THE SUPERSYMMETRIC UNIVERSE

John Ellis

CERN, TH Division

CH 1211 Geneva 23, Switzerland

John.Ellis@cern.ch

Abstract

These lectures provide a phenomenological introduction to supersymmetry, concentrating on the minimal supersymmetric extension of the Standard Model (MSSM). Motivations are provided for thinking that supersymmetry might appear at the TeV scale, including the naturalness of the mass hierarchy, gauge unification and the probable mass of the Higgs boson. Then simple globally supersymmetric field theories are introduced, with the emphasis on features important for model-building. Supersymmetry breaking and local supersymmetry (supergravity) are then introduced, and the structure of sparticle mass matrices and mixing are reviewed. The available experimental and cosmological constraints on MSSM parameters are discussed and combined, and the prospects for discovering supersymmetry in future experiments are previewed. Finally, the observability of leptonic processes violating flavour and CP are discussed, on the basis of the minimal supersymmetric seesaw model of neutrino masses.

1. GETTING MOTIVATED

1.1 Defects of the Standard Model

The Standard Model agrees with all confirmed experimental data from accelerators, but is theoretically very unsatisfactory. It does not explain the particle quantum numbers, such as the electric charge Q , weak isospin I , hypercharge Y and colour, and contains at least 19 arbitrary parameters. These include three independent gauge couplings and a possible CP-violating strong-interaction parameter, six quark and three charged-lepton masses, three generalized Cabibbo weak mixing angles and the CP-violating Kobayashi-Maskawa phase, as well as two independent masses for weak bosons.

As if 19 parameters were insufficient to appall you, at least nine more parameters must be introduced to accommodate neutrino oscillations: three neutrino masses, three real mixing angles, and three CP-violating phases, of which one is in principle observable in neutrino-oscillation experiments and the other two in neutrinoless double-beta decay experiments. Even more parameters would be needed to generate neutrino masses in a credible way, associated with a heavy-neutrino sector and/or additional Higgs particles.

Eventually, one would like to include gravity in a unified theory along with the other particle interactions, which involves introducing at least two more parameters, Newton's constant $G_N = 1/m_P^2 : m_P \sim 10^{19}$ GeV that characterizes the strength of gravitational interactions, and the cosmological constant Λ or some time-varying form of vacuum energy as seems to be required by recent cosmological data. A complete theory of cosmology will presumably also need parameters to characterize the early inflation of the Universe and to generate its baryon asymmetry, which cannot be explained within the Standard Model.

The Big Issues in physics beyond the Standard Model are conveniently grouped into three categories [1]. These include the problem of **Unification**: is there a simple group framework for unifying all the particle interactions, a so-called Grand Unified Theory (GUT), **Flavour**: why are there so many different types of quarks and leptons and why do their weak interactions mix in the peculiar way observed, and **Mass**: what is the origin of particle masses, are they due to a Higgs boson, why are the masses so small? Solutions to all these problems should eventually be incorporated in a Theory of Everything (TOE) that also includes gravity, reconciles it with quantum mechanics, explains the origin of space-time and why it has four dimensions, etc. String theory, perhaps in its current incarnation of M theory, is the best (only?) candidate we have for such a TOE [2], but we do not yet understand it well enough to make clear experimental predictions.

Supersymmetry is thought to play a rôle in solving many of these problems beyond the Standard Model. As discussed later, GUT predictions for the unification of gauge couplings work best if the effects of relatively light supersymmetric particles are included [3]. Also, the hierarchy of mass scales in physics, and particularly the fact that $m_W \ll m_P$, appears to require relatively light supersymmetric particles: $M \lesssim 1$ TeV for its stabilization [4]. Finally, supersymmetry seems to be essential for the consistency of string theory [5], although this argument does not really restrict the mass scale at which supersymmetric particles should appear.

Thus there are plenty of good reasons to study supersymmetry, and we return later to examine in more detail the motivations provided by unification and the mass hierarchy problem.

1.2 The Electroweak Vacuum

Generating particle masses within the Standard Model requires breaking its gauge symmetry, and the only consistent way to do this is by breaking the symmetry of the electroweak vacuum:

$$m_{W,Z} \neq 0 \leftrightarrow \langle 0 | X_{I,I_3} | 0 \rangle \neq 0 \quad (1)$$

where the symbols I, I_3 denote the weak isospin quantum numbers of whatever object X has a non-zero vacuum expectation value. There are a couple of good reasons to think that X must have (predominantly) isospin $I = 1/2$. One is the ratio of the W and Z boson masses [6]:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \simeq 1, \quad (2)$$

and the other reason is to provide non-zero fermion masses. Since left-handed fermions f_L have $I = 1/2$, right-handed fermions f_R have $I = 0$ and fermion mass terms couple them together: $m_f \bar{f}_L f_R$, we must break isospin symmetry by $1/2$ a unit:

$$m_f \neq 0 \leftrightarrow \langle 0 | X_{1/2, \pm 1/2} | 0 \rangle \neq 0. \quad (3)$$

The next question is, what is the nature of X ? Is it elementary or composite? In the initial formulation of the Standard Model, it was assumed that X should be an elementary Higgs-Brout-Englert [7, 8] field H : $\langle 0 | H^0 | 0 \rangle \neq 0$, which would have a physical excitation that manifested itself as a neutral scalar Higgs boson [7]. However, as discussed in more detail later, an elementary Higgs field has problems with quantum (loop) corrections. Those due to Standard Model particles are quadratically divergent, resulting in a large cutoff-dependent contribution to the physical masses of the Higgs boson, W, Z bosons and other particles:

$$\delta m_H^2 \simeq \mathcal{O}\left(\frac{\alpha}{\pi}\right) \Lambda^2, \quad (4)$$

where Λ represents the scale at which new physics appears.

The sensitivity (4) disturbs theorists, and lies at the root of the hierarchy problem [4]. One of the suggestions for avoiding it was to postulate replacing an elementary Higgs-Brout-Englert field H by a composite

field such as a condensate of fermions: $\langle 0|\bar{F}F|0\rangle \neq 0$. This possibility was made more appealing by the fact that fermion condensates are well known in solid-state physics, where Cooper pairs of electrons are responsible for conventional superconductivity, and in strong-interaction physics, where quarks condense in the vacuum: $\langle 0|\bar{q}q|0\rangle \neq 0$.

In order to break the electroweak symmetry at a large enough scale, fermions with new interactions that become strong at a higher mass scale would be required. One suggestion was that the Yukawa interactions of the top quark might be strong enough to condense them: $\langle 0|\bar{t}t|0\rangle \neq 0$ [9], but this would have required the top quark to weigh more than 200 GeV, in simple models. Alternatively, theorists proposed the existence of new fermions held together by completely new interactions that became strong at a scale ~ 1 TeV, commonly called *Technicolour* models [10].

The simplest version of technicolour predicted three massless technipions that would be eaten by the $W^\pm$ and Z^0 to provide their masses via the Higgs-Brout-Englert mechanism, leaving over a single physical scalar state weighing about 1 TeV, that would behave in some ways like a heavy Higgs boson. Unfortunately, this simple technicolour picture must be complicated [11], and runs into various experimental problems, such as the absence of any light technipions [12], the observed suppression of flavour-changing neutral interactions [13], and precision electroweak data, which limit the possible magnitudes of one-loop radiative corrections due to virtual techniparticles [14]. Does this mean that technicolour is dead? Not quite [15], but it has motivated technicolour enthusiasts to pursue epicyclic variations on the original idea, such as walking technicolour [16], in which the technicolour dynamics is not scaled up from QCD in such a naive way.

