Flavor Mass and Mixing and $S_3$ Symmetry
– An $S_3$ Invariant Model Reasonable to All –

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Abstract

We assume that weak bases of flavors $(u, c)^{L,R}$, $(d, s)^{L,R}$, $(e, \mu)^{L,R}$, $(\nu_e, \nu_\mu)^{L,R}$ are the $S_3$ doublet and $t^{L,R}$, $b^{L,R}$, $\tau^{L,R}$, $\nu_\tau^{L,R}$ are the $S_3$ singlet and further there are $S_3$ doublet Higgs ($H_D^1$, $H_D^2$) and $S_3$ singlet Higgs $H_S$. We suggest an $S_3$ invariant Yukawa interaction, in which masses caused from the interaction of $S_3$ singlet flavors and Higgs is very large and masses caused from interactions of $S_3$ doublet flavors and Higgs are very small, and the vacuum expectation value $\langle |H_D^1| \rangle_0$ is rather small compared to the $\langle |H_D^2| \rangle_0$. In this model, we can explain the quark sector mass hierarchy, flavor mixing $V_{\text{CKM}}$ and measure of CP violation naturally. The leptonic sector mass hierarchy and flavor mixing described by $V_{\text{MNS}}$ having one-maximal and one-large mixing character can also be explained naturally with no other symmetry restriction. In our model, an origin of Cabibbo angle is the ratio $\lambda = \langle |H_D^1| \rangle_0 / \langle |H_D^2| \rangle_0$ and an origin of CP violation is the phase of $H_D^1$.

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I. INTRODUCTION

Many authors have succeeded in explaining the quark mass hierarchy and quark mixing characterized by $V_{CKM}$ and neutrino mixing $V_{MNS}$ having the large mixing character confirmed by recent experiments [1, 2], using democratic/universal model [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. However, many works adopting the democratic/universal model assume small mass terms deviated from democratic/universal masses as to be any violations to a symmetry reflecting in democratic/universal character. In Fritzsch-type model, the principle why Fritzsch-type mass matrix is assumed is not clear.

Many authors [17, 18, 19] have considered the $S_3$ symmetry as a symmetry reflecting in democratic/universal character. Especially, Kubo et. al. [19] suggested an outstanding model considering the small mass terms deviated from democratic/universal masses as to be $S_3$ invariant. However, their model assumes an additional symmetry to explain the neutrino mixing having bi-maximal mixing character.

In this work, we suggest a model in which the small mass terms deviated from democratic/universal mass are constructed by an $S_3$ invariant manner, and the quark mass hierarchy and mixing and neutrino mixing having one-maximal and one-large mixing character confirmed by recent experiments [1, 2] is explained without any other symmetry restriction.

II. $S_3$ INvariant MODEL

In the democratic/universal models, the mass matrix for the quark and lepton fields is assumed to be composed of the completely universal term with respect to an basis $v_{L,R} = ^t(v_1, v_2, v_3)_{L,R}$ ($t$ denotes the transpose of matrix or vector) and the small violation term or small gap from the universal term as following form [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16],

$$M = \Gamma \left[ \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right] - \left( \begin{array}{ccc} \delta_4 & \delta_1 & \delta_2 \\ \delta_1 & \delta_5 & \delta_3 \\ \delta_2 & \delta_3 & 0 \end{array} \right), \quad \delta_i \ll 1 \quad (i = 1, 2, 3, 4, 5),$$

(1)

where we assumed a symmetric mass matrix, because it is natural to assume that there is no difference in kinematical quantity producing mass between $u_L$ and $u_R$, then the magnitudes of the Yukawa interaction of, e.g., $u_Ls_R$ and $s_Lu_R$ are same. In next section, we consider the CP violation, then we will treat an Hermitian mass matrix there. In the Fritzsch-type
or models \cite{14,15,16}, the mass matrix has the following form,

\[
M = \begin{pmatrix}
0 & A & 0 \\
A & 0 & B \\
0 & B & C
\end{pmatrix}, \quad A \ll B \ll C.
\] (2)

Mass matrix (1) is transformed by an orthogonal matrix \(T\) with three small angles to the form, in which basis vectors are transformed a little,

\[
\mathbf{v}'_{L,R} = T^{-1}(v_1, v_2, v_3)_{L,R}, \\
M' = T^{-1}MT = \Gamma' \\
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix} - \begin{pmatrix}
0 & \delta'_1 & \delta'_2 \\
\delta'_1 & 0 & \delta'_3 \\
\delta'_2 & \delta'_3 & 0
\end{pmatrix}, \quad \delta'_i \ll 1 \quad (i = 1, 2, 3),
\]

or

\[
\mathbf{v}''_{L,R} = T^{-1}(v''_1, v''_2, v''_3)_{L,R}, \\
M'' = T^{-1}MT = \Gamma'' \\
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix} - \begin{pmatrix}
\delta''_1 & 0 & 0 \\
0 & \delta''_2 & 0 \\
0 & 0 & \delta''_3
\end{pmatrix} . \quad \delta''_i \ll 1 \quad (i = 4, 5, 6).
\]

Thus, the basis vector for flavor fields depends on the mass matrix pattern assumed in the democratic/universal model. This fact suggests that one cannot define a basis vector definitely in the democratic/universal model. We will discuss on this problem later in this section.

Hereafter, we assume the mass matrix pattern (4) for we used this mass matrix in our early works \cite{9,10,11}. We define the eigen vectors \(\mathbf{f}_{L,R} = T^{-1}(f_1, f_2, f_3)_{L,R}\) corresponding to the eigenvalues \((0, 0, 3\Gamma)\) for the first term in (4) (we drop the prime (‘) in (4) hereafter),

\[
\begin{pmatrix}
f_1 \\
f_2 \\
f_3
\end{pmatrix}_{L,R} = T_0^{-1} \begin{pmatrix}
h_1 \\
h_2 \\
h_3
\end{pmatrix}_{L,R},
\]

\[
T_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\
0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}
\end{pmatrix},
\]

\[
= \begin{pmatrix}
\frac{1}{\sqrt{2}}v_1 - \frac{1}{\sqrt{2}}v_2 \\
\frac{1}{\sqrt{6}}v_1 + \frac{1}{\sqrt{6}}v_2 - \frac{2}{\sqrt{6}}v_3 \\
\frac{1}{\sqrt{3}}v_1 + \frac{1}{\sqrt{3}}v_2 + \frac{1}{\sqrt{3}}v_3
\end{pmatrix}_{L,R},
\]

\[
= \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix} \begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}_{L,R}
\]

