Metal Enrichment of the Intergalactic Medium and Production of Massive Black Holes

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ABSTRACT

A model for the chemical evolution of the intergalactic medium (IGM) is presented using theoretical yields of very massive ($M_{\text{VMS}} > 100 \, M_\odot$) stars (VMSs) and Type II supernovae (SNe II). It is shown that if $[\text{Si}/\text{C}]$ is indeed as high as $\sim 0.7$ in the IGM, then VMSs ($M_{\text{VMS}} \approx 140$–260 $M_\odot$) associated with pair-instability supernovae (PI-SNe) in low-mass ($\sim 10^5 \, M_\odot$) halos at high redshift must produce at least 50% of the Si. The remainder is from later galactic outflows of SN II debris, which also provide most of the C and O. Both sources are required to account for the metal inventory in the IGM. The early VMS production must continue until redshift $z \sim 15$ so that the efficiency of VMS formation per low-mass halo is significantly below unity. Contributions from the later galactic outflows mainly occur at $z \sim 4$–6. Using a Salpeter initial mass function, we infer that the number of VMSs ($M_{\text{VMS}} \approx 260$–2000 $M_\odot$) producing massive black holes (MBHs) with an average mass $\langle M_{\text{MBH}} \rangle \sim 270$–550 $M_\odot$ is $\approx 0.72$ times the number of VMSs associated with PI-SNe. The amount of metals (particularly Si) in the IGM that is attributable to PI-SNe is thus closely coupled with the total mass of MBHs produced in epochs prior to galaxy formation. Production of $\sim 50\%$ of the Si in the IGM by PI-SNe corresponds to an early inventory of MBHs that constitutes a fraction $\sim (4$–$8) \times 10^{-5}$ of the total baryonic mass in the universe. This is comparable to the global mass budget of the central supermassive black holes (SMBHs) in present-day galaxies. The corresponding occurrence rates in each halo of $\sim 10^5 \, M_\odot$ during the epoch of VMS formation at $z \gtrsim 15$ are $\sim 0.9$ Gyr$^{-1}$ for VMSs associated with PI-SNe and $\sim 0.6$ Gyr$^{-1}$ for the concomitant more massive stars producing MBHs. These rates may be of use to studies of $H_2$ dissociation and reionization and to models of SMBH formation.

Subject headings: galaxies: formation — intergalactic medium — nuclear reactions, nucleosynthesis, abundances

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1. Introduction

In this paper we explore implications of data on C, O, and Si in the intergalactic medium (IGM) for the “metal” sources. In earlier works (Wasserburg & Qian 2000; Qian & Wasserburg 2002; see also Qian, Sargent, & Wasserburg 2002), we proposed that very massive ($M_{\text{VMS}} > 100 M_\odot$) stars (VMSs) formed from big bang debris produced a prompt metal inventory in the IGM at the level of $[\text{Fe/H}]_{\text{IGM}} \equiv \log (\text{Fe}/H)_{\text{IGM}} - \log (\text{Fe}/H)_\odot \sim -3$, where $(\text{Fe}/H)$ is the number ratio of Fe to H. This was based on observations of metal-poor stars in the Galactic halo that showed a sudden jump in the production of heavy rapid-neutron-capture ($r$-process) elements such as Ba and Eu relative to Fe at $[\text{Fe/H}] \sim -3$ (e.g., McWilliam et al. 1995; Burris et al. 2000). We inferred that the jump was due to the onset of major production of the heavy $r$-process elements by a class of supernovae associated with normal stars, which could predominantly form only after VMSs had provided a metallicity at the level of $[\text{Fe/H}]_{\text{IGM}} \sim -3$ to the IGM. This is supported by theoretical models of star formation, which suggested that a critical metallicity of $\sim 5 \times 10^{-4}$ times the solar value is required for sufficient cooling of gas clouds to form low-mass protostellar aggregates (Bromm et al. 2001; see also Bromm & Loeb 2003). Using VMS yields of Heger & Woosley (2002, hereafter HW02), Oh et al. (2001) showed that there would be sufficient hard UV photons emitted from VMSs to reionize the universe if these stars were to provide $[\text{Si/H}]_{\text{IGM}} \approx -2.3$ (e.g., Cowie & Songaila 1998; Ellison et al. 2000; see also Aguirre et al. 2004). We here consider both early VMSs and later galactic outflows for metal enrichment of the IGM. It is argued that VMSs ($M_{\text{VMS}} \approx 140$–260 $M_\odot$) associated with pair-instability supernovae (PI-SNe; HW02) are the dominant source of Si and also made substantial contributions to C and O. The dominant contributions to C and O are from later galactic outflows governed by Type II supernovae (SNe II). We then show that for a plausible initial mass function (IMF), the metal (particularly Si) inventory in the IGM due to PI-SNe is coupled with production of black holes by VMSs with $M_{\text{VMS}} > 260 M_\odot$. Thus, there is concomitant production of both metals and massive black holes at early epochs.

The present study is an extension of our previous work (Qian & Wasserburg 2005, hereafter QW05), where we sought to explain the data on C, O, and Si in the IGM using a number of VMS and SN II models and considering a wide range of IMFs for VMSs. In that work, we showed from detailed arguments that VMSs associated with PI-SNe are required to account for the Si/C ratio in the IGM and that the preferred scenario for metal enrichment of the IGM is a combination of contributions from early VMSs and later galactic outflows. In the present paper, we assume the basic conclusions of that work and adopt the same input and formalism to develop the general self-consistent implications of the chemical evolution model used there. The new results to be presented include the constraint on the redshift for termination of VMS contributions and the relationship between metal production by PI-SNe
and the early inventory of massive black holes.

We first review the issues. There is a substantial inventory of metals in the general IGM over a wide range of redshift, \( z = 1.5–5.5 \) (e.g., Songaila 2001; Pettini et al. 2003). Detailed studies of quasar absorption spectra have established abundances of the ionic species C III, C IV, O VI, Si III, and Si IV in the Ly\( \alpha \) forest (see Schaye et al. 2003; Aguirre et al. 2004; and Simcoe et al. 2004 for recent analyses). The corresponding elemental abundances have been determined using models of the UV background (UVB; Haardt & Madau 1996, 2001) to account for the fraction of an element in a specific ionization stage. Two typical UVB models are model Q, which has a harder UVB provided by quasars only, and model QG, which has a softer UVB provided by both quasars and galaxies. For both models, Schaye et al. (2003) and Aguirre et al. (2004) found that C and Si abundances are quite variable over different regions. For a given density of the Ly\( \alpha \) forest, the abundance of, say C, appears to follow a lognormal distribution. The net inventory of C over the density range (0.32 to 63 times the cosmic mean) covered by the data is \([C/H]_{\text{IGM}} = -2.3\) for UVB model Q and \(-2.8\) for model QG. For either model, Schaye et al. (2003) and Aguirre et al. (2004) found that neither \([C/H]_{\text{IGM}}\) nor \([\text{Si}/H]_{\text{IGM}}\) show evidence of evolution in the range \( z = 1.8–4.1 \). Using the data at \( z \approx 2.5 \) and similar UVB models, Simcoe et al. (2004) also found lognormal distributions for C and O abundances. The results of the above groups for \([C/H]_{\text{IGM}}\) are in good agreement for either UVB model. The observed variability of abundances in different regions must be a result of varying degrees of mixing. However, there is no apparent variation of the Si/C or C/O ratio (Aguirre et al. 2004; Simcoe et al. 2004). In the models presented below, we will focus on the net IGM inventory and the associated elemental ratios. The distribution of metals is not treated.

