511 keV line from Q balls in the Galactic Center

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The 511 keV photons from the galactic center can be explained by positrons produced through Q-ball decay. In the scheme of gauge-mediated supersymmetry breaking, large Q balls with lepton charge are necessarily long-lived. In particular, the lifetime can be as long as (or even longer than) the age of the universe. If kinematically allowed, such large Q balls decay into positrons, which eventually annihilate with electrons into 511 keV photons. Our scenario is realized within the minimal supersymmetric standard model in the inflationary universe, which is very plausible.

I. INTRODUCTION

The 511 keV gamma-ray line emission from the galactic bulge was observed by the spectrometer SPI on the International Gamma-Ray Astrophysics Laboratory (INTEGRAL) \cite{1}. With the unprecedented resolution, it revealed that the emission comes from spread (or multiple point) sources, which makes it very difficult to explain with traditional astrophysical objects, such as neutron stars, black holes, radioactive nuclei from supernovae, novae, red giants, and Wolf-Rayer stars, cosmic ray instars, black holes, radioactive nuclei from supernovae, point sources, which makes it very difficult to explain.

One of the alternative candidates for the 511 keV line was provided by annihilating or decaying dark matter whose decay products contain positrons \cite{2,3}. In order to have the observed flux of the 511 keV line emission, decaying dark matter (DDM) scenario may be characterized into the two extreme cases. One is that the DDM accounts for the present matter density of the universe ($\rho_{\text{ddm}} \sim \rho_m$), with extremely long lifetime ($\tau_{\text{ddm}} \gg t_0$). The other has lifetime as long as the age of the universe ($\tau_{\text{ddm}} \sim t_0$), while its abundance is very small ($\Omega_{\text{ddm}} \ll 1$). The observed flux of the 511 keV line emission is in general related to the abundance and the lifetime of the decaying particle as

$$\frac{\Phi_{511}}{10^{-3} \text{cm}^2/\text{sec}} \approx \Omega_{\text{ddm}} \left(\frac{\tau_{\text{ddm}}}{10^{27} \text{sec}}\right)^{-1} \left(\frac{m_{\text{ddm}}}{\text{MeV}}\right)^{-1},$$

where we assume spherically symmetric profile for dark matter density with $\rho \propto r^{-1.2}$. In order to achieve this condition, the scenarios proposed thus far need somewhat artificial and \textit{ad hoc} assumptions about couplings, and sometimes huge dilution by late entropy production.

In this article, we show instead more natural candidate for the source of positrons in the galactic bulge: Q balls \cite{4} in the minimal supersymmetric standard model (MSSM). Throughout this paper we assume the gauge-mediated supersymmetry (SUSY) breaking. Our promising candidate is a large Q ball with the charge being the lepton number. The large charge enables Q balls not only to be long-lived but also to have small mass per unit charge. Due to these features, we can successfully explain the origin of the galactic positrons with relatively small energy ($\lesssim O(10)$ MeV) \cite{4}.

The organization of the article is as follows. In the next section, we review several properties of two different types of Q balls in the MSSM with gauge-mediated SUSY breaking. In Sec. II, we estimate how large the charge should be for the Q balls to be enough long-lived. The 511 keV gamma ray flux from the Q-ball decay will be calculated in Sec. III. Sec. IV is devoted for the Q-ball formation processes and a possible way to give the small enough abundance of the Q balls. Finally, we give conclusions in Sec. V.

II. Q BALLS IN THE GAUGE-MEDIATED SUSY BREAKING

A Q ball is a nontopological soliton composed of a complex scalar field $\Phi$, and it minimizes the energy of the scalar field with a fixed $U(1)$ charge $Q$. In MSSM, this charge is actually some combination of baryon and lepton numbers. Since our aim here is to explain the positron flux, we identify the $U(1)$ charge with the lepton number. Such Q balls are often called ‘L balls’.

MSSM contains many flat directions along which the scalar potential vanishes at the level of renormalizable operators in the global SUSY limit. The flat directions generally consist of squarks, sleptons and higgs \cite{5}, and are fully classified in Ref. \cite{7}. Here we consider the leptonic directions such as $eL\ell$. The scalar potential is lifted by the SUSY breaking effects and presumably by nonrenormalizable operators in the superpotential. Since the existence of the nonrenormalizable operators is irrelevant to the following discussion, we will drop them for simplicity. In fact they can be forbidden in the presence of the discrete gauge symmetry. Also, for the moment, let us concentrate on the $U(1)$ conserving part of the scalar potential, since $U(1)$ violating interactions are not important for determining properties of the Q balls (however, see Ref. \cite{8}).