(5)
where the definition of $T_0$ is the inverse of the $T_0$ used in our previous papers \[9, 10, 11\]. In this basis vector $f_{L,R} = {}^t(f_1, f_2, f_3)_{L,R}$, the mass matrix (4) is expressed as

$$M_f = T_0^{-1}MT_0 = \Gamma \begin{pmatrix}
\delta_1 & \frac{1}{\sqrt{3}}(\delta_2 - \delta_3) & -\frac{1}{\sqrt{6}}(\delta_2 - \delta_3) \\
\frac{1}{\sqrt{3}}(\delta_2 - \delta_3) & \frac{1}{3}(\delta_1 + 2\delta_2 + 2\delta_3) & \frac{1}{3\sqrt{2}}(-2\delta_1 + \delta_2 + \delta_3) \\
-\frac{1}{\sqrt{6}}(\delta_2 - \delta_3) & \frac{1}{3\sqrt{2}}(-2\delta_1 + \delta_2 + \delta_3) & \frac{1}{3}(9 - 2\delta_1 - 2\delta_2 - 2\delta_3)
\end{pmatrix}. \quad (6)$$

Diagonalising this mass matrix $M_f$ by the transformation matrix $U(\delta_1, \delta_2, \delta_3)$ nearly equal to identity, the mass eigenvalues $m_1, m_2, m_3$ and eigenvectors corresponding to these masses are obtained as

$$\text{diag}(m_1, m_2, m_3) = U^{-1}(\delta_1, \delta_2, \delta_3)M_fU(\delta_1, \delta_2, \delta_3)$$

$$m_1 \approx \left[\frac{1}{3}(\delta_1 + \delta_2 + \delta_3) - \frac{1}{3}\xi\right] \Gamma \approx \delta_1 \Gamma,$$

$$m_2 \approx \left[\frac{1}{3}(\delta_1 + \delta_2 + \delta_3) + \frac{1}{3}\xi\right] \Gamma \approx \frac{2}{3}(\delta_2 + \delta_3) \Gamma,$$

$$m_3 \approx \left[3 - \frac{2}{3}(\delta_1 + \delta_2 + \delta_3)\right] \Gamma \approx 3\Gamma,$$

$$\xi = [(\delta_2 + \delta_3 - 2\delta_1)^2 + 3(\delta_2 - \delta_3)^2]^{1/2}$$

$${}^t(f_1^m, f_2^m, f_3^m)_{L,R} = U^{-1}(\delta_1, \delta_2, \delta_3){}^t(f_1, f_2, f_3)_{L,R},$$

where $(f_1^m, f_2^m, f_3^m)_{L,R}$ is mass basis. In our paper \[9\], we analyzed the quark sector mass hierarchy and quark mixing matrix $V_{\text{CKM}}$ numerically using this mass matrix (4), and get the numerical results that

$$\delta_1, \delta_3 - \delta_2 \ll \delta_2, \delta_3 \ll 1. \quad (8)$$

The basis ${}^t(f_1, f_2, f_3)_{L,R}$ may be considered as a weak basis because the mass matrix (6) in the basis ${}^t(f_1, f_2, f_3)_{L,R}$ has non zero nondiagonal elements and then produce the flavor mixing matrix not equal to identity. The Fritzsch-type mass matrix (2) is also considered to be a model based on the weak basis because of the parameter hierarchy $A \ll B \ll C$. It should be noted that the weak basis based on above democratic/universal-type model and Fritzsch-type model has an ambiguity depending on which type mass pattern is assumed.

The basic idea of the democratic/universal-type model is as follows; the Yukawa interaction strengths composed of the basis field $(v_1, v_2, v_3)_{L,R}$ are almost same values, then the main part of the mass matrix is invariant under the permutation of the basis fields $(v_1, v_2, v_3)_{L,R}$, i.e., the basis $(v_1, v_2, v_3)_{L,R}$ is considered as the basis field of the permutation group $S_3$ and the main part of the mass matrix is the product of the $S_3$ singlets $f^L=S^R=(v_1+v_2+v_3)_{L,R}/\sqrt{3}$. Mass matrix (4) has the small parameters $\delta_1, \delta_2, \delta_3$ in addition to this main mass term. We assume that these small parameters are produced by the way
in which $S_3$ invariance is satisfied. The basis $(v_1, v_2, v_3)_{L,R}$ are translated to $S_3$ singlet $f^{L,R}_S$ and doublet $f^{L,R}_D$ as

$$
S_3 \text{ singlet } f^{L,R}_S = \frac{1}{\sqrt{3}}(v_1 + v_2 + v_3)_{L,R},
$$

$$
S_3 \text{ doublet } f^{L,R}_D = \begin{pmatrix}
  f^{L,R}_1 \\
  f^{L,R}_2 
\end{pmatrix}
= \begin{pmatrix}
  \frac{1}{\sqrt{2}}(v_1 - v_2)_{L,R} \\
  \frac{1}{\sqrt{6}}(v_1 + v_2 - 2v_3)_{L,R}
\end{pmatrix},
$$

(9)

By using these $S_3$ singlet and doublet of flavors, we can make the following $S_3$ invariant Yukawa interaction;

$$
\Gamma_S \bar{f}^L_S f^R_S H + \Gamma_D \bar{f}^L_D f^R_D H + h.c.,
$$

where $H$ is the neutral part of $SU(2)_L$ Higgs doublet field and is considered as $S_3$ singlet. The mass matrix of this Yukawa interaction is written as

$$
\begin{pmatrix}
  \Gamma_D h & 0 & 0 \\
  0 & \Gamma_D h & 0 \\
  0 & 0 & \Gamma_S h
\end{pmatrix},
$$

where $h = \langle H \rangle_0$ is the vacuum expectation value of $H$. One can get the mass of doublet $f_1$ and $f_2$, but cannot get the mass hierarchy $m_1 \ll m_2 \ll m_3$. Thus, we introduce an $S_3$ doublet of the Higgs field

$$
S_3 \text{ singlet } : H_S, \quad S_3 \text{ doublet } : H_D = (H_D^1, H_D^2),
$$

(10)

and make the $S_3$ invariant Yukawa interactions including the neutral $S_3$ doublet $H_D$ [19];

$$
-\mathcal{L}_D = \Gamma_S \bar{f}^L_S f^R_S H_S + \Gamma_D^1 \bar{f}^L_D f^R_D H_S + \Gamma_D^2 [(\bar{f}^L_1 f^R_2 + \bar{f}^L_2 f^R_1) H_D^1 + (\bar{f}^L_1 f^R_2 - \bar{f}^L_2 f^R_1) H_D^2]