1.3 Why Supersymmetry?

An alternative approach to the hierarchy problem [4] is offered by supersymmetry, if it appears at an accessible energy scale. One may formulate the hierarchy problem [4] by posing the question why is $m_W \ll m_P$, or, equivalently, why is $G_F \sim 1/m_W^2 \gg G_N = 1/m_P^2$? Another equivalent question is why the Coulomb potential in an atom is so much greater than the Newton potential: $e^2 \gg G_N m^2 = m^2/m_P^2$, where m is a typical particle mass?

Your first thought might simply be to set $m_P \gg m_W$ by hand, and forget about the problem. Life is not so simple, because, as already mentioned, quantum corrections to m_H and hence m_W are quadratically

divergent in the Standard Model:

$$\delta m_{H,W}^2 \simeq \mathcal{O}\left(\frac{\alpha}{\pi}\right)\Lambda^2, \quad (5)$$

which is $\gg m_W^2$ if the cutoff Λ , which represents the scale where new physics beyond the Standard Model appears, is comparable to the GUT or Planck scale. For example, if the Standard Model were to hold unscathed all the way up the Planck mass $m_P \sim 10^{19}$ GeV, the radiative correction (5) would be 36 orders of magnitude greater than the physical values of $m_{H,W}^2$!

In principle, this is not a problem from the mathematical point of view of renormalization theory. All one has to do is postulate a tree-level value of m_H^2 that is (very nearly) equal and opposite to the ‘correction’ (5), and the correct physical value may be obtained. However, this strikes many physicists as rather unnatural: they would prefer a mechanism that keeps the ‘correction’ (5) comparable at most to the physical value.

This is possible in a supersymmetric theory, in which there are equal numbers of bosons and fermions with identical couplings. Since bosonic and fermionic loops have opposite signs, the residual one-loop correction is of the form

$$\delta m_{H,W}^2 \simeq \mathcal{O}\left(\frac{\alpha}{\pi}\right)(m_B^2 - m_F^2), \quad (6)$$

which is $\lesssim m_{H,W}^2$ and hence naturally small if the supersymmetric partner bosons B and fermions F have similar masses:

$$|m_B^2 - m_F^2| \lesssim 1 \text{ TeV}^2. \quad (7)$$

This is the best motivation we have for finding supersymmetry at relatively low energies [4].

In addition to this first supersymmetric miracle of removing (6) the quadratic divergence (5), many logarithmic divergences are also absent in a supersymmetric theory, as discussed later. This property is logically distinct from the absence of quadratic divergences in a supersymmetric theory, and is also important for supersymmetric model-building.

1.4 What is Supersymmetry?

The basic idea of supersymmetry is the existence of fermionic charges Q_α that relate bosons to fermions. Recall that all previous symmetries, such as flavour $SU(3)$ or electromagnetic $U(1)$, have involved scalar charges Q that link particles with the same spin into multiplets:

$$Q |\text{Spin} J \rangle = |\text{Spin} J \rangle. \quad (8)$$

Indeed, Coleman and Mandula [17] proved that it was ‘impossible’ to mix internal and Lorentz symmetries: $J_1 \leftrightarrow J_2$. However, their ‘no-go’ theorem assumed implicitly that the prospective charges should have integer spins.

The basic element in their ‘proof’ was the observation that the only possible conserved tensor charges were those with no Lorentz indices, i.e., scalar charges, and the energy-momentum vector P_μ . To see how their ‘proof’ worked, consider two-to-two elastic scattering, $1+2 \rightarrow 3+4$, and imagine that there exists a conserved two-index tensor charge, $\Sigma_{\mu\nu}$. By Lorentz invariance, its diagonal matrix elements between single-particle states $|a\rangle$ must take the general form:

$$\langle a|\Sigma_{\mu\nu}|a\rangle = \alpha P_\mu^{(a)} P_\nu^{(a)} + \beta g_{\mu\nu}, \quad (9)$$

where α, β are arbitrary reduced matrix elements, and $g_{\mu\nu}$ is the metric tensor. For $\Sigma_{\mu\nu}$ to be conserved in a two-to-two scattering process, one must have

$$P_\mu^{(1)} P_\nu^{(1)} + P_\mu^{(2)} P_\nu^{(2)} = P_\mu^{(3)} P_\nu^{(3)} + P_\mu^{(4)} P_\nu^{(4)}, \quad (10)$$

where we assume that the symmetry is local, so that two-particle matrix elements of $\Sigma_{\mu\nu}$ play no rôle. Since Lorentz invariance also requires $P_\mu^{(1)} + P_\mu^{(2)} = P_\mu^{(3)} + P_\mu^{(4)}$, the only possible outcomes are $P_\mu^{(1)} = P_\mu^{(3)}$ or $P_\mu^{(4)}$. Thus the only possibilities are completely forward scattering or completely backward scattering. This disagrees with observation, and is in fact theoretically impossible in any local field theory.

This rules out any non-trivial two-index tensor charge, and the argument can clearly be extended to any higher-rank tensor with more Lorentz indices. But what about a spinorial charge Q_α ? This can have no diagonal matrix element:

$$\langle a|Q_\alpha|a\rangle \neq 0, \quad (11)$$

and hence the Coleman-Mandula argument fails.

So what is the possible form of a ‘supersymmetry’ algebra that includes such spinorial charges Q_α^i ?¹ Since the different Q^i are supposed to generate symmetries, they must commute with the Hamiltonian:

$$[Q^i, H] = 0 : i = 1, 2, \dots, N. \quad (12)$$

So also must the anticommutator of two spinorial charges:

$$[\{Q^i, Q^j\}, H] = 0 : i, j = 1, 2, \dots, N. \quad (13)$$

¹In what follows, I shall suppress the spinorial subscript α whenever it is not essential. The superscripts $i, j, \dots, N$ denote different supersymmetry charges.

However, the part of the anticommutator $\{Q^i, Q^j\}$ that is symmetric in the internal indices i, j cannot have spin 0. Instead, as we discussed just above, the only possible non-zero spin choice is $J = 1$, so that

$$\{Q^i, Q^j\} \propto \delta^{ij} P_\mu + \dots : i, j = 1, 2, \dots, N. \quad (14)$$

In fact, as was proved by Haag, Lopuszanski and Sohnius [18], the only allowed possibility is

$$\{Q^i, Q^j\} = 2\delta^{ij} \gamma^\mu P_\mu \mathcal{C} + \dots : i, j = 1, 2, \dots, N, \quad (15)$$

where $\mathcal{C}$ is the charge-conjugation matrix discussed in more detail in Lecture 2, and the dots denote a possible ‘central charge’ that is anti-symmetric in the indices i, j , and hence can only appear when $N > 1$.

According to a basic principle of Swiss law, anything not illegal is compulsory, so there MUST exist physical realizations of the supersymmetry algebra (15). Indeed, non-trivial realizations of the non-relativistic analogue of (15) are known from nuclear physics [19], atomic physics and condensed-matter physics. However, none of these is thought to be fundamental.