In the gauge-mediated SUSY breaking, the scalar potential of the flat direction is parabolic with the messenger scale $M_S$, while, above that scale, the potential
grows logarithmically:

\[ V_{\text{gauge}}(\Phi) \sim \begin{cases} m_\phi^2 |\Phi|^2 & (|\Phi| \ll M_S) \\ M_F^2 \left( \log \frac{|\Phi|^2}{M_S^2} \right)^2 & (|\Phi| \gg M_S) \end{cases} \tag{2} \]

where \( m_\phi \) is a soft breaking mass \( \sim O(\text{TeV}) \), and \( M_F \) and \( M_S \) are related to the \( F \)- and \( A \)-components of a gauge-singlet chiral multiplet \( S \) in the messenger sector as

\[ M_F^2 = \frac{g^2}{(4\pi)^4} (F_S)^2, \quad M_S = \langle S \rangle. \tag{3} \]

Here \( g \) generically stands for the standard model gauge coupling. The masses of the sparticles in the visible sector is given by \( m \sim g^2 \Lambda_{\text{mess}}/(4\pi)^2 \), where \( \Lambda_{\text{mess}} = \langle F_S \rangle / \langle S \rangle \sim 10^5 \text{ GeV} \). Noting that \( (F_S)^{1/2} \sim \langle S \rangle \) should not be necessarily satisfied, the allowed range for \( M_F \) is

\[ 10^3 \text{GeV} \lesssim M_F \lesssim \frac{g^{1/2}}{4\pi} \sqrt{m_{3/2} M_P}, \tag{4} \]

where \( m_{3/2} \) is the gravitino mass and \( M_P = 2.4 \times 10^{18} \) GeV is the reduced Planck mass.

In addition to the gauge-mediated effects, there is always the gravity, which also mediates the SUSY breaking, leading to

\[ V_{\text{grav}}(\Phi) = m_{3/2}^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M_F^2} \right) \right] |\Phi|^2. \tag{5} \]

Here we include the one-loop corrections to the mass term. For most flat directions, the numerical coefficient \( K \) is negative and varies between \(-0.1\) and \(-0.01\). The scalar potential is given by the sum of \( V_{\text{gauge}} \) and \( V_{\text{grav}} \). Since the gravitino mass is much smaller than the weak scale, \( V_{\text{grav}} \) dominates only at large field amplitudes.

The Q-ball solution exists if and only if \( V(\phi)/\phi^2 \) has a minimum at \( \phi \neq 0 \), where \( \phi \equiv \sqrt{2} |\Phi| \) is the radial part of \( \Phi \). In other words, the effective potential must be shallower than \( \phi^2 \). The gauge-mediation potential \( V_{\text{gauge}} \), as well as the gravity-mediation potential \( V_{\text{grav}} \) with negative \( K \), satisfies this criterion. Therefore, there are two types of the Q ball in the gauge-mediated SUSY breaking models, depending on the value of \( \phi \). If the field amplitude is smaller than \( \phi_{\text{eq}} \sim M_F^2/m_{3/2} \), the gauge-mediation potential dominates. We call this the gauge-mediation (GM) type Q ball. Its typical charge is related to the field amplitude at which the flat direction \( \Phi \) starts to oscillate by \( Q \approx \beta_{\text{gauge}} \left( \frac{\phi_{\text{eq}}}{M_F} \right)^4 \),

where \( \beta_{\text{gauge}} \approx 6 \times 10^{-4} \). The mass and the size of the GM-type Q ball are, respectively,

\[ M_Q \sim M_F Q^{3/4}, \quad R_Q \sim M_F^{-1} Q^{1/4}. \tag{7} \]

The mass per unit charge, \( m_Q = M_Q/Q \), can be very small for large enough \( Q \). If \( m_Q \) exceeds the mass of some particles carrying lepton number, the L ball can decay into those particles. Thus, in contrast to the Q balls having the baryon number, L balls can always decay into neutrinos, unless \( Q \) is so large to render \( m_Q \) smaller than the neutrino mass.