+ \Gamma_D^3 (\bar{f}^L_D H_D^1 f^R_S + \bar{f}^L_S H_D^1 f^R_D) + h.c.,
$$

(11)

where we used the fact that the $S_3$ doublet can be made from the tensor product of $\bar{f}^L_D$ and $f^R_D$ [17, 18, 19] as

$$
\begin{pmatrix}
  \bar{f}^L_1 f^R_2 + \bar{f}^L_2 f^R_1 \\
  \bar{f}^L_1 f^R_2 - \bar{f}^L_2 f^R_1
\end{pmatrix}.
$$

In general $H_S$ and $H_D$ are complex fields, but in this section we assume that these fields are real for simplicity of discussion. In next section discussing the quark sector mass and mixing, we consider the case that $H_S$ and $H_D$ are complex fields for we discuss the CP
violation. The mass matrix corresponding to Yukawa interaction (11) is expressed as

\[ M_f = \begin{pmatrix}
\Gamma_D^1 h + \Gamma_D^2 h_2 & \Gamma_D^2 h_1 & \Gamma_D^3 h_1 \\
\Gamma_D^2 h_1 & \Gamma_D^1 h - \Gamma_D^2 h_2 & \Gamma_D^3 h_2 \\
\Gamma_D^3 h_1 & \Gamma_D^3 h_2 & \Gamma_S h
\end{pmatrix}, \tag{12} \]

\[ h = \langle H_S \rangle_0, \ h_1 = \langle H_D^1 \rangle_0, \ h_2 = \langle H_D^2 \rangle_0. \]

Because \( \Gamma_D \) is considered to be very small compared to \( \Gamma_S \), this mass matrix can be similar to the mass matrix (6), if the following condition is satisfied,

\[ \Gamma_D^1 h + \Gamma_D^2 h_2 \ll \Gamma_D^1 h - \Gamma_D^2 h_2 \approx \Gamma_D^3 h_2 \ll \Gamma_S h, \ h_1 \ll h_2. \tag{13} \]

Here, it should be stressed that the expectation value of \( H_D^1 \) has to be rather small compared to that of \( H_D^2 \) so as to produce the realistic quark mass hierarchy and mixing matrix \( V_{CKM} \).

Next we consider the neutrino mass. For neutrino, we assume that there are very large Majorana masses constructed from the right-handed neutrinos and very small neutrino masses are obtained from the see-saw mechanism \[20\]. The neutrino Dirac mass matrix is obtained from the Yukawa interaction similar to Eq. (11). The Higgs fields are necessary in the Yukawa interaction for Dirac mass because \( SU(2)_L \) doublet left-handed Dirac field could not make \( SU(2)_L \) invariant interactions without \( SU(2)_L \) doublet of Higgs fields. But there is no reason that Higgs field is necessary for the Yukawa interaction producing Majorana mass. Thus we assume that the Majorana mass is obtained from the \( S_3 \) invariant Yukawa interaction containing only right handed neutrino \( \nu^R_D = (\nu^R_1, \nu^R_2, \nu^R_S) \), \( \nu^R_S \) and no Higgs field \[19\] as

\[ -\mathcal{L}_M = \Gamma_S^M \nu^R_S C^{-1} \nu^R_S + \Gamma_D^M \nu^R_D C^{-1} \nu^R_D. \tag{14} \]

If \( \Gamma_D^M \ll \Gamma_S^M \) as case of the Dirac neutrino mass, we can explain the one-maximal and one-large neutrino mixing character in this model without any other symmetry restriction. We will discuss this problem in section 4 in detail.

Finally, we comment on the weak basis, which is the flavor basis in weak interaction. In \( SU(3)_C \times SU(2)_L \times U(1)_Y \) standard model, there is no definite definition determining the weak basis because we have no definite criterion for assuming a mass matrix pattern and then Yukawa interaction constructing the flavor masses. However, if one assume the \( S_3 \) symmetry for flavor and define the weak basis using the \( S_3 \) singlet and doublet defined in
Eq. (9) as follows;

\[
\left( \begin{array}{c}
    f_{1L,R} \\
    f_{2L,R} \\
    f_{3L,R}
\end{array} \right) = \left( \begin{array}{c}
    u^{L,R} \\
    d^{L,R} \\
    e^{L,R}
\end{array} \right), \quad \left( \begin{array}{c}
    c^{L,R} \\
    s^{L,R} \\
    \mu^{L,R}
\end{array} \right), \quad \left( \begin{array}{c}
    \tau^{L,R} \\
    \nu^{L,R} \\
    \nu^{L,R}
\end{array} \right),
\]

(15)

the \(S_3\) invariant Yukawa interaction is determined uniquely as Eqs. (11) and (14), then the weak basis defined Eq. (15) is unique.

III. QUARK MASS AND MIXING

In this section, we consider the case that the neutral Higgs fields \(H_S\) and \(H_D\) are complex and then have phases. We assume the Yukawa interaction describing the masses of the \(d-\) and \(u\)-quark sector similar to Eq. (11);

\[
-\mathcal{L}_D^d = \Gamma_D^{dL} f_{1L}^R \bar{H}_S H_D + \Gamma_D^{dL} f_{2L}^R \bar{H}_S H_D + \Gamma_D^{dL} [ (f_{1L}^R f_{1L}^R H_D^1 + f_{2L}^R f_{2L}^R H_D^2) + (f_{1L}^R f_{1L}^R - f_{2L}^R f_{2L}^R) H_D^2 ] \\
+ \Gamma_D^{dL} [ (f_{1L}^H H_D^1 + f_{2L}^H H_D^2) f_{1L}^R + f_{2L}^H H_D^1 f_{1L}^R + H_D^2 f_{2L}^R ] + \text{h.c.}, \quad \text{for } d\text{-quark sector}
\]

\[
-\mathcal{L}_D^u = \Gamma_D^{uL} f_{1L}^R \bar{H}_S H_D + \Gamma_D^{uL} f_{2L}^R \bar{H}_S H_D + \Gamma_D^{uL} [ (f_{1L}^R H_D^1 + f_{2L}^R H_D^2) f_{1L}^R + f_{2L}^R H_D^1 f_{1L}^R + H_D^2 f_{2L}^R ] + \text{h.c.}, \quad \text{for } u\text{-quark sector}
\]

(16)

\[
\left( \begin{array}{c}
    f_{1L,R}^d \\
    f_{2L,R}^d \\
    f_{3L,R}^d
\end{array} \right) = \left( \begin{array}{c}
    u^{L,R} \\
    d^{L,R} \\
    e^{L,R}
\end{array} \right), \quad \left( \begin{array}{c}
    c^{L,R} \\
    s^{L,R} \\
    \mu^{L,R}
\end{array} \right), \quad \left( \begin{array}{c}
    \tau^{L,R} \\
    \nu^{L,R} \\
    \nu^{L,R}
\end{array} \right), \quad \left( \begin{array}{c}
    H_D \\
    H_D \\
    H_D
\end{array} \right).
\]

This Yukawa interaction produces an hermitian mass matrix, as shown in just below.