From the observations there is no apparent evolution of the IGM inventory over \( z = 1.8–4.1 \). Thus, this inventory must have been produced prior to \( z \sim 4 \). The plausible sources are VMSs and SNe II. The Si/C and C/O ratios\(^1\) are potential diagnostics of the sources. These ratios are sensitive to the UVB model. As both of these ratios are available for models Q and QG, we discuss these two UVB models first. While model Q gives \([\text{Si}/C]_{\text{IGM}} = 1.45\) and \([\text{C}/O]_{\text{IGM}} = 0\), model QG gives much lower values of \([\text{Si}/C]_{\text{IGM}} = 0.74\) and \([\text{C}/O]_{\text{IGM}} = -0.50\) (Aguirre et al. 2004; Simcoe et al. 2004). For both models \([\text{Si}/C]_{\text{IGM}}\) is quite

\(^1\)For both the IGM data and the results from stellar models, we use the same reference solar abundances as in QW05: \( \log \epsilon_{\odot}(\text{Si}) = \log(\text{Si}/\text{H})_{\odot} + 12 = 7.55 \) (Anders & Grevesse 1989) adopted by Aguirre et al. (2004), and \( \log \epsilon_{\odot}(\text{C}) = 8.52 \) and \( \log \epsilon_{\odot}(\text{O}) = 8.83 \) (Grevesse & Sauval 1998) adopted by Simcoe et al. (2004). As Schaye et al. (2003) and Aguirre et al. (2004) adopted a slightly different solar C abundance of \( \log \epsilon_{\odot}(\text{C}) = 8.55 \) (Anders & Grevesse 1989), we have made appropriate small adjustments to their \([C/H]_{\text{IGM}}\) and \([\text{Si}/C]_{\text{IGM}}\) values for consistency.
high and points to VMS sources (Schaerer 2002; Aguirre et al. 2004). For UVB model QG, $[\text{Si}/\text{C}]_{\text{IGM}}$ is well above the yield ratio of SN II sources but below that of pure VMS sources (see Table 2 of QW05). The value of $[\text{Si}/\text{C}]_{\text{IGM}}$ for model Q is so high that it can be barely matched by the yield ratio of a pure VMS source. We have shown earlier (QW05) that the values of $[\text{Si}/\text{C}]_{\text{IGM}}$ and $[\text{C}/\text{O}]_{\text{IGM}}$ for UVB model Q cannot be accounted for simultaneously by any existing stellar models. However, both elemental ratios for model QG can be matched using a combination of VMS and SN II sources. Results on $[\text{Si}/\text{C}]_{\text{IGM}}$ only were also given by Aguirre et al. (2004) for models QGS and QGS3.2, both of which have an even softer UVB than model QG. Model QGS gives $[\text{Si}/\text{C}]_{\text{IGM}} = 0.23$ while model QGS3.2 gives $[\text{Si}/\text{C}]_{\text{IGM}} = 0.43$. These $[\text{Si}/\text{C}]_{\text{IGM}}$ values are consistent with the yield ratio of SN II sources but significantly below that of pure VMS sources (see Table 2 of QW05). Consequently, were the composition of the IGM characterized by these $[\text{Si}/\text{C}]_{\text{IGM}}$ values, there would be no requirement of VMS contributions. However, Schaye et al. (2003) showed that for UVB model QGS, the C abundance strongly decreases with time for all densities, which is clearly unphysical. Furthermore, model QGS3.2 predicts a jump in the optical depth of Si IV relative to C IV due to the sudden transition in its UVB at $z = 3.2$. This is disfavored by the observations (Aguirre et al. 2004). In contrast, UVB model QG appears compatible with all the observations so far and also gives physical results regarding the evolution of metallicity with time and density (Schaye et al. 2003; Aguirre et al. 2004). Therefore, it seems unlikely that this model could be seriously in error (Aguirre et al. 2004). The results on the IGM inventory for UVB model QG (see Table 1) will be used in the subsequent presentation. As shown by QW05, VMS contributions are required to match $[\text{Si}/\text{C}]_{\text{IGM}}$ for this model and both $[\text{Si}/\text{C}]_{\text{IGM}}$ and $[\text{C}/\text{O}]_{\text{IGM}}$ for this model can be accounted for by a blend of VMS and SN II contributions. Insofar as $[\text{Si}/\text{C}]_{\text{IGM}}$ is approximately at or above the value given by this model, these results appear robust. The high value of $[\text{Si}/\text{C}]_{\text{IGM}}$ was not discussed in the recent critique of VMS models by Tumlinson, Venkatesan, & Shull (2004) or in another study by Daigne et al. (2004). In particular, Daigne et al. (2004) concluded that “there is no evidence, nor any need, for a hypothesized primordial population of very massive stars in order to account for the chemical abundances of extremely metal-poor halo stars or of the intergalactic medium.” We consider that further improvements in measurements and analyses of the IGM abundances, as well as further theoretical studies of the conditions under which stars of extremely low metallicity can form, will clarify the choice between the metal enrichment scenarios with and without VMSs.

Based on the high value of $[\text{Si}/\text{C}]_{\text{IGM}}$ for UVB model QG, we use the VMS model of HW02 and the low-metallicity SN II model of Woosley & Weaver (1995, hereafter WW95) to quantify VMS contributions to the IGM. The VMSs relevant for metal enrichment are in the mass range $M_{\text{VMS}} \approx 140–260 \, M_\odot$ and produce PI-SNe (HW02). A Salpeter IMF is
assumed for both VMSs and SNe II and the corresponding yield ratios are given in Table 1 (models 2A and 6 in Table 2 of QW05). Using these yield ratios, we can find mixtures of VMS and SN II contributions that can account for $[\text{Si/C}]_{\text{IGM}}$ and $[\text{C/O}]_{\text{IGM}}$ simultaneously. As both $[\text{C/O}]_{\text{VMS}} = -0.57$ and $[\text{C/O}]_{\text{SN II}} = -0.42$ are within 0.1 dex of $[\text{C/O}]_{\text{IGM}} = -0.50$, the appropriate mixture is determined by matching $[\text{Si/C}]_{\text{IGM}}$. The required fraction $f_{\text{VMS}}$ of Si contributed by VMSs can be calculated from

$$\left( \frac{\text{C}}{\text{Si}} \right)_{\text{IGM}} = f_{\text{VMS}} \left( \frac{\text{C}}{\text{Si}} \right)_{\text{VMS}} + (1 - f_{\text{VMS}}) \left( \frac{\text{C}}{\text{Si}} \right)_{\text{SN II}}.$$  (1)