In the relativistic limit, supermultiplets consist of massless particles with spins differing by half a unit. In the case of simple $N = 1$ supersymmetry, the basic building blocks are *chiral supermultiplets*:

$$\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \text{ e.g., } \begin{pmatrix} \ell \text{ (lepton)} \\ \tilde{\ell} \text{ (slepton)} \end{pmatrix} \text{ or } \begin{pmatrix} q \text{ (quark)} \\ \tilde{q} \text{ (squark)} \end{pmatrix} \quad (16)$$

gauge supermultiplets:

$$\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \text{ e.g., } \begin{pmatrix} \gamma \text{ (photon)} \\ \tilde{\gamma} \text{ (photino)} \end{pmatrix} \text{ or } \begin{pmatrix} g \text{ (gluon)} \\ \tilde{g} \text{ (gluino)} \end{pmatrix} \quad (17)$$

and the *graviton supermultiplet* consisting of the spin-2 graviton and the spin-3/2 gravitino.

Could any of the known particles in the Standard Model be linked together in supermultiplets? Unfortunately, none of the known fermions q, ℓ can be paired with any of the known bosons $\gamma, W^\pm, Z^0, g, H$, because their internal quantum numbers do not match [20]. For example, quarks q sit in triplet representations of colour, whereas the known bosons are either singlets or octets of colour. Then again, leptons ℓ have non-zero lepton number $L = 1$, whereas the known bosons have $L = 0$. Thus, the only possibility seems to be to introduce new supersymmetric partners (spartners) for all the known particles: quark $\rightarrow$ squark, lepton $\rightarrow$ slepton, photon $\rightarrow$ photino, $Z \rightarrow$ Zino, $W \rightarrow$ Wino, gluon $\rightarrow$ gluino, Higgs $\rightarrow$ Higgsino, as suggested in (16, 17) above.

The best that one can say for supersymmetry is that it economizes on principle, not on particles!

1.5 (S)Experimental Hints

By now, you may be wondering whether it makes sense to introduce so many new particles just to deal with a paltry little hierarchy or naturalness problem. But, as they used to say during the First World War, ‘if you know a better hole, go to it.’ As we learnt above, technicolour no longer seems to be a viable hole, and I am not convinced that theories with large extra dimensions really solve the hierarchy problem, rather than just rewrite it. Fortunately, there are two hints from the high-precision electroweak data that supersymmetry may not be such a bad hole, after all.

One is the fact that there probably exists a Higgs boson weighing less than about 200 GeV [21]. This is perfectly consistent with calculations in the minimal supersymmetric extension of the Standard Model (MSSM), in which the lightest Higgs boson weighs less than about 130 GeV [22], as we discuss later in more detail.

The other hint is provided by the strengths of the different gauge interactions, as measured at LEP [3]. These may be run up to high energy scales using the renormalization-group equations, to see whether they unify as predicted in a GUT. The answer is no, if supersymmetry is not included in the calculations. In that case, GUTs would require

$$\sin^2 \theta_W = 0.214 \pm 0.004, \quad (18)$$

whereas the experimental value of the effective neutral weak mixing parameter at the Z^0 peak is $\sin^2 \theta = 0.23149 \pm 0.00017$ [21]. On the other hand, minimal supersymmetric GUTs predict

$$\sin^2 \theta_W \sim 0.232, \quad (19)$$

where the error depends on the assumed sparticle masses, the preferred value being around 1 TeV, as suggested completely independently by the naturalness of the electroweak mass hierarchy. This argument is also discussed later in more detail.

2. SIMPLE MODELS

2.1 Deconstructing Dirac

In this Section, we tackle some unavoidable spinorology. The most familiar spinors used in four-dimensional field theories are four-component Dirac spinors ψ . You may recall that it is possible to introduce projec-

tion operators

$$P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5), \quad (20)$$

where $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, and the γ_μ can be written in the forms

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad (21)$$

where $\sigma_\mu \equiv (1, \sigma_i)$, $\bar{\sigma}_\mu \equiv (1, -\sigma_i)$. Then γ_5 can be written in the form $\text{diag}(-1, 1)$, where $-1, 1$ denote 2×2 matrices. Next, we introduce the corresponding left- and right-handed spinors

$$\psi_{L,R} \equiv P_{L,R}\psi, \quad (22)$$

in terms of which one may decompose the four-component spinor into a pair of two-component spinors:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \quad (23)$$

These will serve as our basic fermionic building blocks.

Antifermions can be represented by adjoint spinors

$$\bar{\psi} \equiv \psi^\dagger \gamma^0 = (\bar{\psi}_R, \bar{\psi}_L) \quad (24)$$

where the γ^0 factor has interchanged the left- and right-handed components $\psi_{L,R}$. We can now decompose in terms of these the conventional fermion kinetic term

$$\bar{\psi}\gamma_\mu\partial^\mu\psi = \bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + \bar{\psi}_R\gamma_\mu\partial^\mu\psi_R \quad (25)$$

and the conventional mass term

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R. \quad (26)$$

We see that the kinetic term keeps separate the left- and right-handed spinors, whereas the mass term mixes them.

The next step is to introduce the charge-conjugation operator $\mathcal{C}$, which changes the overall sign of the vector current $\bar{\psi}\gamma^\mu\psi$. It transforms spinors into their conjugates:

$$\psi^c \equiv \mathcal{C} \bar{\psi}^T = \mathcal{C}(\psi^\dagger \gamma^0)^T = \begin{pmatrix} \bar{\psi}_R \\ \bar{\psi}_L \end{pmatrix}, \quad (27)$$

and operates as follows on the γ matrices:

$$\mathcal{C}^{-1}\gamma^\mu\mathcal{C} = -\gamma^{\mu T}. \quad (28)$$

A convenient representation of $\mathcal{C}$ is:

$$\mathcal{C} = i\gamma^0\gamma^2. \quad (29)$$

It is apparent from the above that the conjugate of a left-handed spinor is right-handed:

$$(\psi_L)^c = \begin{pmatrix} 0 \\ \bar{\psi}_L \end{pmatrix}, \quad (30)$$

so that the combination

$$\bar{\psi}_L^c \psi_L = \psi_L \sigma_2 \psi_L \quad (31)$$

mixes left- and right-handed spinors, and has the same form as a mass term (26).

It is apparent from (30) that we can construct four-component Dirac spinors entirely out of two-component left-handed spinors and their conjugates:

$$\psi = \begin{pmatrix} \psi_i \\ \psi_j^c \end{pmatrix}, \quad (32)$$

a trick that will be useful later in our supersymmetric model-building. As examples, instead of working with left- and right-handed quark fields q_L and q_R , or left- and right-handed lepton fields ℓ_L and ℓ_R , we can write the theory in terms of left-handed antiquarks and antileptons: $q_R \rightarrow q_L^c$ and $\ell_R \rightarrow \ell_L^c$.