The phase of \(H_S\) is cancel out from the phase transformation of phase transformation of \(f_{1L}^R\) in the first term and \(f_{2L}^D\) in the second term caused by the \(SU(2)_L\) gauge freedom and then we can take the \(H_S\) to be real. Here we assume that the phases of \(f_{1L}^R\) and \(f_{2L}^D\) after the \(SU(2)_L\) gauge transformation are the same as that of \(f_{1L}^R\) and \(f_{2L}^D\), respectively. Thus the \(H_S\) can be real and the masses produced from the first and second term are real. The phase of \(H_D^2\) in third term has to be the same to that of the \(H_S\), because the term containing \(H_D^2\) in this forth term represents the diagonal mass and has to be real and then the \(SU(2)_L\) gauge freedom of \(f_{1L}^L\) and \(f_{2L}^L\) have to be cancelled out by the phase of \(H_D^2\). From the same procedure, terms \(f_{1L}^L H_D^1 f_{1L}^R\) and \(f_{2L}^L H_D^2 f_{2L}^R\) in the forth term can be real. However, \(H_D^1\) does not have to be the same phase to that of \(H_S\), then the terms \(f_{1L}^L f_{2L}^R H_D^1\) and \(f_{2L}^L f_{1L}^R H_D^1\) in the third term and the
terms $f_1^LF_D^f f_1^R$ and $f_1^L H_D^1 f_1^R$ in the forth term may be complex. Thus we get the hermitian mass matrices for $d$ and $u$ quark sector as follows:

\[
M_d = \begin{pmatrix}
\mu_1^d + \mu_2^d & \lambda \mu_2^d e^{i\phi} & \lambda \mu_3^d e^{i\phi} \\
\lambda \mu_2^d e^{-i\phi} & \mu_1^d - \mu_2^d & \mu_3^d \\
\lambda \mu_3^d e^{-i\phi} & \mu_3^d & \mu_0^d
\end{pmatrix},
\]

\[
M_u = \begin{pmatrix}
\mu_1^u + \mu_2^u & \lambda \mu_2^u e^{-i\phi} & \lambda \mu_3^u e^{-i\phi} \\
\lambda \mu_2^u e^{i\phi} & \mu_1^u - \mu_2^u & \mu_3^u \\
\lambda \mu_3^u e^{i\phi} & \mu_3^u & \mu_0^u
\end{pmatrix},
\]

(17)

where we use the following parameterization,

\[
\mu_0^{d,u} = \langle H_S^d, H_S^u \rangle_{SS}, \quad \mu_1^{d,u} = \langle H_{D_1}^d, H_{D_1}^u \rangle_{11,22}, \quad \mu_2^{d,u} = \langle H_{D_2}^d, H_{D_2}^u \rangle_{11,22},
\]

\[
\lambda_\mu^{d,u} = \langle H_{D_1}^d H_{D_1}^u \rangle_{12,21}, \quad \mu_3^{d,u} = \langle H_{D_2}^d H_{D_2}^u \rangle_{2S,S2}, \quad \lambda \mu_3^{d,u} = \langle H_{D_1}^d H_{D_1}^u \rangle_{1S,S1},
\]

\[
\lambda = \frac{\langle H_{D_2}^1 \rangle_0}{\langle H_{D_2}^2 \rangle_0}, \quad \phi = \text{phase of } H_D^1.
\]

Diagonalising the mass matrix (17) by unitary matrices $U(\mu_i^d, \phi)$ and $U(\mu_i^u, \phi)$, the CKM quark mixing matrix $V_{CKM}$ is defined as

\[
\text{diag}[m_d, m_s, m_b] = U^{-1}(\mu_i^d, \phi) M_d U(\mu_i^d, \phi), \quad t(\langle d^m, s^m, b^m \rangle)_{L,R} = U^{-1}(\mu_i^d, \phi) t(\langle d, s, b \rangle)_{L,R},
\]

\[
\text{diag}[m_u, m_c, m_t] = U^{-1}(\mu_i^u, \phi) M_u U(\mu_i^u, \phi), \quad t(\langle u^m, c^m, t^m \rangle)_{L,R} = U^{-1}(\mu_i^u, \phi) t(\langle u, c, t \rangle)_{L,R},
\]

\[
V_{CKM} = U^\dagger(\mu_i^u, \phi) U(\mu_i^d, \phi),
\]

(18)

where $(\langle d^m, s^m, b^m \rangle)$ and $(\langle u^m, c^m, t^m \rangle)$ are the mass eigen states for the weak basis $(d, s, b)$ and $(u, c, t)$, respectively. Here, we count the number of parameters appearing in present model for quark sector: $10 = 8$ mass parameters $\mu_i^d$, $\mu_i^u$, $+1$ ratio of Higgs $|H_D^1|$ and $|H_D^2|$, $+1$ phase parameters $\phi = \text{phase of } H_D^1$. Observable parameter number is 10: 6 for quark masses and 3 mixing angles and 1 phase in CKM matrix.

We represent approximate expressions analytically for eigenvalues of quark masses and diagonalising unitary matrix. For the $d$-quark sector, masses and the diagonalising matrix
are expressed as

\[ m_d \approx \mu_1^d - \frac{1 + \lambda^2 \mu_3^{d2}}{2 \frac{\mu_3}{\mu_0}} + \left( \mu_2^d + \frac{1 - \lambda^2 \mu_3^{d2}}{2 \frac{\mu_3}{\mu_0}} \right) \frac{1}{\cos^2 \theta_d - \sin^2 \theta_d} \approx \mu_1^d + \mu_2^d - \frac{\lambda^2 \mu_3^{d2}}{2 \frac{\mu_3}{\mu_0}}, \]

\[ m_s \approx \mu_1^d - \frac{1 + \lambda^2 \mu_3^{s2}}{2 \frac{\mu_3}{\mu_0}} - \left( \mu_2^d + \frac{1 - \lambda^2 \mu_3^{s2}}{2 \frac{\mu_3}{\mu_0}} \right) \frac{1}{\cos^2 \theta_d - \sin^2 \theta_d} \approx \mu_1^d - \mu_2^d - \frac{\lambda^2 \mu_3^{s2}}{2 \frac{\mu_3}{\mu_0}}, \]

\[ m_b \approx \mu_0^d, \]

\[ \tan 2\theta_d = \frac{\lambda \mu_2^d}{\mu_2^d + \frac{1 - \lambda^2 \mu_3^{d2}}{2 \frac{\mu_3}{\mu_0}}}, \]

\[ U(\mu_1^d, \phi) \approx \begin{pmatrix} \cos \theta_d & -\sin \theta_d e^{i\phi} & \frac{\lambda \mu_2^d}{\mu_0} e^{i\phi} \\ -\sin \theta_d e^{-i\phi} & \cos \theta_d & \frac{\mu_1^d}{\mu_0} \\ (\lambda \sin \theta_d - \cos \theta_d) e^{-i\phi} \frac{\mu_2^d}{\mu_0} & (\lambda \sin \theta_d - \cos \theta_d) \frac{\mu_1^d}{\mu_0} & 1 \end{pmatrix} . \] (20)