With $[\text{Si/C}]_{\text{VMS}} = 1.18$ and $[\text{Si/C}]_{\text{SN II}} = 0.42$, $f_{\text{VMS}} = 0.63$ is required to match $[\text{Si/C}]_{\text{IGM}} = 0.74$ exactly and $f_{\text{VMS}} = 0.5$ gives $[\text{Si/C}] = 0.65$. Therefore, VMSs must be the dominant source of Si. This result does not depend on the specific stellar models adopted here. In fact, as other SN II models have lower values of $[\text{Si/C}]_{\text{SN II}}$ (see Table 2 of QW05), using them only strengthens the above result. We conclude that a minimum value of $f_{\text{VMS}} \sim 0.5$ is required to account for the observed elemental ratios in the IGM. For the stellar models adopted here, $f_{\text{VMS}} \sim 0.5$ corresponds to $f_{\text{C}} \sim 0.15$ and $f_{\text{O}} \sim 0.19$, so SNe II are the dominant source of C and O. This conclusion can be generalized to all stellar models as well.

## 2. Metal Enrichment of the IGM by VMSs

We now discuss metal enrichment of the IGM by VMSs. The formation of these stars is supposed to precede that of lower-mass stars and occur in halos where big bang debris can cool by H$_2$ molecules (see Abel et al. 2002; Bromm & Larson 2004 for reviews). This requires a minimum halo mass of $\sim 10^5 M_\odot$. The efficiency of VMS formation in such low-mass halos determines how early VMSs could provide substantial enrichment to the IGM. For a simple estimate, we consider that all halos with mass $M > M_{\text{min}}$ are formed from “building-block” halos with mass $M_{\text{min}}$. We assume that a VMS would have formed in every “building-block” halo and ejected all its nucleosynthetic products into the IGM. Then at redshift $z$, the abundance of element E in the IGM would be

$$Z_{\text{IGM}}^E \equiv \frac{(E/H)_{\text{IGM}}}{(E/H)_{\odot}} \approx \frac{\langle Y_{\text{VMS}}^E \rangle F(M > M_{\text{min}} | z)}{X_E^\odot f_b M_{\text{min}}},$$  (2)

where $\langle Y_{\text{VMS}}^E \rangle$ is the average mass yield of E per VMS event, $F(M > M_{\text{min}} | z)$ is the fraction of all matter residing in halos with $M > M_{\text{min}}$ at redshift $z$, $X_E^\odot$ is the solar mass fraction of E, and $f_b$ is the baryonic fraction of the halo mass. Consider Si as an example. Using $\langle Y_{\text{Si}}^{\text{VMS}} \rangle = 16.6 M_\odot$ (HW02), $X_{\text{Si}}^\odot = 7.1 \times 10^{-4}$ (Anders & Grevesse 1989), $f_b = 0.15$, and $M_{\text{min}} = 10^5 M_\odot$, we obtain $Z_{\text{Si}}^{\text{IGM}} \approx 1.6 F(M > 10^5 M_\odot | z)$. The function $F(M > 10^5 M_\odot | z)$...
can be estimated using the Press-Schechter formalism (Press & Schechter 1974) and ranges from $\sim 0.3\%$ to $3\%$ for $z \sim 24$ to 17. Thus, if VMSs are to provide $50\%$ of the measured inventory of $[\text{Si}/\text{H}]_{\text{IGM}} = \log Z_{\text{Si}}^{\text{IGM}} = -2.0$ (Aguirre et al. 2004), this requires that a VMS form in every “building-block” halo by $z \sim 24$ or more plausibly, in $\sim 10\%$ of such halos by $z \sim 17$.

For a formal and self-consistent model of the chemical evolution of the IGM using the rate of occurrence of a VMS event in a halo, we consider a large reference region of the universe and treat it as a closed homogeneous system (QW05). The evolution of $(\text{E}/\text{H})_{\text{IGM}}$ in the system as a function of time $t$ is governed by

$$\frac{d(\text{E}/\text{H})_{\text{IGM}}}{dt} = \frac{Q_{\text{E}}}{(\text{H})_{\text{IGM}}},$$

where $Q_{\text{E}}$ is the net rate for ejection of E atoms into the IGM and $(\text{H})_{\text{IGM}}$ is the number of H atoms in the IGM. The rate $Q_{\text{E}}$ depends on the rate of occurrence of a VMS event and the efficiency of gas expulsion by the VMS explosion. We consider that a VMS may form in a halo when the virial temperature $T_{\text{vir}}$ of the gas reaches a minimum value of $T_{\text{vir},0} = 300$ K for $\text{H}_2$ cooling. For $z \gg 1$,

$$T_{\text{vir}} \approx 211 \left( \frac{\mu}{1.22} \right) \left( \frac{M}{10^5 M_\odot} \right)^{2/3} \left( \frac{1 + z}{10} \right) \text{K},$$

where $\mu$ is the mean atomic weight and $\mu = 1.22$ or 0.6 for a neutral or ionized gas, respectively, with a primordial composition of H and He. We assume that all the gas would be expelled from the halo by the VMS explosion if the gravitational binding energy of the gas $E_{\text{bi, gas}}$ is less than the VMS explosion energy $E_{\text{exp}}$ (e.g., Bromm, Yoshida, & Hernquist 2003) and retained for $E_{\text{bi, gas}} \geq E_{\text{exp}}$. For $z \gg 1$,

$$E_{\text{bi, gas}} \approx 4.31 \times 10^{47} \left( \frac{M}{10^5 M_\odot} \right)^{5/3} \left( \frac{1 + z}{10} \right) \text{erg}.$$ 

For a given $z$, a halo associated with an $n_0 \sigma$ density fluctuation (an $n_0 \sigma$ halo) would have the mass $M_0$ corresponding to $T_{\text{vir}} = T_{\text{vir},0}$ for the onset of VMS formation, while an $n_{\text{bi}} \sigma$ halo would have the mass $M_{\text{bi}}$ corresponding to $E_{\text{bi, gas}} = E_{\text{exp}}$ for the onset of gas retention. Then we have $Q_{\text{E}} = \sum_{n=n_0}^{n_{\text{bi}}} N_n \sigma P_{E,n \sigma}$, where $N_n \sigma$ is the total number of halos with a specific $n$ value in the system and $P_{E,n \sigma}$ is the number production rate of E in a single $n \sigma$ halo.