2.2 Simplest Supersymmetric Field Theories

Let us now consider a field theory [23] containing just a single left-handed fermion ψ_L and a complex boson ϕ , without any interactions, as described by the Lagrangian

$$L_0 = i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + |\partial\phi|^2. \quad (33)$$

We consider the simplest possible non-trivial transformation law for the free theory (33):

$$\phi \rightarrow \phi + \delta\phi, \text{ where } \delta\phi = \sqrt{2}\bar{E}\psi_L, \quad (34)$$

where E is some constant right-handed spinor. In parallel with (34), we also consider the most general possible transformation law for the fermion ψ :

$$\psi_L \rightarrow \psi_L + \delta\psi_L, \text{ where } \delta\psi_L = -a i \sqrt{2}(\gamma_\mu \partial^\mu \phi)E - F E^c, \quad (35)$$

where a and F are constants to be fixed later, and we recall that E^c is a left-handed spinor. We can now consider the resulting transformation of the full Lagrangian (33), which can easily be checked to take the form

$$\delta L_0 = \sqrt{2}\partial_\mu[\bar{\psi}E\partial^\mu\phi + \bar{E}\gamma^\mu\phi^*\gamma_\nu\partial^\nu\psi], \quad (36)$$

if and only if we choose

$$a = 1 \text{ and } F = 0 \quad (37)$$

in this free-field model. With these choices, and the resulting total-derivative transformation law (36) for the free Lagrangian, the free action A_0 is invariant under the transformations (34,35), since

$$\delta A_0 = \delta \int d^4x L_0 = 0. \quad (38)$$

Fine, you may say, but is this symmetry actually supersymmetry? To convince yourself that it is, consider the sequences of pairs (34,35) of transformations starting from either the boson ϕ or the fermion ψ :

$$\phi \rightarrow \psi \rightarrow \partial\phi, \quad \psi \rightarrow \partial\phi \rightarrow \partial\psi. \quad (39)$$

In both cases, the action of two symmetry transformations is equivalent to a derivative, i.e., the momentum operator, corresponding exactly to the supersymmetry algebra. A free boson and a free fermion together realize supersymmetry: like the character in Molière, we have been talking prose all our lives without realizing it!

Now we look at interactions in a supersymmetric field theory [24]. The most general interactions between spin-0 fields ϕ^i and spin-1/2 fields ψ^i that are at most bilinear in the latter, and hence have a chance of being renormalizable in four dimensions, can be written in the form

$$L = L_0 - V(\phi^i, \phi_j^*) - \frac{1}{2}M_{ij}(\phi, \phi^*)\bar{\psi}^{ci}\psi^j \quad (40)$$

where V is a general effective potential, and M_{ij} includes both mass terms and Yukawa interactions for the fermions. Supersymmetry imposes strong constraints on the allowed forms of V and M , as we now see. Suppose that M depended non-trivially on the conjugate fields ϕ^* : then the supersymmetric variation $\delta(M\bar{\psi}^c\psi)$ would contain a term

$$\frac{\partial M}{\partial \phi^*}\psi^*\bar{\psi}^c\psi \quad (41)$$

that could not be compensated by the variation of any other term. We conclude that M must be independent of ϕ^* , and hence $M = M(\phi)$ alone.

Another term in the variation of the last term in (40) is

$$\frac{\partial M_{ij}}{\partial \phi^k} \bar{E} \psi^k \bar{\psi}^{ci} \psi^j. \quad (42)$$

This term cannot be cancelled by the variation of any other term, but can vanish by itself if $\partial M_{ij}/\partial \phi^k$ is completely symmetric in the indices i, j, k . This is possible only if

$$M_{ij} = \frac{\partial W}{\partial \phi^i \partial \phi^j} \quad (43)$$

for some function $W(\phi)$ called the *superpotential*. If the theory is to be renormalizable, W can only be cubic. The trilinear term of W determines the Yukawa couplings, and the bilinear part the mass terms.

We now re-examine the form of the supersymmetric transformation law (35) itself. Yet another term in the variation of the second term in (40) has the form

$$i M_{jk} \bar{\psi}^{cj} \gamma_\mu \partial^\mu \phi^k E + (\text{Herm. Conj.}). \quad (44)$$

This can cancel against an F -dependent term in the variation of the fermion kinetic term

$$-i \bar{\psi}_i \gamma_\mu \partial^\mu F^i E^c + (\text{Herm. Conj.}), \quad (45)$$

if the following relation between F and M holds: $\frac{\partial F_i^*}{\partial \phi^j} = M_{ij}$, which is possible if and only if

$$F_i^* = \frac{\partial W}{\partial \phi^i}. \quad (46)$$

Thus the form of W also determines the required form of the supersymmetry transformation law.

The form of W also determines the effective potential V , as we now see. One of the terms in the variation of V is

$$\frac{\partial V}{\partial \phi^i} \bar{E} \psi^i + (\text{Herm. Conj.}), \quad (47)$$

which can only be cancelled by a term in the variation of $M_{ij} \bar{\psi}^{ci} \psi^j$, which can take the form $M_{ij} \bar{\psi}^{ci} F^j E^c$ if

$$\frac{\partial V}{\partial \phi^i} = M_{ij} F^j, \quad (48)$$

which is in turn possible only if

$$V = \left| \frac{\partial W}{\partial \phi^i} \right|^2 = |F^i|^2. \quad (49)$$

We now have the complete supersymmetric field theory for interacting chiral (matter) supermultiplets [24]:

$$L = i\bar{\psi}_i\gamma_\mu\partial^\mu\psi^i + |\partial_\mu\phi^i|^2 - \left|\frac{\partial W}{\partial\phi^i}\right|^2 - \frac{1}{2}\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\bar{\psi}^{ci}\partial\psi^j + (\text{Herm.Conj.}). \quad (50)$$

This Lagrangian is invariant (up to a total derivative) under the supersymmetry transformations

$$\delta\phi^i = \sqrt{2}\bar{E}\psi^i, \quad \delta\psi^i = -i\sqrt{2}\gamma_\mu\partial^\mu\phi^i E - F^i E^c : F^i = \left(\frac{\partial W}{\partial\phi^i}\right)^*. \quad (51)$$

The simplest non-trivial superpotential involving a single superfield ϕ is

$$W = \frac{\lambda}{3}\phi^3 + \frac{m}{2}\phi^2. \quad (52)$$

It is a simple exercise for you to verify using the rules given above that the corresponding Lagrangian is

$$L = i\bar{\psi}\gamma_\mu\partial^\mu\psi + |\partial_\mu\phi|^2 - |m\phi + \lambda\phi^2|^2 - m\bar{\psi}^c\psi - \lambda\phi\bar{\psi}^c\psi. \quad (53)$$

We see explicitly that the bosonic component ϕ of the supermultiplet has the same mass as the fermionic component ψ , and that the Yukawa coupling λ fixes the effective potential.