For u-quark sector, just similar expressions are obtained replacing suffix d with suffix u,

\[ m_u \approx \mu_1^u - \frac{1 + \lambda^2 \mu_3^{u2}}{2 \frac{\mu_3}{\mu_0}} + \left( \mu_2^u + \frac{1 - \lambda^2 \mu_3^{u2}}{2 \frac{\mu_3}{\mu_0}} \right) \frac{1}{\cos^2 \theta_u - \sin^2 \theta_u} \approx \mu_1^u + \mu_2^u - \frac{\lambda^2 \mu_3^{u2}}{2 \frac{\mu_3}{\mu_0}}, \]

\[ m_c \approx \mu_1^u - \frac{1 + \lambda^2 \mu_3^{c2}}{2 \frac{\mu_3}{\mu_0}} - \left( \mu_2^u + \frac{1 - \lambda^2 \mu_3^{c2}}{2 \frac{\mu_3}{\mu_0}} \right) \frac{1}{\cos^2 \theta_u - \sin^2 \theta_u} \approx \mu_1^u - \mu_2^u - \frac{\lambda^2 \mu_3^{c2}}{2 \frac{\mu_3}{\mu_0}}, \]

\[ m_t \approx \mu_0^u, \]

\[ \tan 2\theta_u = \frac{\lambda \mu_2^u}{\mu_2^u + \frac{1 - \lambda^2 \mu_3^{u2}}{2 \frac{\mu_3}{\mu_0}}}, \]

\[ U(\mu_1^u, \phi) \approx \begin{pmatrix} \cos \theta_u & -\sin \theta_u e^{i\phi} & \frac{\lambda \mu_2^u}{\mu_0} e^{-i\phi} \\ -\sin \theta_u e^{i\phi} & \cos \theta_u & \frac{\mu_1^u}{\mu_0} \\ (\lambda \sin \theta_u - \cos \theta_u) e^{i\phi} \frac{\mu_2^u}{\mu_0} & (\lambda \sin \theta_u - \cos \theta_u) \frac{\mu_1^u}{\mu_0} & 1 \end{pmatrix} . \] (22)

The CKM matrix is also written analytically using the expressions Eqs. (20) and (22) as

\[ V_{CKM} = U^\dagger(\mu_1^u, \phi) U(\mu_1^d, \phi) \]

\[ \approx \begin{pmatrix} \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d e^{i2\phi} \\ -\sin \theta_u \cos \theta_d e^{i\phi} + \cos \theta_u \sin \theta_d e^{-i\phi} \\ (\lambda \cos \theta_u e^{i\phi} + \sin \theta_d) \frac{\mu_2^d}{\mu_0} - (\lambda \cos \theta_d + \sin \theta_d) \frac{\mu_2^u}{\mu_0} e^{-i\phi} \\ -\sin \theta_u \cos \theta_u e^{i\phi} + \sin \theta_u \cos \theta_u e^{-i\phi} \\ \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d e^{i2\phi} \\ (\lambda \sin \theta_u e^{i2\phi} + \cos \theta_d e^{i\phi}) \frac{\mu_2^d}{\mu_0} + (\lambda \sin \theta_d - \cos \theta_d) \frac{\mu_2^u}{\mu_0} e^{-i\phi} \\ (\lambda \cos \theta_u e^{i\phi} + \sin \theta_d) \frac{\mu_2^d}{\mu_0} - (\lambda \cos \theta_u + \sin \theta_d) \frac{\mu_2^u}{\mu_0} e^{-i\phi} \\ (\lambda \sin \theta_u e^{i2\phi} + \cos \theta_d e^{i\phi}) \frac{\mu_2^d}{\mu_0} + (\lambda \sin \theta_u - \cos \theta_d) \frac{\mu_2^u}{\mu_0} e^{-i\phi} \\ 1 \end{pmatrix} . \] (23)
We examine our model numerically. The present experimental values for the quark masses and CKM matrix are given in the PDG 2004 [21];

\[
\begin{align*}
\frac{m_d}{m_s} &= 0.057 \pm 0.023, \quad \frac{m_s}{m_b} = 0.024 \pm 0.006, \quad m_b = 4.25 \pm 0.15 \text{GeV}, \\
\frac{m_u}{m_c} &= 0.0022 \pm 0.0010, \quad \frac{m_c}{m_t} = 0.0070 \pm 0.0007, \quad m_t = 178^{+10.4}_{-8.3} \text{GeV}, \\
|V_{CKM}| &= \begin{pmatrix}
0.9739 \text{ to } 0.9751 & 0.221 \text{ to } 0.227 & 0.0029 \text{ to } 0.0045 \\
0.221 \text{ to } 0.227 & 0.9730 \text{ to } 0.9744 & 0.039 \text{ to } 0.044 \\
0.0048 \text{ to } 0.014 & 0.037 \text{ to } 0.043 & 0.9990 \text{ to } 0.9992
\end{pmatrix},
\end{align*}
\]

(24)

vertex coordinate of unitarity triangle \(\bar{\rho} = 0.20 \pm 0.09\), \(\bar{\eta} = 0.33 \pm 0.05\),

invariant measure of CP violation \(J = (2.88 \pm 0.33) \times 10^{-5}\).

Using the computer simulation, we search the allowed values of 10 parameters satisfying the experimental values (24), and then we obtain the following values;

\[
\begin{align*}
\mu_0^d &= 4.25 \pm 0.15 \text{GeV}, \quad \frac{\mu_1^d}{\mu_0^d} = 0.0133 \pm 0.0027, \quad \frac{\mu_2^d}{\mu_0^d} = -0.0113 \pm 0.0027, \\
\frac{\mu_3^d}{\mu_0^d} &= 0.0260 \pm 0.0017, \\
\mu_0^u &= 178^{+10.4}_{-8.3} \text{GeV}, \quad \frac{\mu_1^u}{\mu_0^u} = 0.00393 \pm 0.00003, \quad \frac{\mu_2^u}{\mu_0^u} = -0.00380 \pm 0.00003, \\
\frac{\mu_3^u}{\mu_0^u} &= -0.0150 \pm 0.0003, \\
\lambda &= 0.219 \pm 0.005, \quad \phi = -(76.8 \pm 1.8)^\circ.
\end{align*}
\]

(25)

Thus, our model can produce all experimental values exactly.