The rate $P_{E,n \sigma}$ is related to the rate of occurrence of a VMS event $R_{n \sigma}$ in an $n \sigma$ halo. We assume that these rates are proportional to the number $(\text{H})_{n \sigma}$ of H atoms in the gas of
the halo and write

\[ P_{E,n} = \langle y_{E}^{\text{VMS}} \rangle R_{n} = \Lambda_{E}^{\text{VMS}} (E/H)_{0} (H)_{n} \sigma, \]

\[ R_{n} \sigma \approx \Lambda_{E}^{\text{VMS}} f_{b} n_{\sigma} \frac{\int_{\gamma_{E}}^{\nu_{E}} M_{\sigma} \sigma}{(Y_{E}^{\text{VMS}})^{3}}, \]

where \( \langle y_{E}^{\text{VMS}} \rangle \) is the average number yield of E per VMS event corresponding to the mass yield \( \langle Y_{E}^{\text{VMS}} \rangle \) and \( \Lambda_{E}^{\text{VMS}} \) is a rate constant. The above prescription of \( P_{E,n} \propto R_{n} \propto (H)_{n} \sigma \) is meant to imply that it is more probable for a VMS to form in a larger halo, and we will discuss the limit on the occurrence of a VMS event in a typical halo below. Substituting \( Q_{E} \) in equation (3) and noting that for \( z \gg 1 \) the majority of the H atoms of the system resides in the IGM and the majority of the H atoms in a halo resides in the gas, we obtain

\[ \frac{dZ_{E}^{\text{IGM}}}{dt} = \Lambda_{E}^{\text{VMS}} \sum_{n=n_{0}}^{n_{bi}} N_{n} \sigma (H)_{n} (H)_{\text{IGM}} \approx \Lambda_{E}^{\text{VMS}} F(M_{0} < M < M_{\text{bi}} | z), \]

\[ Z_{E}^{\text{IGM}}(t) \approx \Lambda_{E}^{\text{VMS}} \int_{0}^{t(z)} F(M_{0} < M < M_{\text{bi}} | z') dt', \]

where

\[ F(M_{0} < M < M_{\text{bi}} | z) = \sqrt{\frac{2}{\pi}} \int_{n_{0}}^{n_{bi}} \exp \left(-\frac{x^{2}}{2}\right) dx \]

(10) is the fraction of all matter residing in \( n \sigma \) halos with \( n_{0} < n < n_{bi} \) as given by the Press-Schechter formalism and \( t(z) = 0.538[10/(1 + z)]^{3/2} \) Gyr for the adopted cosmology (see discussion in Barkana & Loeb 2001).

The evolution of \( Z_{E}^{\text{IGM}} \) is thus only dependent on \( \Lambda_{E}^{\text{VMS}} \) and \( F(M_{0} < M < M_{\text{bi}} | z) \). As \( Z_{E}^{\text{IGM}} \) scales with \( \Lambda_{E}^{\text{VMS}} \), we will use \( \Lambda_{E}^{\text{VMS}} = 0.1 \) Gyr\(^{-1} \) as a reference value (for elements produced by SNe II only, the average Galactic value of \( \Lambda_{E}^{\text{SN II}} \) is \( \sim 0.1 \) Gyr\(^{-1} \) over a period of \( \sim 10 \) Gyr prior to solar system formation; QW05). To determine \( F(M_{0} < M < M_{\text{bi}} | z) \), we use \( T_{\text{vir,0}} = 300 \) K (\( \mu = 1.22 \)) to find \( M_{0} \) and a typical VMS explosion energy of \( E_{\text{exp}} = 4 \times 10^{52} \) erg (HW02) to find \( M_{\text{bi}} \) for which \( E_{\text{bi, gas}} = E_{\text{exp}} \). The relevant range of \( n \sigma \) halos with \( M_{0} < M < M_{\text{bi}} \) \( (n_{0} < n < n_{bi}) \) corresponds to \( 5.5 \times 10^{4} M_{\odot} < M < 6.1 \times 10^{7} M_{\odot} \) \( (2.3 < n < 3.8) \) at \( z = 20 \) and \( 8.3 \times 10^{4} M_{\odot} < M < 7.2 \times 10^{7} M_{\odot} \) \( (1.8 < n < 2.9) \) at \( z = 15 \). Although the range \( M_{0} < M < M_{\text{bi}} \) is rather wide, the integral for \( F(M_{0} < M < M_{\text{bi}} | z) \) is dominated by the contributions from \( n \sigma \) halos with \( M \sim M_{0} \) \( (n \sim n_{0}); \text{see eq.}[10]) \. Thus, a typical halo relevant for metal-enrichment of the IGM has \( M \sim 10^{5} M_{\odot} \) for \( z \sim 15-20 \) and an exact treatment of halos of much higher masses, which requires a detailed description of their merger history, is not crucial to our model. [In particular, \( F(M_{0} < M < M_{\text{bi}} | z) \) is not sensitive to \( E_{\text{exp}} \) for \( E_{\text{exp}} \gtrsim 10^{51} \) erg as halos with \( E_{\text{bi, gas}} \gtrsim 10^{51} \) erg are extremely rare for
The evolution of \( [E/H]_{\text{IGM}} = \log Z_{E}^{\text{IGM}} \) for \( \Lambda_{E}^{\text{VMS}} \) is shown as the dot-dashed curve in Figure 1. This gives \( [E/H]_{\text{IGM}} = -3.46 \) at \( z = 15 \). Thus, to provide 50% of the measured inventory of \( [\text{Si}/H]_{\text{IGM}} = -2.0 \) by \( z = 15 \) (point A) requires \( \Lambda_{\text{Si}}^{\text{VMS}} = 1.4 \) Gyr\(^{-1}\), which corresponds to \( R_{n} \sigma_{t_{15}} \sim 0.2 \) VMS event over \( t_{15} \equiv t(z = 15) = 0.27 \) Gyr for a halo with \( M_{n} \sigma \sim 10^{5} M_{\odot} \) and \( R_{n} \sigma \sim 0.9 \) Gyr\(^{-1}\) (eq. [7]). In Figure 2 we show the values of \( \Lambda_{\text{Si}}^{\text{VMS}} \) necessary to achieve \( [\text{Si}/H] = -2.3 \) (solid curve) or \( -2.0 \) (dashed curve) by a given \( z \) as calculated from equation (9). The dot-dashed curve indicates formation of \( R_{n} \sigma t(z) \sim 1 \) VMS by time \( t(z) \) in a halo of mass \( M_{n} \sigma \sim 10^{5} M_{\odot} \) and is taken as a bound. It is evident that a major part (\( \gtrsim 50\% \)) of the Si in the IGM, which must come from VMSs, can only be provided for \( z \) substantially below 19 and with plausible production rates, for \( z < 16 \). We consider \( \Lambda_{\text{Si}}^{\text{VMS}} = 1.4 \) Gyr\(^{-1}\) a reasonable rate and show the corresponding evolution of \( [\text{Si}/H]_{\text{IGM}} \) for \( z > 15 \) as the solid curve in Figure 1. The evolution of \( [C/H]_{\text{IGM}} \) and \( [O/H]_{\text{IGM}} \) is fixed by the yield ratios of the VMS model (see Table 1) and shown as the dot-dashed and short-dashed curves, respectively.

