \\
\lambda^6 & \lambda^4 & 1
\end{pmatrix}
m_t,
\]
(4)
\[
M_d \simeq \begin{pmatrix}
0 & \lambda^3 & \lambda^3 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}
m_b,
\]
(5)
where $\lambda \simeq 0.2$ and coefficients (not written) are close to one. According to the quark-lepton symmetry, as within the $SO(10)$ model, $M_e \sim M_d$, $M_D \sim M_u$, and including the $-3$ factor of Georgi and Jarlskog [12], which gives better the charged lepton masses, we get the following lepton mass matrices,

$$M_e \simeq \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & -3\lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_b,$$

(6)

$$M_D \simeq \begin{pmatrix} 0 & \lambda^6 & \lambda^6 \\ \lambda^6 & -3\lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} m_t,$$

(7)

where $M_D$ is the Dirac neutrino mass matrix. The $-3$ factor is due to the contribution of the $126$ representation. The other entries are due to the $10$ representation.

It was noted some years ago [5] that in the case of normal hierarchy of neutrinos the inverse of the neutrino mass matrix has all entries of the same order of magnitude. Then, by inverting the type I seesaw formula, $M_R \simeq M_D M_\nu^{-1} M_D^T$, we can determine the (Majorana) mass matrix of the right-handed neutrinos, and we yield

$$M_R \simeq \begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^6 \\ \lambda^{10} & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} m_R.$$

(8)

By applying again the direct seesaw formula we obtain the effective neutrino mass matrix

$$M_\nu^I \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \frac{m_2}{m_R},$$

(9)

These mass matrices form our framework for the type I seesaw mechanism. In the next section we are going to study its prediction and include the type II term.
III. ANALYSIS

In order to explore the consequences of the foregoing framework, we should consider the following form of the type I term

\[
M^I_\nu \simeq \begin{pmatrix}
\lambda^4 & \lambda^2 & \lambda^2 \\
\lambda^2 & 1 + \frac{4}{2} & 1 - \frac{4}{2} \\
\lambda^2 & 1 - \frac{4}{2} & 1 + \frac{4}{2}
\end{pmatrix} \frac{m_i^2}{m_R}.
\] (10)

In fact, the matrix (9) is of course approximate, and may provide different values for the mixing angles. Nevertheless, we first assume maximal 2-3 mixing, and then the bimaximal mixing matrix \(U_\nu\) (that is the 1-2 mixing also maximal) is obtained for \(A = \lambda^4\). When we include the effect of the charged lepton mass matrix on the mixing, \(U_e^\dagger U_\nu\), we get

\[
U_{12} \simeq \frac{1}{\sqrt{2}} + \frac{\lambda}{6}
\] (11)

\[
U_{23} \simeq \frac{1}{\sqrt{2}} - \frac{\lambda^2}{\sqrt{2}}
\] (12)

\[
U_{13} \simeq \frac{\lambda}{3\sqrt{2}}
\] (13)

so that \(U_{12}\) is out of the experimental range and in particular is not consistent with the quark-lepton complementarity relation (3).

However, we have also to include the impact of the triplet term \(M^{II}_\nu = M_L\), and we would like to take \(M_L\) proportional to \(M_R\), so that

\[
M^{II}_\nu = M_L = \frac{m_L}{m_R} M_R.
\] (14)

This relation is indeed present in left-right models (see for example \[17\]), including the \(SO(10)\) model (see \[9\]). In fact, both \(M_R\) and \(M_L\) are generated by the \(126\) representation. We set \(m^{II}_\nu/m^I_\nu = k\), the ratio between the overall scales or the type I and type II terms,

\[
k = \frac{m_L m_R}{m_i^2} = \gamma \frac{m_R}{m_T}.
\] (15)

Then we get \(U_\nu\) from the diagonalization of \(M^I_\nu + M^{II}_\nu\), and again \(U_e\) from the diagonalization of \(M_e\).

The effect of \(M^{II}_\nu\) is that to decrease the mixings \[18\] in such a way that, with the contribution from \(U_e\), for a certain range of \(k\), \(U_{12}\) falls again within the experimental
range (the impact on $U_{23}$ and $U_{13}$ of our triplet term is almost negligible). The range of $k$ we found is in fact

$$0.08 < k < 0.18.$$  \hspace{1cm} (16)

For larger $k$, $U_{12}$ is too small, and for smaller $k$, $U_{12}$ is too large. We predict $\vartheta_{23}$ nearly maximal, and $\vartheta_{13} \simeq \theta_{12}/3\sqrt{2} \simeq 0.05$, which can be checked by future experiments.

**IV. CONCLUSION**

We have proposed that a contribution from the triplet term in the type II seesaw mechanism is important to reconcile the observed quark-lepton complementarity (3) with the realistic quark-lepton symmetry. We have assumed that the type I term gives the bimaximal neutrino mixing and the triplet term is proportional to the right-handed neutrino mass matrix. This framework is well compatible with the unified $SO(10)$ model. Our study is in some sense the opposite of the one performed in Ref.\cite{17}, where the bimaximal mixing comes from the type II term. Also other choices of the two seesaw terms have been considered, see for example Ref.\cite{19}. Finally we note that the explicit inclusion of phases in the charged lepton mass matrix has an impact on the quark-lepton complementarity \cite{20}, thus the contribution of the triplet term may be reduced or enhanced by this presence.
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