On the other hand, if the field amplitude is larger than \( \phi_{\text{eq}} \), the gravity-mediation potential dominates, and ‘new-type’ Q balls are generated. The typical charge is

\[ Q \approx \beta_{\text{grav}} \left( \frac{\phi_{\text{eq}}}{m_{3/2}} \right)^2, \tag{8} \]

where \( \beta_{\text{grav}} \approx 6 \times 10^{-3} \). The mass and the size are

\[ M_Q \approx m_{3/2} Q, \quad R_Q \approx m_{3/2}^{-1} |K|^{-1/2}, \tag{9} \]

respectively. The mass per unit charge is now independent of \( Q \), and given by the gravitino mass: \( m_Q \sim m_{3/2} \). The decay particle species now depend on the gravitino mass. To produce positrons in the galactic center, \( m_{3/2} \) should be larger than the positron mass, 511 keV. Since there is a variety of the gauge-mediated SUSY breaking scenarios with the gravitino mass varying from \( O(\text{GeV}) \) well down to \( O(\text{keV}) \), such a requirement can be easily satisfied.

### III. Q-BALL DECAY AND POSITRON PRODUCTION

As mentioned in the previous section, we are interested in the positron production from L balls. The decay products generally contain neutrinos, charged leptons, and their antiparticles. In our case, the mass per unit lepton number, \( m_Q \), should be larger than 511 keV, in order to produce positrons. In this section let us derive a condition on each type of Q-balls to have lifetimes longer than the present age of the universe, with \( m_Q \sim \text{MeV} \) fixed.

The Q-ball decay takes place only through its surface because of the Pauli blocking and the large effective masses of the coupled particles well inside the Q ball. In the case of L balls, the decay proceeds via gaugino exchange into a pair of leptons. The charge decreasing rate is constrained from above,

\[ -\frac{dQ}{dt} \leq \frac{m_Q^3}{192\pi^2 A}, \tag{10} \]

‡ We describe the dynamics of the flat direction \( \Phi \) and Q-ball formation in Sec. 5.

† Branching ratio to photons can be extremely suppressed due to the very small left-right slepton mixings.
where $A = 4\pi R_Q^2$ is the surface area of the Q ball. The actual decay rate of the L balls should be close to this upper bound. We consider the two types of Q balls one by one.

**A. GM-type Q balls**

The decay rate of the GM-type Q ball reads

$$\Gamma_Q = -\frac{1}{Q}\frac{dQ}{dt} \sim \frac{M_F}{48\pi Q^{5/4}}.$$  \hspace{1cm} (11)

As mentioned in Introduction, the life time of the L ball must be as long as, or longer than, the present age of the universe, $t_0 \sim 13$ Gyr. To this end, a huge charge is necessary:

$$Q \gtrsim 10^{36} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{4/5}. \hspace{1cm} (12)$$

In addition, the mass per unit charge must be around $O(\text{MeV})$, in order for the L balls to emit positrons:

$$Q \sim \left( \frac{M_F}{\text{MeV}} \right)^4 \sim 10^{36} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^4. \hspace{1cm} (13)$$

These two conditions meet when $Q \sim 10^{36}$ and $M_F \sim 10^6$ GeV. Notice that the value coincides with the upper limit of $M_F$ in Eq. (11).

On the other hand, $\phi_{\text{osc}}$ should be smaller than $\phi_{\text{eq}}$ to produce GM-type Q-balls:

$$\phi_{\text{osc}} \leq \phi_{\text{eq}} \sim \frac{M_F}{m_{3/2}} \lesssim 10^{-3} M_F, \hspace{1cm} (14)$$

where we used Eq. (11) in the last inequality. Thus, the charge of the Q ball is bounded from above $^5$

$$Q \lesssim \beta_{\text{gauged}} \left( \frac{\phi_{\text{eq}}}{M_F} \right)^4 \lesssim 6 \times 10^{32}, \hspace{1cm} (15)$$

where we have substituted $M_F \sim 10^6$ GeV. It is therefore impossible to obtain large enough charge of $10^{36}$, forcing us to conclude that the GM-type of the Q ball cannot account for the observed 511 keV gamma ray, because of its too short lifetime.