3. Termination of VMS Contributions

Figure 1 shows that for a reasonable formation efficiency, VMSs would provide \( [C/H] = -3.5 \) and \( [O/H] = -2.9 \) to the IGM by \( z = 15 \). This is consistent with the proposal by Bromm & Loeb (2003) that VMSs can no longer be formed when \( [C/H] = -3.5 \pm 0.1 \) and \( [O/H] = -3.05 \pm 0.2 \) are reached in the IGM. They showed that for these metallicities, gas clouds initially cooled by \( \text{H}_{2} \) molecules could continue to cool by C and O atoms and fragment into smaller clumps. However, the termination of VMS formation is a complex matter. The soft UV radiation from VMSs would dissociate \( \text{H}_{2} \) molecules in low-mass halos, thus suppressing later VMS formation. The history of \( \text{H}_{2} \) dissociation is not well known but a theoretical study by Ciardi et al. (2000) suggested that universal dissociation has not yet occurred for \( z \sim 20 \). The UVB produced by VMSs may also play an important role in reionization, which could have taken place at \( z \sim 17 \) (Kogut et al. 2003). As shown by Oh et al. (2001), there should be sufficient hard UV photons emitted from VMSs to reionize the universe if these stars were to provide \( [\text{Si}/H]_{\text{IGM}} \sim -2.3 \) (see Tumlinson et al. 2004; Daigne et al. 2004 for a different view). The above estimates for the onset of C and O cooling, universal \( \text{H}_{2} \) dissociation, and reionization appear to roughly coincide. Clearly, there must be a transition region in which ongoing VMS formation increases the soft UVB for \( \text{H}_{2} \) dissociation and begins to suppress further VMS formation, thus resulting in a global decrease in \( \Lambda_{E}^{\text{VMS}} \). There will also be an accompanying increase in the hard UVB that begins to initiate reionization and raise the IGM temperature. The balance between these processes is not well understood and cannot be addressed here. It is possible that both effects
are operating to end the regime of H$_2$ cooling at roughly similar $z$ to reionization. Combining the consideration of the increasing UVB with the metallicity condition for termination of VMS formation proposed by Bromm & Loeb (2003), we will assume that VMS contributions to the IGM will cease at $z \sim 15$ and transition to formation of lower-mass stars will occur universally.

4. Enrichment of the IGM by Galactic Outflows

For simplicity we assume that VMS production stops sharply at $z = 15$ and take VMS contributions to the IGM as shown in Figure 1. Then the bulk of the C and O and the remainder of the Si must be provided by SNe II at $15 > z \gtrsim 4$. The formalism resulting in equation (8) can be modified to treat the chemical evolution of the IGM in this regime, where formation of regular stars including SN II progenitors requires $T_{\text{vir},0} = 10^4$ K ($\mu = 0.6$) for cooling by atomic species. This corresponds to halos with $M \sim 10^8 M_\odot$ and $E_{\text{bi,gas}}$ far exceeding the typical SN II explosion energy of $\sim 10^{51}$ erg. Thus only a fraction $\epsilon$ of the debris from an SN II will be ejected into the IGM. While the dependence of $\epsilon$ on the halo mass may be complicated, we will treat it as a constant for halos below some cut-off mass $M_1$. Our goal is to estimate how efficient SN II-driven galactic outflows must be in order to provide a substantial part of the IGM inventory at $15 > z \gtrsim 4$. Equation (8) may be rewritten in this regime as

\[
\frac{dZ_{\text{E,IGM}}}{dt} \approx \epsilon \Lambda_{\text{E,SN II}} \int_{M_0}^{M_1} F(M | z) \, dt,
\]

where $Z_{\text{E,IGM}}(t_{15})$ is the IGM inventory of E at $z = 15$ resulting from just VMS production.

Note that $Z_{\text{E,IGM}}(t) - Z_{\text{E,IGM}}(t_{15})$ scales with $\epsilon \Lambda_{\text{E,SN II}}$. For convenience we use $\epsilon \Lambda_{\text{E,SN II}} = 0.01$ Gyr$^{-1}$ as a reference value. This corresponds to an outflow efficiency of $\epsilon = 0.1$ for a Galactic SN II production rate of $\Lambda_{\text{E,SN II}} = 0.1$ Gyr$^{-1}$. Using $T_{\text{vir},0} = 10^4$ K and $M_1 = 10^{10} M_\odot$, we show the contributions from just galactic outflows as the long-dashed curve in Figure 1. The value of $M_1 = 10^{10} M_\odot$ is consistent with observations of outflows from Lyman break and dwarf galaxies (e.g., Pettini et al. 2001; Martin et al. 2002). The results will not change much if $M_1$ is varied from this value by a factor of several either way (QW05). As can be seen from Figure 1, galactic outflows can only provide $[E/H]_{\text{IGM}} = -2.92$ by $z = 4$ for $\epsilon \Lambda_{\text{E,SN II}} = 0.01$ Gyr$^{-1}$. To provide $[\text{Si/H}] = -2.3$ (50% of the IGM inventory) by $z = 4$ would require $\epsilon \Lambda_{\text{Si,SN II}} = 0.042$ Gyr$^{-1}$. This corresponds to a rather high outflow efficiency of $\epsilon = 0.42$ for intermediate-mass ($\sim 10^8 - 10^{10} M_\odot$) halos with a Galactic production rate.
If galactic outflows are to provide \([\text{Si}/\text{H}] = -2.3\) by \(z = 3\), then \(\epsilon \Lambda_{\text{SN II}}^{\text{Si}} = 0.024\) Gyr\(^{-1}\) is required and \(\epsilon = 0.24\) is sufficient for a Galactic production rate.

Figure 1 shows an example of a self-consistent model for the chemical evolution of the IGM with contributions from VMSs for \(z > 15\) and galactic outflows for \(z < 15\). The overall evolution of \([\text{Si}/\text{H}]_{\text{IGM}}, [\text{O}/\text{H}]_{\text{IGM}}, \text{and } [\text{C}/\text{H}]_{\text{IGM}}\) is shown assuming that VMSs provided 50% of the Si inventory by \(z = 15\) (point A) and galactic outflows the other half by \(z = 4\) (point B) and using the yield ratios of the sources (see Table 1). It can be seen that galactic outflows cannot provide significant contributions until \(z \sim 6\). Thus, the model implies that there should be very little change in the net IGM inventory between \(z = 15\) and \(z \sim 6\). The respective contributions to C, O, Si, and Fe from VMSs and galactic outflows associated with SNe II are given in Table 1.