We now turn to the possible form of a supersymmetric gauge theory [25]. Clearly, it must contain vector fields A_μ^a and fermions χ^a in the same adjoint representation of the gauge group. Once one knows the gauge group and the fermionic matter content, the form of the Lagrangian is completely determined by gauge invariance:

$$L = \frac{i}{2}\bar{\chi}^a\gamma^\mu D_{ab}^\mu\chi^b - \frac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} \left[-\frac{1}{2}(D^a)^2 \right]. \quad (54)$$

Here, the gauge-covariant derivative

$$D_{ab}^\mu \equiv \delta_{ab}\partial^\mu - gf_{abc}A_c^\mu, \quad (55)$$

and the gauge field strength is

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c, \quad (56)$$

as usual. We return later to the D term at the end of (54). Yet another of the miracles of supersymmetry is that the Lagrangian (54) is automatically supersymmetric, without any further monkeying around. The

corresponding supersymmetry transformations may be written as

$$\delta A_\mu^a = -\bar{E}\gamma_\mu\chi^a, \quad (57)$$

$$\delta\chi^a = -\frac{i}{2}F_{\mu\nu}^a\gamma^\mu\gamma^\nu E + D^a E, \quad (58)$$

$$\delta D^a = -i\bar{E}\gamma_5\gamma^\mu D_\mu^{ab}\chi^b. \quad (59)$$

What about the D term in (54)? It is a trivial consequence of equations of motion derived from (54) that $D^a = 0$. However, this is no longer the case if matter is included. Then, it turns out, one must add to (54) the following:

$$\Delta L = -\sqrt{2}g\chi^a\phi_i^*(T^a)_j^i\psi^j + (\text{Herm.Conj.}) + g(\phi_i^*(T^a)_j^i\phi^j)D^a, \quad (60)$$

where T^a is the group representation matrix for the matter fields ϕ^i . With this addition, the equation of motion for D^a tells us that

$$D^a = g\phi_i^*(T^a)_j^i\phi^j, \quad (61)$$

and we find a D term in the full effective potential:

$$V = \Sigma_i|F_i|^2 + \Sigma_a\frac{1}{2}(D^a)^2, \quad (62)$$

where the form of D^a is given in (61).

2.3 Further Aspects of Supersymmetric Field Theories

So far, we have taken a relatively unsophisticated approach to supersymmetry. However, one of the reasons why theorists are so enthusiastic about supersymmetry is because it is not just a new type of symmetry, but extends the concept of space-time itself. Recall the basic form of the supersymmetry algebra:

$$2\delta^{ij}\gamma_\mu P^\mu \mathcal{C} = \{Q^i, Q^j\}. \quad (63)$$

The reason this is written backwards here is to emphasize that one can regard supersymmetric charges Q^i as square roots of the translation operator. Recall how the translation operator acts on a bosonic field:

$$\phi(x+a) = e^{ia.P}\phi(x)e^{-ia.P}, \quad (64)$$

where the momentum operator P is the generator of infinitesimal translations:

$$i[P_\mu, \phi(x)] = \partial_\mu\phi(x). \quad (65)$$

Expanding the formula (64), we find the following expression for a small finite translation:

$$\delta_a \phi(x) \equiv \phi(x+a) - \phi(x) \simeq a_\mu \partial^\mu \phi(x) = i a_\mu [P^\mu, \phi(x)]. \quad (66)$$

Following this deconstruction of translations, we now can see better how the supersymmetric charge can be regarded, in some sense, as the square root: ' $Q \sim \sqrt{P}$ ', just as the Dirac equation can be regarded as the square root of the Klein-Gordon equation. There is an exact supersymmetric analogue of (66):

$$\delta_{\bar{E}} \phi(x) = \sqrt{2} \bar{E} \psi(x) = i \sqrt{2} \bar{E} [Q, \phi(x)]. \quad (67)$$

By analogy with (64,66), one may consider the spinor $\bar{E}$ as a sort of 'superspace' coordinate, and one can combine the bosonic field $\phi(x)$ and its fermionic partner $\psi(x)$ into a superfield:

$$\delta_{\bar{E}} \Phi(x) = \Phi(x, \bar{E}) - \Phi(x) : \Phi(x, \bar{E}) \equiv \phi(x) + \sqrt{2} \bar{E} \psi(x). \quad (68)$$

At this level, the introduction of superspace and superfields may appear superfluous, but it gives deeper insights into the theory and facilitates the derivation of many important results, such as the non-renormalization theorems of supersymmetry, that we discuss next. In some sense, the next generation of accelerators such as the LHC is 'guaranteed' to discover extra dimensions, either bosonic ones [26], or fermionic.

Many remarkable no-renormalization theorems can be proved in supersymmetric field theories [27]. First and foremost, they have no quadratic divergences. One way to understand this is to compare the renormalizations of bosonic and fermionic mass terms:

$$m_B^2 |\phi|^2 \leftrightarrow m_F \bar{\psi} \psi. \quad (69)$$

We know well that fermion masses m_F can only be renormalized logarithmically. Since supersymmetry guarantees that $m_B = m_F$, it follows that there can be no quadratic divergence in m_B . Going further, chiral symmetry guarantees that the one-loop renormalization of a fermion mass has the general multiplicative form:

$$\delta m_F = \mathcal{O}\left(\frac{\alpha}{\pi}\right) m_F \ln\left(\frac{\mu_1}{\mu_2}\right), \quad (70)$$

where $\mu_{1,2}$ are different renormalization scales. This means that if m_F (and hence also m_B) vanish at the tree level in a supersymmetric theory, then both m_F and m_B remain zero after renormalization. This is one example of the reduction in the number of logarithmic divergences in a supersymmetric theory.

In general, there is no intrinsic renormalization of any superpotential parameters, including the Yukawa couplings λ , apart from overall multiplicative factors due to wave-function renormalizations:

$$\Phi \rightarrow Z\Phi, \quad (71)$$

which are universal for both the bosonic and fermionic components ϕ, ψ in a given superfield Φ . However, gauge couplings *are* renormalized, though the β -function is changed:

$$\beta(g) \neq 0: -11N_c \rightarrow -9N_c \quad (72)$$

at one-loop order in an $SU(N_c)$ supersymmetric gauge theory with no matter, as a result of the extra gaugino contributions.

There are even fewer divergences in theories with more supersymmetries. For example, there is only a finite number of divergent diagrams in a theory with $N = 2$ supersymmetries, which may be cancelled by imposing a few simple relations on the spectrum of supermultiplets. Finally, there are no divergences at all in theories with $N = 4$ supersymmetries, which obey automatically the necessary finiteness conditions.

Many theorists from Dirac onwards have found the idea of a completely finite theory attractive, so it is natural to ask whether theories with $N \geq 2$ supersymmetries could be interesting as realistic field theories. Unfortunately, the answer is ‘not immediately’, because they do not allow the violation of parity. To see why, consider the simplest possible extended supersymmetric theory containing an $N = 2$ matter multiplet, which contains both left- and right-handed fermions with helicities $\pm 1/2$. Suppose that the left-handed fermion with helicity $+1/2$ sits in a representation R of the gauge group. Now act on it with either of the two supersymmetry charges $Q_{1,2}$: they each yield bosons, that each sit in the same representation R . Now act on either of these with the other supercharge, to obtain a right-handed fermion with helicity $-1/2$: this must also sit in the same representation R of the gauge group. Hence, left- and right-handed fermions have the same interactions, and parity is conserved. There is no way out using gauginos, because they are forced to sit in adjoint representations of the gauge group, and hence also cannot distinguish between right and left.

Thus, if we want to make a supersymmetric extension of the Standard Model, it had better be with just $N = 1$ supersymmetry, and this is what we do in the next Section.