We consider the meaning of the Cabibbo angle and CP violation phase in our model. \(|V_{CKM}|_{12}\) elements is expressed approximately in Eq.(23), as \(|-\cos \theta_u \sin \theta_d e^{i\phi} + \sin \theta_u \cos \theta_d e^{-i\phi}|\). This is estimated approximately by the approximate expressions (19) and (21) for \(\theta_d\) and \(\theta_u\), as

\[
\begin{align*}
[(\cos \theta_u \sin \theta_d - \sin \theta_u \cos \theta_d)^2 \cos^2 \phi + (\cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d)^2 \sin^2 \phi]^1/2 \\
\approx |\sin(\theta_d + \theta_u) \sin \phi| \approx |\lambda \sin \phi|.
\end{align*}
\]

We can say that the origin of the Cabibbo angle is the ratio \(\lambda = \langle |H_D^1|_0 \rangle / \langle |H_D^2|_0 \rangle\) and the phase of \(H_D^1\), and the origin of the CP violation phase is the phase of \(H_D^1\).
IV. LEPTON MASS AND MIXING

In this section, we consider the lepton mass hierarchy and leptonic mixing having the one-maximal and one-large character. We assume the Yukawa interaction as Eq. (11) for masses of charged lepton (e, μ, τ) and Dirac masses of (ν<sub>e</sub>, ν<sub>μ</sub>, ν<sub>τ</sub>) and the Yukawa interaction as Eq. (14) for Majorana mass;

\[
-\mathcal{L}_D^{L}\nu = \Gamma_{S}^{l,\nu} f_{S}^L f_{S}^R H_{S} + \Gamma_{D}^{l,\nu} f_{D}^L f_{D}^R H_{D} + \Gamma_{D}^{2l,\nu} [(f_{1}^L f_{2}^R + f_{2}^L f_{1}^R) H_{D} + (f_{1}^L f_{1}^R - f_{2}^L f_{2}^R) H_{D}^2] + \Gamma_{D}^{3l,\nu} (f_{D}^L H_{D} f_{S}^R + f_{S}^L H_{D} f_{D}^R) + h.c.,
\]

\[
-\mathcal{L}_M = \Gamma_{S}^{M,\nu} \nu_{S}^R C^{-1} \nu_{S}^R + \Gamma_{D}^{M,\nu} \nu_{D}^R C^{-1} \nu_{D}^R.
\]

(26)

where \( C \) is the charge conjugation matrix. In lepton sector, we do not consider the CP violation, then the phases of the fields related are not considered. Thus we get the mass matrices for charged lepton (e, μ, τ) and Dirac mass for neutrino (ν<sub>e</sub>, ν<sub>μ</sub>, ν<sub>τ</sub>) and Majorana mass as follows;

\[
M_{l} = \begin{pmatrix}
\mu_{1} + \mu_{2} & \lambda \mu_{2} & \lambda \mu_{3} \\
\lambda \mu_{2} & \mu_{1} - \mu_{2} & \mu_{3} \\
\lambda \mu_{3} & \mu_{3} & \mu_{0}
\end{pmatrix},
\]

\[
M_{\nu}^{D} = \begin{pmatrix}
\mu_{\nu} + \mu_{\nu}' & \lambda \mu_{\nu}' & \lambda \mu_{\nu} \\
\lambda \mu_{\nu}' & \mu_{\nu} - \mu_{\nu}' & \mu_{\nu}' \\
\lambda \mu_{\nu} & \mu_{\nu}' & \mu_{0}
\end{pmatrix},
\]

\[
M_{M} = \begin{pmatrix}
M_{1} & 0 & 0 \\
0 & M_{1} & 0 \\
0 & 0 & M_{0}
\end{pmatrix}.
\]

(27)

Through the see-saw mechanism, neutrino grows up Majorana neutrino and masses fall down extremely small because of the GUT scale of Majorana mass, and then the mass matrix for left-handed Majorana neutrino is expressed as

\[
M_{\nu}^{M} = M_{\nu}^{D} M_{M}^{-1} M_{\nu}^{D} =
\begin{pmatrix}
\frac{\mu_{\nu} + \mu_{\nu}'}{M_{1}} + \frac{\lambda \mu_{\nu}'}{M_{0}} & \lambda \left( \frac{2\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\nu}^2}{M_{0}} \right) & \lambda \left( \frac{\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\nu}^2}{M_{0}} \right) \\
\lambda \left( \frac{2\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\nu}^2}{M_{0}} \right) & \frac{\mu_{\nu}^2}{M_{1}} + \frac{\mu_{\tau}^2}{M_{0}} & \lambda \left( \frac{\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\tau}^2}{M_{0}} \right) \\
\lambda \left( \frac{\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\tau}^2}{M_{0}} \right) & \lambda \left( \frac{2\mu_{\nu} \mu_{\nu}'}{M_{1}} + \frac{\mu_{\tau}^2}{M_{0}} \right) & \frac{\mu_{\tau}^2}{M_{1}} + \frac{\mu_{\tau}^2}{M_{0}}
\end{pmatrix}.
\]

(28)
Diagonalising the mass matrix (27) of charged lepton and (28) of neutrino by unitary matrices $U(\mu_0^e)$ and $U(\mu_i^\nu, M_i)$, the MNS leptonic mixing matrix $V_{MNS}$ is defined as

\[
\text{diag}(m_e, m_\mu, m_\tau) = U^{-1}(\mu_0^e)M_i U(\mu_0^e), \quad t(e^m, \mu^m, \tau^m)_{L,R} = U^{-1}(\mu_0^e)t(e, \mu, \tau)_{L,R},
\]

\[
\text{diag}(m_\nu_e, m_\nu_\mu, m_\nu_\tau) = U^{-1}(\mu_i^\nu, M_i)M_\nu^M U(\mu_i^\nu, M_i), \quad t(\nu^m_e, \nu^m_\mu, \nu^m_\tau)_{L} = U^{-1}(\mu_i^\nu, M_i)t(\nu_e, \nu_\mu, \nu_\tau)_{L},
\]

\[
V_{MNS} = U(\mu_i^\nu)^\dagger U(\mu_i^\nu, M_i),
\]

where $(e^m, \mu^m, \tau^m)$ and $(\nu^m_e, \nu^m_\mu, \nu^m_\tau)$ are mass eigen states of charged lepton $(e, \mu, \tau)$ and Majorana neutrino $(\nu_e, \nu_\mu, \nu_\tau)$.