**B. New-type Q balls**

For the new-type Q ball, the decay rate is given by

$$\Gamma_Q \sim \frac{m_{3/2}}{48\pi |K| Q}. \hspace{1cm} (16)$$

In order for the new-type Q balls to live longer than the age of the universe, its charge should satisfy

$$Q \gtrsim 4 \times 10^{37} \left( \frac{0.1}{|K|} \right) \left( \frac{m_{3/2}}{\text{MeV}} \right). \hspace{1cm} (17)$$

Equivalently, from Eq. (15), the field amplitude at which the flat direction starts to oscillate is bounded below,

$$\phi_{\text{osc}} \gtrsim 10^{17}\text{GeV} \left( \frac{0.1}{|K|} \right) \left( \frac{m_{3/2}}{\text{MeV}} \right)^{1/2}. \hspace{1cm} (18)$$

Such a large field value can be naturally attained if non-renormalizable terms in the superpotential are forbidden by some symmetry. In the following argument, let us concentrate on the new-type Q-ball as a possible source for the positrons in the galactic bulge.

**IV. POSITRONS FLUX FROM GALACTIC CENTER**

The relation between the 511 keV gamma-ray flux and the properties (i.e., lifetime, mass, and abundance) of the decaying dark matter is given by Eq. (14). If the charge $Q$ is as large as $10^{47}$, the new-type Q ball has lifetime as long as $10^{27} \text{sec}$ and accounts for most of the dark matter. However, the maximal possible charge of the new-type Q ball is attained when $\phi_{\text{osc}} \sim M_F$:

$$Q_{\text{max}} = \beta_{\text{grav}} \left( \frac{M_F}{m_{3/2}} \right)^2 \sim 4 \times 10^{40} \left( \frac{m_{3/2}}{\text{MeV}} \right)^{-2}. \hspace{1cm} (19)$$

which is much smaller than $10^{47}$. In other words, the lifetime is too short for the Q balls to be the dominant component of the dark matter. Using the relation (14), the range of the charge and the abundance of the Q ball are

$$4 \times 10^{37} \lesssim Q \lesssim 4 \times 10^{40},$$

$$4 \times 10^{-10} \lesssim \Omega_Q \lesssim 4 \times 10^{-7},$$

respectively. This corresponds to the lifetime: $4 \times 10^{17}\text{sec} \lesssim \tau_Q \lesssim 4 \times 10^{20}\text{sec}$.

While the charge in the above range can be easily realized, the abundance of such large Q balls tends to be too large, unless we introduce some mechanism to suppress it. One of the simplest solutions is to dilute them by generating large entropy at later epoch; it can be realized by e.g., thermal inflation $^{13}$ or unstable domain-wall network $^{17}$. However, since the large entropy production makes it difficult to generate enough baryon asymmetry, it is desirable if there is another way to reduce the Q-ball density. In fact, as discussed in the next section, nonperturbative decay of the flat direction could serve as another solution, which can be naturally implemented in our scheme.

$^5$ If $K$ is positive, another type of Q balls dubbed “delayed-type” Q-balls are formed $^{12}$. The properties of the delayed-type Q-balls are same as the GM-type Q-balls, except for $\beta_{\text{gauged}} \sim 1$. But even in this case, the maximum charge is marginally too small.
V. DYNAMICS OF THE FLAT DIRECTION

Here we first explain the dynamics of flat directions and subsequent Q-ball formation. Second, we briefly discuss a possible way to reduce the Q-ball density via nonperturbative decay of the flat direction.

A. Q-ball formation

During inflation the flat direction is assumed to develop a large vacuum expectation value $\phi_0$. We further assume that $\phi_0$ is larger than $\phi_{eq}$ so that new-type Q balls are formed. When the Hubble parameter becomes comparable to the gravitino mass, the flat direction starts to oscillate. At the same time, it is kicked into the phase direction due to $U(1)$-violating interactions dubbed A-term. Without nonrenormalizable interactions in the superpotential, it comes from higher order lepton number violating terms in the Kähler potential:

$$V_A(\Phi) \sim \frac{m_3^2}{M_*^2} \Phi^n + \text{h.c.},$$  \hspace{1cm} (22)

where $M_*$ is a cut-off scale of this interaction.

Once the rotation starts, the flat direction feels spatial instabilities due to negative pressure; the potential of the flat direction to other fields (say $\chi$), Appendix B. Rapid decay of the flat direction

In the previous subsection, we did not include the coupling of the flat direction to other fields (say $\chi$). In fact, the flat direction decays into $\chi$ particles very efficiently if the rotational orbit is eccentric enough. $\chi$ particles are created nonperturbatively while the flat direction passes near the origin. In general, $\chi$ has further couplings to other particles, symbolically denoted by $X$. Through the interactions, the created $\chi$ particles may decay into $X$ immediately after their production. Such mechanism is known as instant preheating.