2.4 Building Supersymmetric Models

Any supersymmetric model is based on a Lagrangian that contains a supersymmetric part and a supersymmetry-breaking part:

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{susy\times}. \quad (73)$$

We discuss the supersymmetry-breaking part $\mathcal{L}_{susy\times}$ in the next Lecture: here we concentrate on the supersymmetric part $\mathcal{L}_{susy}$. The minimal supersymmetric extension of the Standard Model (MSSM) has the same gauge interactions as the Standard Model, and Yukawa interactions that are closely related. They are based on a superpotential W that is a cubic function of complex superfields corresponding to left-handed fermion fields. Conventional left-handed lepton and quark doublets are denoted L, Q , and right-handed fermions are introduced via their conjugate fields, which are left-handed, $e_R \rightarrow E^c, u_R \rightarrow U^c, d_R \rightarrow D^c$. In terms of these,

$$W = \Sigma_{L,E^c} \lambda_L L E^c H_1 + \Sigma_{Q,U^c} \lambda_U Q U^c H_2 + \Sigma_{Q,D^c} \lambda_D Q D^c H_1 + \mu H_1 H_2. \quad (74)$$

A few words of explanation are warranted. The first three terms in (74) yield masses for the charged leptons, charge-(+2/3) quarks and charge-(-1/3) quarks respectively. All of the Yukawa couplings $\lambda_{L,U,D}$ are 3×3 matrices in flavour space, whose diagonalizations yield the mass eigenstates and Cabibbo-Kobayashi-Maskawa mixing angles.

Note that two distinct Higgs doublets $H_{1,2}$ have been introduced, for two important reasons. One reason is that the superpotential must be an analytic polynomial: as we saw in (74), it cannot contain both H and H^* , whereas the Standard Model uses both of these to give masses to all the quarks and leptons with just a single Higgs doublet. The other reason is to cancel the triangle anomalies that destroy the renormalizability of a gauge theory. Ordinary Higgs boson doublets do not contribute to these anomalies, but the fermions in Higgs supermultiplets do, and two doublets are required to cancel each others' contributions. Once two Higgs supermultiplets have been introduced, there is the possibility, even the necessity, of a bilinear term $\mu H_1 H_2$ coupling them together.

Once the MSSM superpotential (74) has been specified, the effective potential is also fixed:

$$V = \Sigma_i |F^i|^2 + \frac{1}{2} \Sigma_a (D^a)^2 : F_i^* \equiv \frac{\partial W}{\partial \phi^i}, D^a \equiv g_a \phi_i^* (T^a)_j^i \phi^j, \quad (75)$$

according to the rules explained earlier in this Lecture, where the sums run over the different chiral fields i and the $SU(3), SU(2)$ and $U(1)$ gauge-group factors a .

There are important possible variations on the MSSM superpotential (74), which are impossible in the Standard Model, but are allowed by the gauge symmetries of the MSSM supermultiplets. These are additional superpotential terms that violate the quantity known as R parity:

$$R \equiv (-1)^{3B+L+2S}, \quad (76)$$

where B is baryon number, L is lepton number, and S is spin. It is easy to check that $R = +1$ for all the particles in the Standard Model, and $R = -1$ for all their spartners, which have identical values of B and L , but differ in spin by half a unit. Clearly, R would be conserved if both B and L were conserved, but this is not automatic. Consider the following superpotential terms:

$$\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \epsilon_i H L_i, \quad (77)$$

which are visibly $SU(3) \times SU(2) \times U(1)$ symmetric. The first term in (77) would violate L , causing for example $\bar{\ell} \rightarrow \ell + \ell$, the second would violate both B and L , causing for example $\bar{q} \rightarrow q + \ell$, the third would violate B , causing for example $\bar{q} \rightarrow \bar{q} + \bar{q}$, and the last would violate L by causing $H \leftrightarrow L_i$ mixing. These interactions would provide many exciting signatures for supersymmetry, such as dilepton events, jets plus leptons and multijet events. Such interactions are constrained by direct searches, by the experimental limits on flavour-changing interactions and other rare processes, and by cosmology: they would tend to wipe out the baryon asymmetry of the Universe if they are too strong [28]. They would also cause the lightest supersymmetric particle to be unstable, not necessarily a disaster in itself, but it would remove an excellent candidate for the cold dark matter that apparently abounds throughout the Universe. For simplicity, the conservation of R parity will be assumed in the rest of these Lectures.

We now look briefly at the construction of supersymmetric GUTs, of which the minimal version is based on the group $SU(5)$ [29]. As in the transition from the Standard Model to the MSSM, one simply extends the conventional GUT multiplets to supermultiplets, so that matter particles are assigned to $\bar{\mathbf{5}}$ representations $\bar{F}$ and $\mathbf{10}$ representations T , one doubles the electroweak Higgs fields to include both $H, \bar{H}$ in $\mathbf{5}, \bar{\mathbf{5}}$ representations, and one postulates a $\mathbf{24}$ representation Φ to break the $SU(5)$ GUT symmetry down to $SU(3) \times SU(2) \times U(1)$. The superpotential for the Higgs sector takes the general form

$$W_5 = (\mu + \frac{3}{2}\lambda M) H \bar{H} + \lambda H \Phi \bar{H} + f(\Phi). \quad (78)$$

Here, $f(\Phi)$ is chosen so that the vacuum expectation value of Φ has the form

$$\langle 0|\Phi|0 \rangle = M \times \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}). \quad (79)$$

The coefficient of the $H\bar{H}$ term has been chosen so that it almost cancels with the term $\propto H \langle 0|\Phi|0 \rangle \bar{H}$ coming from the second term in (78), *for the last two components*. In this way, the triplet components of $H, \bar{H}$ acquire large masses $\propto M$, whilst the last two may acquire a vacuum expectation value: $\langle 0 | H | 0 \rangle = \text{column}(0, 0, 0, 0, v)$, $\langle 0 | \bar{H} | 0 \rangle = \text{column}(0, 0, 0, 0, \bar{v})$, once supersymmetry breaking and radiative corrections are taken into account, as in the next Lecture.

In order that $v, \bar{v} \sim 100$ GeV, it is necessary that the residual $H\bar{H}$ mixing term $\mu \lesssim 1$ TeV. Since, as we recall shortly, $M \sim 10^{16}$ GeV, this means that the parameters of W_5 (78) must be tuned finely to one part in 10^{13} . This fine-tuning may appear very unreasonable, but it is technically natural, in the sense that there are no big radiative corrections. Thanks to the supersymmetric no-renormalization theorem for superpotential parameters, we know that $\delta\lambda, \delta\mu = 0$, apart from wave-function renormalization factors. Thus, if we adjust the input parameters of (78) so that μ is small, it will stay small. However, this begs the more profound question: how did μ get to be so small in the first place?

As already mentioned, a striking piece of circumstantial evidence in favour of the idea of supersymmetric grand unification is provided by the measurements of low-energy gauge couplings at LEP and elsewhere [3]. The three gauge couplings of the Standard Model are renormalized as follows:

$$\frac{dg_a^2}{dt} = b_a \frac{g_a^4}{16\pi^2} + \dots, \quad (80)$$

at one-loop order, and the corresponding value of the electroweak mixing angle $\sin^2 \theta_W(m_Z)$ is given at the one-loop level by:

$$\sin^2 \theta_W(m_Z) = \frac{g'^2}{g_2^2 + g'^2} = \frac{3}{5} \frac{g_1^2(m_Z)}{g_2^2(m_Z) + \frac{3}{5}g_1^2(m_Z)} \quad (81)$$

$$= \frac{1}{1+8x} \left[3x + \frac{\alpha_{em}(m_Z)}{\alpha_3(m_Z)} \right], \quad (82)$$

where

$$x \equiv \frac{1}{5} \left(\frac{b_2 - b_3}{b_1 - b_2} \right). \quad (83)$$

One can distinguish the predictions of different GUTs by their different values of the renormalization coefficients b_i , which are in turn determined