Here, we show the leptonic mixing $V_{MNS}$ has one-maximal and one-large mixing character. As shown later, $\mu_1^e, \mu_2^e, \mu_3^e$ are very small compared to $\mu_0^e$, then $U(\mu_i^\nu)$ is nearly equal to identity. Thus leptonic mixing is almost equal to neutrino mixing: $V_{MNS} \approx U(\mu_i^\nu, M_i)$. Now, if $M_0 \gg M_1$ in Eq. (28), second terms in every elements are negligible. The parameter values for $\mu_i^\nu$ can be assumed $\mu_1^\nu \approx -\mu_2^\nu, \mu_3^\nu \ll \mu_0$ and $\lambda \approx 0.22$ as estimated in previous quark sector, then, if we set $\mu_1^\nu - \mu_2^\nu \approx \mu_3^\nu, \mu_1^\nu + \mu_2^\nu = \delta \ll \mu_3^\nu$ and neglect the $\lambda^2$ term, we can get the following expression

\[
M_\nu^M \approx \left( \begin{array}{ccc}
\frac{\delta^2}{M_1} & -\lambda \frac{\mu_3^\nu}{2M_1} & -\lambda \frac{\mu_3^\nu - 3\mu_2^\nu}{2M_1} \\
-\lambda \frac{\mu_3^\nu}{2M_1} & \frac{\mu_3^\nu}{M_1} & \frac{\mu_3^\nu}{M_1} \\
-\lambda \frac{\mu_3^\nu - 3\mu_2^\nu}{2M_1} & \frac{\mu_3^\nu}{M_1} & \frac{\mu_3^\nu}{M_1}
\end{array} \right).
\]

This matrix can be diagonalized by the unitary matrix $U(\mu_i^\nu, M_i)$ as

\[
\text{diag} \left( \begin{array}{c}
\frac{\mu_1^\nu}{M_1} \\
\frac{\mu_2^\nu}{M_1} \\
\frac{\mu_3^\nu}{M_1}
\end{array} \right) = U(\mu_i^\nu, M_i)^{-1} M_\nu^M U(\mu_i^\nu, M_i),
\]

\[
U(\mu_i^\nu, M_i) \approx \left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & 1/\sqrt{2} \\
0 & -1/\sqrt{2} & 1/\sqrt{2}
\end{array} \right) \left( \begin{array}{ccc}
1 & 0 & \eta \\
0 & 1 & 0 \\
0 & -\eta & 1
\end{array} \right) \left( \begin{array}{ccc}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{array} \right),
\]

\[
\eta \approx -\frac{\lambda}{2\sqrt{2}} (1 - \frac{\delta}{2\mu_3}), \quad \tan 2\theta \approx \frac{3\lambda \mu_3}{\sqrt{2} \delta}.
\]
From this approximate expression, it is recognized that $\nu_\mu - \nu_\tau$ mixing becomes maximal and $\nu_e - \nu_\mu$ mixing angle can be large, for example, if $\delta \approx \lambda \mu_3$, $\tan 2\theta \approx \frac{3}{\sqrt{2}}$. Furthermore, it is recognized that $|V_{\text{MNS}}|_{13} \approx \eta \approx \lambda/2\sqrt{2}$.

Next, we examine our model numerically. Firstly, we analyze the charged lepton sector; estimate $\mu_i^l$ and $U(\mu_i^l)$ satisfying experimental data using Eqs. (27) and (29). Using the mass of $(e, \mu, \tau)$:

$$\frac{m_e}{m_\mu} = 0.004835 \pm 0.000005, \frac{m_\mu}{m_\tau} = 0.05946 \pm 0.00001, m_\tau = 1776.99^{+0.29}_{-0.26} \text{MeV},$$

(33)

we can get the values of parameters and charged lepton mixing matrix:

$$\frac{\mu_0}{\mu_0} = 1777 \text{MeV}, \lambda = 0.219 \pm 0.005 \text{ (in-put determined from quark sector analysis),}$$

$$\frac{\mu_1^l}{\mu_0} = 0.03007 \pm 0.00005, \frac{\mu_2^l}{\mu_0} = -(0.02900 \pm 0.00005), \frac{\mu_3^l}{\mu_0} = 0.0250 \pm 0.0017,$$

$$U(\mu_i^l) = \begin{pmatrix}
0.993 \sim 0.994 & -(0.113 \sim 0.114) & 0.005 \sim 0.006 \\
0.113 \sim 0.114 & 0.993 & 0.025 \sim 0.028 \\
-(0.008 \sim 0.009) & -(0.024 \sim 0.027) & 1.00
\end{pmatrix}.$$  (34)

Present experimental status of neutrino mixing is summarized as $\sin^2 2\theta_{\text{atm}} > 0.9$, $1.3 \times 10^{-3} \text{eV}^2 < \Delta m_{\text{atm}}^2 < 3.0 \times 10^{-3} \text{eV}^2$,

$$3.0 \times 10^{-1} < \tan^2 \theta_\odot < 5.8 \times 10^{-1},$$

$$5.9 \times 10^{-5} \text{eV}^2 < \Delta m_\odot^2 < 9.3 \times 10^{-5} \text{eV}^2,$$

(35)

$$|V_{\text{MNS}}|_{13}^2 < 0.067.$$

We assume that $\Delta m_{\text{atm}}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2$, $\Delta m_\odot^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2$, $\theta_{\text{atm}} = \nu_\mu - \nu_\tau$ mixing angle, $\theta_\odot = \nu_e - \nu_\mu$ mixing angle, then the ratios of neutrino masses and magnitudes of elements of $V_{\text{MNS}}$ are restricted as

$$0 < \frac{m_{\nu_e}}{m_{\nu_\mu}} < 0.6, \ 0.14 < \frac{m_{\nu_\mu}}{m_{\nu_\tau}} < 0.24, \ 0.036 \text{eV} < m_{\nu_\tau} < 0.055 \text{eV}$$

$$|V_{\text{MNS}}| = \begin{pmatrix}
|c_\odot c_{13}| & |s_\odot c_{13}| & |s_{13}|
\end{pmatrix} = \begin{pmatrix}
|c_\odot c_{13}| - c_\odot s_\odot s_{13} & |c_\odot c_{13}| - c_\odot s_{23} s_{13} & |c_\odot c_{13}| - c_\odot s_{13} c_{13}
\end{pmatrix},$$

$$\begin{pmatrix}
0.77 \sim 0.88 & 0.46 \sim 0.61 & 0.00 \sim 0.26 \\
0.10 \sim 0.49 & 0.47 \sim 0.78 & 0.57 \sim 0.81 \\
0.28 \sim 0.61 & 0.34 \sim 0.71 & 0.57 \sim 0.81
\end{pmatrix},$$