5. Metals in the IGM and Inventory of Massive Black Holes

In considering metal enrichment of the IGM at high \(z\) we have focused on VMSs with \(M_{\text{VMS}} \approx 140-260 \, M_\odot\) in low-mass halos using the yields of HW02. These workers showed that this narrow range of stellar masses produce PI-SNe, which would efficiently eject the associated nucleosynthetic products into the IGM. They also showed that all VMSs with \(M_{\text{VMS}} > 260 \, M_\odot\) would form black holes and therefore not contribute any metals. Thus, the first stellar sources contributing metals are restricted to PI-SNe. However, if one considers any plausible IMF, the occurrence of PI-SNe implies the production of more massive stars that will thus give massive black holes (MBHs). It follows that the IGM inventory of metals that is attributable to PI-SNe must be coupled with the production of MBHs. This then relates the metal (particularly Si) content of the IGM to the inventory of MBHs at early epochs.

Before discussing the coupling between the amount of metals in the IGM and the total mass of MBHs produced in epochs prior to galaxy formation, we note the work by Madau & Rees (2001), who estimated the inventory of MBHs based on considerations other than metal enrichment. Motivated by simulations of Bromm, Coppi, & Larson (1999) and Abel, Bryan, & Norman (2000), which suggested that the first stars may have been very massive, Madau & Rees (2001) recognized the significance of possible early production of MBHs in an episode of pregalactic star formation using earlier stellar models (Bond, Arnett, & Carr 1984; Fryer, Woosley & Heger 2001), which showed that VMSs could produce MBHs. In particular, they found that if one MBH of mass \(M_{\text{MBH}} \gtrsim 150 \, M_\odot\) formed in each \(3 \sigma\) halo (with \(3 \times 10^5 h^{-1} \, M_\odot\) in dark and baryonic matter) collapsing at \(z \approx 20\), then a fraction \(f_{\text{MBH}} \gtrsim 8 \times 10^{-5} h^3\) of all baryonic matter would be in MBHs, where \(h\) is the Hubble
parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\) and we take \(h = 0.7\) throughout this paper. This estimate is comparable to the total mass fraction of the central supermassive black holes (SMBHs) found in most nearby galaxies (see eq. [18] below).

In the earlier sections of the present paper, we have inferred that VMSs with \(M_{\text{VMS}} \approx 140-260 M_\odot\) provided \([\text{Si/H}]_{\text{IGM}} = -2.3\) (\(\sim 50\%\) of the Si in the IGM) and we have derived the rate of VMS formation required to achieve this. We now explore the implications of this metal production for the inventory of MBHs that would be produced in low-mass halos at the same epoch and show that it is in accord with the possibility considered by Madau & Rees (2001). For a quantitative estimate, we consider a large reference region of the IGM with a total mass \(M_{\text{IGM}}\) and use a Salpeter IMF for VMSs. To enrich this IGM with \([\text{Si/H}]_{\text{IGM}} = \log Z_{\text{Si}}^{\text{IGM}}\), the required number \(N_{\text{VMS,si}}\) of VMSs with \(M_{\text{VMS}} \approx 140-260 M_\odot\) is

\[
N_{\text{VMS,si}} \approx \frac{Z_{\text{Si}}^{\text{IGM}} X_{\odot} M_{\text{IGM}}}{\langle Y_{\text{VMS}}^{\text{Si}} \rangle}. \tag{13}
\]

As the total baryonic mass of a hosting halo is \(\sim 10^4 M_\odot\), we take the upper mass limit for VMSs to be 2000 \(M_\odot\). We have checked that our results are not sensitive to this assumed upper mass limit. The number \(N_{\text{MBH}}\) of MBHs resulting from VMSs with \(M_{\text{VMS}} = 260-2000 M_\odot\) is

\[
N_{\text{MBH}} \approx \frac{\int_{260}^{2000} m^{-2.35} dm}{\int_{260}^{140} m^{-2.35} dm} N_{\text{VMS,si}} \approx 0.72 N_{\text{VMS,si}}, \tag{14}
\]

where \(m\) is the VMS mass in units of \(M_\odot\). The average mass of these MBHs is

\[
\langle M_{\text{MBH}} \rangle \approx \alpha \frac{\int_{260}^{2000} m^{-1.35} dm}{\int_{260}^{2000} m^{-2.35} dm} M_\odot \approx 547 \alpha M_\odot, \tag{15}
\]

where \(\alpha \sim 0.5-1\) is the average mass fraction of a VMS progenitor in the relevant mass range that ends up in the MBH (Fryer et al. 2001). The fractional contribution of MBHs to the baryonic mass of the universe can be estimated as

\[
f_{\text{MBH}} \approx \frac{N_{\text{MBH}} \langle M_{\text{MBH}} \rangle}{M_{\text{IGM}}} \approx Z_{\text{Si}}^{\text{IGM}} X_{\odot} \left( \frac{N_{\text{MBH}}}{N_{\text{VMS,si}}} \right) \left( \frac{\langle M_{\text{MBH}} \rangle}{\langle Y_{\text{VMS}}^{\text{Si}} \rangle} \right) \tag{16}
\]

\[
\approx 8.4 \times 10^{-5} \alpha \left( \frac{Z_{\text{Si}}^{\text{IGM}}}{5 \times 10^{-3}} \right). \tag{17}
\]

The above estimate\(^2\) of \(f_{\text{MBH}}\) may be compared with the fraction \(f_{\text{SMBH}}\) of all baryonic

\(^2\)As stated above, this result assumes a Salpeter IMF (\(\propto m^{-2.35}\)) for VMSs. For a steeper IMF of the form \(\propto m^{-3}\), the numerical coefficient in eq. (17) for \(f_{\text{MBH}}\) is reduced by a factor of \(\approx 2\).
matter contributed by SMBHs in the present universe,

$$f_{\text{SMBH}} = \frac{\rho_{\text{SMBH}}}{\Omega_b \rho_{\text{cri}}} = 7.5_{-2.3}^{+3.1} \times 10^{-5},$$  \hspace{1cm} (18)

where $\rho_{\text{SMBH}} = 4.6^{+1.9}_{-1.4} \times 10^5 M_\odot \text{Mpc}^{-3}$ is the present mass density of SMBHs obtained from galactic observations (see the recent analyses by Marconi et al. 2004), $\rho_{\text{cri}} = 1.36 \times 10^{11} M_\odot \text{Mpc}^{-3}$ is the present critical density, and $\Omega_b = 0.045$ is the fractional contribution of baryons to $\rho_{\text{cri}}$. It can be seen that the mass fraction of MBHs calculated from the Si abundance in the IGM of the early universe (eq. [17]) and that of SMBHs determined for the present epoch by observations (eq. [18]) are in remarkably good accord. The possibility considered by Madau & Rees (2001) is now shown to be supported by the required VMS contribution to produce the Si in the IGM. We note that Schneider et al. (2002) derived a wide range of formation efficiency for VMSs with $M_{\text{VMS}} \approx 140–260 M_\odot$ using $f_{\text{MBH}} \leq f_{\text{SMBH}}$ and other constraints. Their results can be easily accommodated by the rate of VMS formation derived here. We also note that accreting MBHs may contribute to the soft X-ray background (SXRB). Based on the unaccounted SXRB flux, Salvaterra, Haardt, & Ferrara (2005) estimated that $f_{\text{MBH}}$ cannot exceed $\sim 10^{-4}$. This is consistent with our result. During the epoch of VMS formation at $z \gtrsim 15$, the rate of occurrence of an MBH-producing VMS event is $(N_{\text{MBH}}/N_{\text{VMS,SI}}) R_{\text{a, M}} \sim 0.6 \text{Gyr}^{-1}$ for a halo of $\sim 10^5 M_\odot$ (eq. [7] with $\Lambda_{\text{VMS}} = 1.4 \text{Gyr}^{-1}$ and eq. [14]). This rate can be used to calculate the contribution from MBHs to the SXRB.