In order to produce $\chi$ particles, the effective mass of $\chi$ must change nonadiabatically. The particle production occurs each time $\Phi$ passes near the origin. Assuming the following interaction$^*$,

$$\mathcal{L}_{\text{int}} = g^2 |\Phi|^2 |\chi|^2,$$  \hspace{1cm} (24)

the adiabicity is violated if

$$\dot{\omega}_k \approx \omega_k^2,$$  \hspace{1cm} (25)

where $\omega_k^2 = \sqrt{k^2 + g^2 \dot{\phi}^2(t)}$. This inequality is satisfied only in the vicinity of the origin; $\phi \lesssim \phi_*$ with $\phi_* \sim (m_3/2\phi_{osc}/g)^{1/2}$. For $m_3/2 \sim \text{MeV}$, $\phi_{osc} \sim 10^{17}$ GeV and $g \sim 0.3$, we have $\phi_* \sim 10^7$ GeV. The nonperturbative particle production via the interaction (24) thus occurs if the ratio of major and minor axes of the elliptical orbit, $\varepsilon$, is smaller than $\varepsilon_c \equiv \phi_*/\phi_{osc} \sim 10^{-10}$. Let us assume that the flat direction acquires quantum fluctuations during inflation. If the averaged value of $\varepsilon$ is slightly smaller than $\varepsilon_c$, the flat direction rapidly decays in most parts of the entire universe. In the meantime, the Q-balls with very large $\varepsilon$ are formed in those patches where $\varepsilon$ is larger than $\varepsilon_c$ due to fluctuations and the instantaneous preheating does not occur **. If this is the case, we can suppress the Q-ball density while keeping the charge of the Q balls very large.

For the A-term given by Eq. (22), $\varepsilon$ is

$$\varepsilon \sim \theta \left( \frac{\phi_{osc}}{M_*} \right)^{n-2},$$  \hspace{1cm} (26)

where $\theta$ is the CP phase. Such small $\varepsilon \lesssim 10^{-10}$ can be realized, for example, when $n = 6, M_* \sim M_P$, and $\theta \sim 10^{-5}$. The quantum fluctuations must be so large that $\delta \varepsilon / \varepsilon \sim H_1 / (\phi_{osc} \theta)$ is of order unity, where $H_1$ is the Hubble parameter during inflation. To this end, $H_1$ should be around $10^{12}$ GeV for the exemplified values of $\phi_{osc}$ and $\theta$.

Lastly let us comment on the asymmetry of the flat direction. It is easy to see that $\delta \varepsilon / \varepsilon$ gives a rough estimate of the isocurvature fluctuations in the lepton number of the flat direction concerned. Therefore it is difficult

$^*$ Such interaction comes from the D-term potential. Although lepton asymmetry cannot be transmitted via this interaction, it is transmitted through, e.g., Yukawa-type interactions with higgsino or gauginos. In the latter case, the efficiency of the $\chi$ production is more or less the same as that considered in the text.

** Notice that the Q-ball formation will proceed if the coherence is extended over $|K|^{-1/2} m_{3/2}$, which is $\sim$ 10 times larger than the horizon at the beginning of the oscillations.
to associate the lepton asymmetry of the flat direction responsible for 511keV line with the primordial baryon asymmetry. However, since our model does not require late-time entropy production, any successful baryogenesis, including the Affleck-Dine mechanism using a flat direction compatible with the leptonic direction we considered, should work.

VI. CONCLUSIONS

We have shown that the annihilation of positrons from the Q-ball decay can explain the observed 511 keV line emission from the galactic center observed by SPI/INTEGRAL. The setup is natural and minimal in the sense that we have used the MSSM Q balls in the gauge-mediated SUSY breaking. One of the advantages using Q balls is that they naturally have both the small mass per unit charge of $\mathcal{O}(\text{MeV})$ and the lifetime longer than the present age of the universe. This distinctive feature makes our scenario appealing. In addition, the desired Q-ball abundance can be realized without resort to large entropy production; the nonperturbative decay processes drastically suppress the abundance.

Finally we comment on the dark matter. Since the present energy density of the L ball cannot account for the dark matter, $\Omega_m \sim 0.3$, we need other candidates. Among all, here we propose Q ball dark matter, which may co-exist. Since the tiny fraction of the large L balls does not change the thermal history of the universe so much, the B-ball dark matter (and possibly related baryogenesis) is as the same in Ref. [10].

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