(36)

where $c_\odot = \cos \theta_\odot$, $s_\odot = \sin \theta_\odot$, $c_{\text{atm}} = \cos \theta_{\text{atm}}$, $s_{\text{atm}} = \sin \theta_{\text{atm}}$.
Using the Eqs. (27), (28), (29) and the numerical result (34) for charged lepton and experimental data (36) for neutrino, we estimate the allowed values for parameters \( \mu_e' \):

\[
\lambda = 0.219 \pm 0.005 \text{ (input determined from quark sector analysis),}
\frac{\mu_d}{\mu_d^0} = 0.050 \pm 0.007, \quad \frac{\mu_e'}{\mu_e'} = -(0.021 \pm 0.007), \quad \frac{\mu_u'}{\mu_u'} = 0.052 \pm 0.010,
\]
\[
\frac{M_1}{M_0} = 0.0020 \pm 0.0007.
\]

For these allowed parameters, the mass of \( \nu_e \) and \( |V_{MNS}|_{13} \) are rather restricted as

\[
\frac{m_{\nu_e}}{m_{\nu_e}} = 0.36 \sim 0.49, \quad |V_{MNS}|_{13} = 0.04 \sim 0.06.
\]

V. CONCLUSION

We assumed the weak bases of flavors \((u, c)_{L,R}, (d, s)_{L,R}, (e, \mu)_{L,R}\) and Dirac neutrino \((\nu_e, \nu_\mu)_{L,R}\) are the \(S_3\) doublet and \(t_{L,R},b_{L,R},\tau_{L,R},\nu_{\tau_{L,R}}\) are the \(S_3\) singlets. Further, we assumed the Higgs \(S_3\) doublet \((H^1_D, H^2_D)\) and Higgs \(S_3\) singlet \(H_S\). In general, though \(H^1_D, H^2_D, H_S\) are complex, \(H^2_D, H_S\) can be made real by the \(SU(2)_L\) gauge freedom of \((u, c)_L, (d, s)_L, (e, \mu)_L, (\nu_e, \nu_\mu)_L\). From these \(S_3\) doublets and singlets, we constructed \(S_3\) invariant Yukawa interactions and hermitian mass matrices for weak basis of flavor. In our model, because the way to construct an \(S_3\) invariant Yukawa interaction is unique, we can define the weak basis of flavor unambiguously.

Obtained mass matrices for quark sector are

\[
M_d = \begin{pmatrix}
\mu_1^d + \mu_2^d & \lambda \mu_3^d e^{i\phi} & \lambda \mu_3^d e^{i\phi} \\
\lambda \mu_2^d e^{-i\phi} & \mu_1^d - \mu_2^d & \mu_3^d \\
\lambda \mu_3^d e^{-i\phi} & \mu_3^d & \mu_0^d
\end{pmatrix},
M_u = \begin{pmatrix}
\mu_1^u + \mu_2^u & \lambda \mu_2^u e^{-i\phi} & \lambda \mu_3^u e^{-i\phi} \\
\lambda \mu_2^u e^{i\phi} & \mu_1^u - \mu_2^u & \mu_3^u \\
\lambda \mu_3^u e^{i\phi} & \mu_3^u & \mu_0^u
\end{pmatrix},
\]

where \(\lambda = \langle |H^1_D|_0^2 / |H^2_D|_0^2 \rangle\) and \(\phi = \text{phase of } \langle H^1_D \rangle_0\). From the present experimental data for quark masses and \(V_{CKM}\) matrix involving the CP violation [21], we can get the results;

\[
\frac{\mu_1^d}{\mu_0^d} = 0.0133 \pm 0.0027, \quad \frac{\mu_1^u}{\mu_0^u} = -0.0113 \pm 0.0027, \quad \frac{\mu_3^d}{\mu_0^d} = 0.0260 \pm 0.0017,
\]
\[
\frac{\mu_3^u}{\mu_0^u} = 0.00393 \pm 0.00003, \quad \frac{\mu_2^u}{\mu_0^u} = -0.00380 \pm 0.00003, \quad \frac{\mu_3^u}{\mu_0^u} = -0.0150 \pm 0.0003,
\]
\[
\lambda = 0.219 \pm 0.005, \quad \phi = -(76.8 \pm 1.8)^\circ.
\]

In our model, the origin of the Cabibbo angle is the ratio \(\lambda = \langle |H^1_D|_0^2 / |H^2_D|_0^2 \rangle\) and the origin of the CP violation is the phase of \(H_1\).
For lepton sector, mass matrices are obtained as

$$M_l, M_D^\nu = \begin{pmatrix} 
\mu_{1l} + \mu_{2l} & \lambda_{\mu_{1l}} & \lambda_{\mu_{3l}} \\
\lambda_{\mu_{2l}} & \mu_{1l} - \mu_{2l} & \mu_{3l} \\
\lambda_{\mu_{3l}} & \mu_{3l} & \mu_{0l}
\end{pmatrix}, \quad M_M = \begin{pmatrix} 
M_1 & 0 & 0 \\
0 & M_1 & 0 \\
0 & 0 & M_0
\end{pmatrix}.$$ 

In our model, one-maximal and one-large mixing angle character of the lepton mixing matrix $V_{\text{MNS}}$ is obtained naturally from the hierarchy of mass parameters $\mu_1^\nu + \mu_2^\nu \ll \mu_1^\nu, -\mu_2^\nu, \mu_3^\nu \ll \mu_0^\nu, M_1 \ll M_0$ and smallness of $\lambda \sim 0.22$ without any other symmetry restriction. From the present experimental data for charged lepton mass \[21\] and neutrino mass and mixing $V_{\text{MNS}}$ \[1, 2, 22\], we obtained the allowed values for mass parameters;

$$\frac{\mu_1^l}{\mu_0^l} = 0.03007 \pm 0.00005, \quad \frac{\mu_2^l}{\mu_0^l} = -(0.02900 \pm 0.00005), \quad \frac{\mu_3^l}{\mu_0^l} = 0.0250 \pm 0.0017,$$

$$\frac{\mu_1^\nu}{\mu_0^\nu} = 0.050 \pm 0.007, \quad \frac{\mu_2^\nu}{\mu_0^\nu} = -(0.021 \pm 0.007), \quad \frac{\mu_3^\nu}{\mu_0^\nu} = 0.052 \pm 0.010,$$

$$\frac{M_1}{M_0} = 0.0020 \pm 0.0007.$$ 

In this allowed values, the $\nu_e$ mass are estimated rather large as $\frac{m_{\nu_e}}{m_{\nu_i}} = 0.36 \sim 0.49$ and $|V_{\text{MNS}}|_{13}$ is restricted as 0.04 $\sim$ 0.06.


   S. Pakvasa and H. Sugawara, Phys. Lett. 82B(1979), 105.