The remarkable accord between the MBH inventory calculated from the Si content of the early IGM (eq. [17]) and the present global mass budget of SMBHs obtained from galactic observations (eq. [18]) cannot be easily dismissed as an accident. It is expected that the early-formed MBHs would cluster near galactic centers (Madau & Rees 2001). As a fraction $\sim (4–8) \times 10^{-5}$ of all baryonic matter is in MBHs (eq. [17] with $\alpha \sim 0.5–1$), a typical galaxy with a baryonic mass of $\sim 10^{11} M_\odot$ would have a total mass of $\sim (4–8) \times 10^6 M_\odot$ in MBHs. This is close to the mass of $(3.7 \pm 0.4) \times 10^6 M_\odot$ for the SMBH at the Galactic center (e.g., Ghez 2004). It is possible that the central SMBHs in typical galaxies of the present universe are formed by simply merging the MBH seeds without much gas accretion. This would explain why the total mass of SMBHs observed at the present epoch corresponds approximately (within a factor of $\sim 2$) to that of the MBH seeds.

However, there appears to be an obvious conflict. It is widely accepted that quasars are powered by gas accretion onto SMBHs. The total gas mass accreted by all optical quasars is inferred to be also comparable to that of the SMBHs observed at the present epoch (e.g., Yu & Tremaine 2002; Marconi et al. 2004). This is supported by the simulations of Hopkins et al. (2005), who proposed a unified model for producing SMBHs, quasars, and galaxy spheroids. In their model, SMBHs were formed predominantly by gas accretion.
Both the global mass budget and the relative distribution of SMBHs over the mass range \( M_{\text{SMBH}} \sim 10^6 - 10^{10} M_\odot \) given by their model are in agreement with those derived by Marconi et al. (2004) from galactic observations. It appears that the model of Hopkins et al. (2005) is rather satisfactory. However, within the uncertainties of their model, it is still plausible to consider that the more common SMBHs of \(< 10^8 M_\odot\) might represent aggregates of the MBH seeds without much gas accretion but large amounts of gas accretion must occur to explain the rarer SMBHs of \(> 10^8 M_\odot\). Thus, in accounting for the approximate agreement between the global SMBH mass budget determined from galactic and quasar observations and the MBH inventory, there appears to be a “rule” that only the rarer higher-mass SMBHs can efficiently accrete gas but the more common lower-mass ones cannot. This matter clearly requires attention. In the model of Hopkins et al. (2005), mergers of halos enhance the infall of gas toward the SMBHs. However, as mergers must have also occurred to form galaxies hosting SMBHs of \(< 10^8 M_\odot\), which constitute \(\approx 50\%\) of the global SMBH mass budget (Marconi et al. 2004), there seem to be some other processes that inhibit gas accretion onto such lower-mass SMBHs.

The complex problem of forming SMBHs from MBH seeds was studied by Volonteri, Haardt, & Madau (2003). However, the total mass of MBHs in their scenarios is only a small fraction (typically \(\sim 10^{-3}\)) of that of the SMBHs at the present epoch as cited above. We urge that the problem of forming SMBHs from MBH seeds be revisited with the initial conditions presented here. For a formation rate of \(\sim 0.6\) Gyr\(^{-1}\) in each halo of \(\sim 10^5 M_\odot\), an MBH with an average mass \(\langle M_{\text{MBH}} \rangle \sim 270-550 M_\odot\) (eq. [15] with \(\alpha \sim 0.5-1\)) should have been produced in \(\sim 16\%\) of such halos by \(z = 15\). If the amount of gas accretion onto black holes in galactic centers is typically comparable to or smaller than the total mass of the original inherited MBHs, then there would not be a serious difficulty in reconciling the global SMBH mass budget determined from galactic observations with that from quasar observations and with the inventory of MBHs calculated from the VMS contribution to the Si in the IGM.

The evolution of an initial distribution of MBH seeds was studied extensively by Islam, Taylor, & Silk (2003, 2004) using a semi-analytical model to follow the hierarchical merging of halos. These authors focused on the dynamics of the MBH inventory and presented a clear analysis of what might be expected from assemblage of MBHs without gas accretion. Amongst their conclusions are: (1) Hierarchical merging of MBH seeds that formed in \(\sim 3\) \(\sigma\) halos collapsing at \(z \sim 25\) can contribute \(\gtrsim 10\%\) of the present global mass budget of SMBHs. A central SMBH of \(\sim 3 \times 10^9 M_\odot\) in a galaxy like ours could be the result of seed accumulation without gas accretion. However, gas accretion is necessary for higher-mass SMBHs. (2) For a present-day galaxy of \(\sim 10^{10}-10^{13} M_\odot\) (mostly in dark matter), the total mass of the inherited MBHs in the galactic halo is comparable to or greater than
the mass contributed by MBHs to the central SMBH. It would be of interest to see what a new calculation of the type presented by Hopkins et al. (2005) would give if the mass and production history of MBHs as provided here were used. If the majority of MBHs are stored in galactic halos but not bulges, then we might expect gas accretion to account for the growth of central SMBHs while the MBHs in the low-density halos might not accrete. In this case, it is accidental that the total mass of MBHs calculated from the VMS contribution to the Si in the IGM is close to the present global mass budget of SMBHs.

6. Conclusions

A two-stage model for the chemical evolution of the IGM is proposed considering VMSs in low-mass ($\sim 10^5 M_\odot$) halos during early epochs ($z \gtrsim 15$). This early stage ends when universal H$_2$ dissociation and reionization occur and is followed by an extended quiescent period of little metal production until sufficient SNe II could occur in intermediate-mass ($\sim 10^8$–$10^{10} M_\odot$) halos to drive significant outflows. Galactic outflows contribute mainly at $z \sim 4$–6. The requirement of early VMS contributions follows from the high value of [Si/C] $\sim 0.7$ inferred for the IGM (Aguirre et al. 2004; see discussion therein regarding possible uncertainties) and model yields of VMSs and SNe II (e.g., HW02; WW95). The bulk ($\gtrsim 50\%$) of the Si must come from VMSs, while the C and O are predominantly from the later galactic outflows. The required VMS formation efficiency corresponds to $\sim 0.2$ VMS per low-mass halo. The requirement on galactic outflows implies efficient ($\sim 40\%$) loss of SN II debris from intermediate-mass halos with a Galactic SN II rate. Contributions from galactic outflows would be diminished as the halo masses and the amount of baryonic matter stored in low-mass stars increase. This may explain the lack of evolution of the IGM inventory for $z = 1.8$–4.1.