by the spectra of light particles around the electroweak scale. In the cases of the Standard Model and the MSSM, these are:

$$\frac{4}{3}N_G - 11 \leftarrow b_3 \rightarrow 2N_G - 9 = -3 \quad (84)$$

$$\frac{1}{6}N_H + \frac{4}{3}N_G - \frac{22}{3} \leftarrow b_2 \rightarrow \frac{1}{2}N_H + 2N_G - 6 = +1 \quad (85)$$

$$\frac{1}{10}N_H + \frac{4}{3}N_G \leftarrow b_1 \rightarrow \frac{3}{10}N_H + 2N_G = \frac{33}{5} \quad (86)$$

$$\frac{23}{218} = 0.1055 \leftarrow x \rightarrow \frac{1}{7}. \quad (87)$$

If we insert the best available values of the gauge couplings:

$$\alpha_{em} = \frac{1}{128}; \alpha_3(m_Z) = 0.119 \pm 0.003, \sin^2 \theta_W(m_Z) = 0.2315, \quad (88)$$

we find the following value:

$$x = \frac{1}{6.92 \pm 0.07}. \quad (89)$$

We see that experiment strongly favours the inclusion of supersymmetric particles in the renormalization-group equations, as required if the effective low-energy theory is the MSSM (87), as in a simple supersymmetric GUT such as the minimal $SU(5)$ model introduced above.

3. TOWARDS REALISTIC MODELS

3.1 Supersymmetry Breaking

This is clearly necessary: $m_e \neq m_{\tilde{e}}, m_\gamma \neq m_{\tilde{\gamma}}$, etc. The Big Issue is whether the breaking of supersymmetry is explicit, i.e., present already in the underlying Lagrangian of the theory, or whether it is spontaneous, i.e., induced by a non-supersymmetric vacuum state. There are in fact several reasons to disfavour explicit supersymmetry breaking. It is ugly, it would be unlike the way in which gauge symmetry is broken, and it would lead to inconsistencies in supergravity theory. For these reasons, theorists have focused on spontaneous supersymmetry breaking.

If the vacuum is not to be supersymmetric, there must be some fermionic state χ that is coupled to the vacuum by the supersymmetry charge Q :

$$\langle 0|Q|\chi \rangle \equiv f_\chi^2 \neq 0. \quad (90)$$

The fermion χ corresponds to a Goldstone boson in a spontaneously broken bosonic symmetry, and therefore is often termed a Goldstone fermion or a Goldstino.

There is just one small problem in globally supersymmetric models, i.e., those without gravity: spontaneous supersymmetry breaking necessarily entails a positive vacuum energy E_0 . To see this, consider the vacuum expectation value of the basic supersymmetry anticommutator:

$$\{Q, Q\} \propto \gamma_\mu P^\mu. \quad (91)$$

According to (90), there is an intermediate state χ , so that

$$\langle 0 | \{Q, Q\} | 0 \rangle = | \langle 0 | Q | \chi \rangle |^2 = f_\chi^4 \propto \langle 0 | P_0 | 0 \rangle = E_0, \quad (92)$$

where we have used Lorentz invariance to set the spatial components $\langle 0 | P_i | 0 \rangle = 0$. Spontaneous breaking of global supersymmetry (90) requires

$$E_0 = f_\chi^4 \neq 0. \quad (93)$$

The next question is how to generate non-zero vacuum energy. Hints are provided by the effective potential in a globally supersymmetric theory:

$$V = \sum_i \left| \frac{\partial W}{\partial \phi^i} \right|^2 + \frac{1}{2} \sum_\alpha g_\alpha^2 |\phi^* T^\alpha \phi|^2. \quad (94)$$

It is apparent from this expression that either the first ‘ F term’ or the second ‘ D term’ must be positive definite.

The option $D > 0$ requires constructing a model with a $U(1)$ gauge symmetry [30]. The simplest example contains just one chiral (matter) supermultiplet with unit charge, for which the effective potential is:

$$V_D = \frac{1}{2} (\xi + g \phi^* \phi)^2. \quad (95)$$

the extra constant term ξ is not allowed in a non-Abelian theory, which is why one must use a $U(1)$ theory. We see immediately that the minimum of the effective potential (95) is reached when $\langle 0 | \phi | 0 \rangle = 0$, in which case $V_F = 1/2 \xi^2 > 0$ and supersymmetry is broken spontaneously. Indeed, it is easy to check that, in this vacuum:

$$m_\phi = g\xi, \quad m_\psi = 0, \quad m_V = m_{\tilde{V}} = 0, \quad (96)$$

exhibiting explicitly the boson-fermion mass splitting in the (ϕ, ψ) supermultiplet. Unfortunately, this example cannot be implemented with the $U(1)$ of electromagnetism in the Standard Model, because there are fields with both signs of the hypercharge Y , enabling V_D to vanish. So, one needs a new $U(1)$ gauge group factor, and many new fields in order to cancel triangle anomalies. For these reasons, D -breaking models did not attract much attention for quite some time, though they have had a revival in the context of string theory [31].

The option $F > 0$ also requires additional chiral (matter) fields with somewhat ‘artificial’ couplings [32]: again, those of the Standard Model do not suffice. The simplest example uses three chiral supermultiplets A, B, C with the superpotential

$$W = \alpha AB^2 + \beta C(B^2 - m^2). \quad (97)$$

using the rules given in the previous Lecture, it is easy to calculate the corresponding F terms:

$$F_A = \alpha B^2, F_B = 2B(\alpha A + \beta C), F_C = \beta(B^2 - m^2), \quad (98)$$

and hence the effective potential

$$V_F = \sum_i |F_i|^2 = 4|B(\alpha A + \beta C)|^2 + |\alpha B^2|^2 + |\beta(B^2 - m^2)|^2. \quad (99)$$

Likewise, it is not difficult to check that the three different positive-semidefinite terms in (99) cannot all vanish simultaneously. Hence, necessarily $V_F > 0$, and hence supersymmetry *must* be broken.

The principal outcome of this brief discussion is that there are no satisfactory models of global supersymmetry breaking, which provided some of the motivation for studying local supersymmetry, i.e., supergravity theory.

3.2 Supergravity and Local Supersymmetry Breaking

So far, we have considered global supersymmetry transformations, in which the infinitesimal transformation spinor E is constant throughout space. Now we consider the possibility of a space-time-dependent field $E(x)$. Why?

This step of making symmetries local has become familiar with bosonic symmetries, where it leads to gauge theories, so it is natural to try the analogous step with fermionic symmetries. Moreover, as we see shortly, it leads to an elegant mechanism for spontaneous supersymmetry breaking, again by analogy with gauge theories, the super-Higgs mechanism. Further, as we also see shortly, making supersymmetry local necessarily involves gravity, and even opens the prospect of unifying all the particle interactions and matter fields with extended supersymmetry transformations:

$$G(J=2) \rightarrow \tilde{G}(J=3/2) \rightarrow V(J=1) \rightarrow q, \ell(J=1/2) \rightarrow H(J=0) \quad (100)$$

in supergravity with $N > 1$ supercharges. In (100), G denotes the graviton, and $\tilde{G}$ the spin-3/2 gravitino, which accompanies it in the

graviton supermultiplet:

$$\begin{pmatrix} G \\ \tilde{G} \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \quad (101)$$

Supergravity is in any case an essential ingredient in the discussion of gravitational interactions of supersymmetric particles, needed, for example, for any meaningful discussion of the cosmological constant.