The VMS contributions are consistent with the level of metallicities at which VMS formation would be suppressed (Bromm & Loeb 2003). They also correspond to a sufficient number of UV photons from VMSs to reionize the universe (Oh et al. 2001). However, how the increasing photon production by VMSs leads to H$_2$ dissociation and reionization, which then terminates VMS formation, is a complex issue and not addressable here. There is a hint that all these events occur at similar $z$. It would be a useful test if H$_2$ dissociation and reionization could be modeled with the VMS formation rates proposed here.

In producing [Si/H] $= -2.3$, VMSs also provide [Fe/H] $= -2.9$ to the IGM (see Table 1). This Fe abundance was considered by us (e.g., Wasserburg & Qian 2000; Qian & Wasserburg 2002) as the onset of regular star formation based on observations of low-metallicity Galactic halo stars. There is a basic issue regarding the metallicity at which regular stars may be
formed. As C and O atoms provide the important cooling, Bromm & Loeb (2003) proposed conditions for regular star formation in terms of threshold C and O abundances. These abundances coincide with the VMS contributions in our model. In this sense the threshold Fe abundance proposed by us is equivalent to the threshold C and O abundances. However, the low-mass star HE 0107–5240 cannot be explained by our scenario. This special star has $[\text{C/H}]$ well above and $[\text{O/H}]$ at or below the threshold but $[\text{Fe/H}] = -5.3$ (Christlieb et al. 2004; Bessell, Christlieb, & Gustafsson 2004). Whether some low-mass stars such as this may form along with VMSs remains an open issue.

A number of Galactic halo stars are observed to have $-4 \lesssim [\text{Fe/H}] < -3$. If they formed at $z \lesssim 4$, this would pose a problem as our model gives a high value of $[\text{Fe/H}]_{\text{IGM}} = -2.3$ (see Table 1). However, the Fe abundances of these stars may be explained if they formed at $6 \lesssim z < 15$. The IGM then would have a net inventory of $[\text{Fe/H}] \sim -2.9$ but the Fe abundances in different regions would follow a lognormal distribution with a scatter of $\sim 0.75$ dex (Schaye et al. 2003; Simcoe et al. 2004). This leads us to suggest that some parts of the Galaxy formed from under-enriched regions of the IGM.

In conjunction with the required metal production by VMSs in the narrow mass range $M_{\text{VMS}} \approx 140–260 M_\odot$, there would also be MBHs produced by more massive stars (HW02). Both of these processes should occur in low-mass halos for $z \gtrsim 15$. This leads to a relationship between the metal (particularly Si) content of the early IGM and the inventory of MBHs ($\langle M_{\text{MBH}} \rangle \sim 270–550 M_\odot$) at the same epoch. The total mass of MBHs calculated from the VMS contribution to the Si in the early IGM using a Salpeter IMF is in remarkable agreement with that of SMBHs observed at the present epoch. These early-formed MBHs could cluster near galactic centers during the later epochs of galaxy formation when galactic outflows contributed to the IGM. Such a cluster of MBHs would, if coalesced, form an SMBH of $\sim (4–8) \times 10^6 M_\odot$ in a typical galaxy with a baryonic mass of $\sim 10^{11} M_\odot$. There is a hint of quasi-conservation of black hole masses, which implies that only the rarer SMBHs of $> 10^8 M_\odot$ can efficiently accrete gas but the more common ones of $< 10^8 M_\odot$ cannot. It remains to be investigated whether a mechanism exists to inhibit gas accretion onto lower-mass SMBHs. In conclusion, early “chemistry” appears to provide considerable insights into aspects of cosmological problems and offer many intriguing possibilities regarding larger cosmological issues.

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Fig. 1.— Model for evolution of abundances of Si (solid curve), O (short-dashed curve), and C (dot-dashed curve). The sources are taken to be VMSs (yields from HW02) in low-mass halos for $z > 15$ and galactic outflows of SN II products (yields from WW95) from intermediate-mass halos for $z < 15$. The evolution is governed by the rates $\Lambda_{VMS}^{\text{SN II}}$ and $\epsilon \Lambda_{E}^{\text{SN II}}$. For Si these are chosen to give $[\text{Si/H}] = -2.3$ at $z = 15$ (point A) and the full IGM inventory of $[\text{Si/H}] = -2.0$ at $z = 4$ (point B). The rates for O and C are fixed by the yields of the sources. The long-dashed curve represents the evolution resulting from just galactic outflows.
Fig. 2.— Production rates of Si required to provide 50% (solid curve) and all (dashed curve) of the IGM inventory of $[\text{Si/H}] = -2.0$ by a given $z$ as calculated from equation (9). The dot-dashed curve corresponds to the condition that every halo of mass $M_n \sigma \sim 10^5 M_\odot$ would have $R_n \sigma t(z) \sim 1$ VMS by a given $z$. If VMS formation occurs at $\sim 10\%$ of the time in such halos, the bulk of the Si inventory can only be obtained at $z \lesssim 15$. 
Table 1. IGM Data, Stellar Models, and Example Results

<table>
<thead>
<tr>
<th></th>
<th>[Si/C]</th>
<th>[C/O]</th>
<th>[O/Fe]</th>
<th>[Si/H]</th>
<th>[C/H]</th>
<th>[O/H]</th>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
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<td>IGM</td>
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<td>-0.50</td>
<td>⋯</td>
<td>-2.0</td>
<td>-2.8</td>
<td>-2.3</td>
<td>⋯</td>
</tr>
<tr>
<td>VMSs</td>
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<td>-0.57</td>
<td>-0.02</td>
<td>-2.3</td>
<td>-3.5</td>
<td>-2.9</td>
<td>-2.9</td>
</tr>
<tr>
<td>SNe II</td>
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<td>-0.42</td>
<td>0.17</td>
<td>-2.3</td>
<td>-2.7</td>
<td>-2.3</td>
<td>-2.5</td>
</tr>
<tr>
<td>Mixture</td>
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<td>-0.45</td>
<td>0.13</td>
<td>-2.0</td>
<td>-2.7</td>
<td>-2.2</td>
<td>-2.3</td>
</tr>
</tbody>
</table>

Data on the IGM inventory are for UVB model QG and are from Schaye et al. 2003, Aguirre et al. 2004, and Simcoe et al. 2004. Yield ratios for the VMS model of HW02 and the SN II model of WW95 assuming a Salpeter IMF (models 2A and 6 in Table 2 of QW05) are given in cols. (2)–(4). The reference solar abundances are the same as adopted in QW05. A mixture is calculated where VMSs and SNe II contribute equally to the Si. The respective contributions from VMSs and SNe II are given in cols. (5)–(8).