The mechanism for the spontaneous breaking of local supersymmetry is known as the super-Higgs effect [33, 34]. You recall that, in the conventional Higgs effect in spontaneously broken gauge theories, a massless Goldstone boson is ‘eaten’ by a gauge boson to provide it with the third polarization state it needs to become massive:

$$(2 \times V_{m=0}) + (1 \times GB) = (3 \times V_{m \neq 0}). \quad (102)$$

In a locally supersymmetric theory, the two polarization states of the massless Goldstone fermion (Goldstino) are ‘eaten’ by a massless gravitino, giving it the total of four polarization states it needs to become massive:

$$(2 \times \psi_{m=0}^\mu) + (2 \times GF) = (4 \times \psi_{m \neq 0}^\mu). \quad (103)$$

This process clearly involves the breakdown of local supersymmetry, since the end result is to give the gravitino a different mass from the graviton: $m_G = 0 \neq m_{\tilde{G}} \neq 0$. It is indeed the only known consistent way of breaking local supersymmetry, just as the Higgs mechanism is the only consistent way of breaking gauge symmetry. We shall not go here through all the details of the super-Higgs effect, but there is one noteworthy feature: this local breaking of supersymmetry can be achieved with zero vacuum energy:

$$\langle 0|V|0 \rangle = 0 \leftrightarrow \Lambda = 0. \quad (104)$$

There is no inconsistency between local supersymmetry breaking and a vanishing cosmological constant Λ , unlike the case of global supersymmetry breaking that we discussed earlier.

3.3 Effective Low-Energy Theory

The coupling of matter particles to supergravity is more complicated than the globally supersymmetric case discussed in the previous lecture. Therefore, it is not developed here in detail. Instead, a few key results are presented without proof. The form of the effective low-energy theory suggested by spontaneous supersymmetry breaking in supergravity is:

$$-\frac{1}{2}\Sigma_a m_{1/2a} \tilde{V}_a \tilde{V}_a - \Sigma_i m_{0i}^2 |\phi^i|^2 - (\Sigma_\lambda A_\lambda \lambda \phi^3 + \Sigma_\mu B_\mu \mu \phi^2 + \text{Herm. Conj.}), \quad (105)$$

which contains many free parameters and phases. The breaking of supersymmetry in the effective low-energy theory (105) is explicit but ‘soft’, in the sense that the renormalization of the parameters $m_{1/2a}, m_{0i}, A_\lambda$ and B_μ is logarithmic. Of course, these parameters are not considered to be fundamental, and the underlying mechanism of supersymmetry breaking is thought to be spontaneous, for the reasons described at the beginning of this lecture.

The logarithmic renormalization of the parameters means that one can calculate their low-energy values in terms of high-energy inputs from a supergravity or superstring theory, using standard renormalization-group equations [35]. In the case of the low-energy gaugino masses M_a , the renormalization is multiplicative and identical with that of the corresponding gauge coupling α_a at the one-loop level:

$$\frac{M_a}{m_{1/2a}} = \frac{\alpha_a}{\alpha_{GUT}} \quad (106)$$

where we assume GUT unification of the gauge couplings at the input supergravity scale. In the case of the scalar masses, there is both multiplicative renormalization and renormalization related to the gaugino masses:

$$\frac{\partial m_{0i}^2}{\partial t} = \frac{1}{16\pi^2} [\lambda^2(m_0^2 + A_\lambda^2) - g_a^2 M_a^2] \quad (107)$$

at the one-loop level, where $t \equiv \ln(Q^2/m_{GUT}^2)$, and the $\mathcal{O}(1)$ group-theoretical coefficients have been omitted. In the case of the first two generations, the first terms in (107) are negligible, and one may integrate (107) trivially to obtain effective low-energy parameters

$$m_{0i}^2 = m_0^2 + C_i m_{1/2}^2, \quad (108)$$

where universal inputs are assumed, and the coefficients C_i are calculable in any given model. The first terms in (107) are, however, important for the third generation and for the Higgs bosons of the MSSM, as we now see.

Notice that the signs of the first terms in (107) are positive, and that of the last term negative. This means that the last term tends to *increase* m_{0i}^2 as the renormalization scale Q *decreases*, an effect seen in Fig. 1. The positive signs of the first terms mean that they tend to *decrease* m_{0i}^2 as Q *decreases*, an effect seen for a Higgs squared-mass in Fig. 1. Specifically, the negative effect on H_u seen in Fig. 1 is due to its large Yukawa coupling to the t quark: $\lambda_t \sim g_{2,3}$. The exciting aspect of this observation is that spontaneous electroweak symmetry breaking is possible [35] when $m_H^2(Q) < 0$, as occurs in Fig. 1. Thus

Figure 1. The renormalization-group evolution of the soft supersymmetry-breaking parameters in the MSSM, showing the increase in the squark and slepton masses as the renormalization scale decreases, whilst the Higgs squared-mass may become negative, triggering electroweak symmetry breaking.

the spontaneous breaking of supersymmetry, which normally provides $m_0^2 > 0$, and renormalization, which then drive $m_H^2(Q) < 0$, conspire to make spontaneous electroweak symmetry breaking possible. Typically, this occurs at a renormalization scale that is exponentially smaller than the input supergravity scale:

$$\frac{m_W}{m_P} = \exp\left(\frac{-\mathcal{O}(1)}{\alpha_t}\right) : \quad \alpha_t \equiv \frac{\lambda_t^2}{4\pi}. \quad (109)$$

Typical dynamical calculations find that $m_W \sim 100$ GeV emerges naturally if $m_t \sim 60$ to 200 GeV, and this was in fact one of the first suggestions that m_t might be as high as was subsequently observed.

To conclude this section, let us briefly review the reasons why soft supersymmetry breaking might be universal, at least in some respects. There are important constraints on the mass differences of squarks and sleptons with the same internal quantum numbers, coming from flavour-changing neutral interactions [36]. These are suppressed in the Standard Model by the Glashow-Iliopoulos-Maiani mechanism [37], which limits them to magnitudes $\propto \Delta m_q^2/m_W^2$ for small squared-mass differences Δm_q^2 . Depending on the process considered, it is either necessary or desirable that particle exchange contributions, which would have ex-

pected magnitudes $\sim \Delta m_q^2/m_q^2$, be suppressed by a comparable factor. In particular, one would like

$$m_0^2(\text{first generation}) - m_0^2(\text{second generation}) \sim \delta m_q^2 \times \frac{m_q^2}{m_W^2}. \quad (110)$$

The limits on third-generation sparticle masses from flavour-changing neutral interactions are less severe, and the first/second-generation degeneracy could be relaxed if $m_q^2 \gg m_W^2$, but models with physical values of m_0^2 degenerate to $\mathcal{O}(m_q^2)$ are certainly preferred. However, this restriction is not respected in many low-energy effective theories derived from string models.

The desirability of degeneracy between sparticles of different generations help encourage some people to study models in which this property would emerge naturally, such as models of gauge-mediated supersymmetry breaking or extra dimensions [26]. However, for the rest of these lectures we shall mainly stick to familiar old supergravity.

3.4 Sparticle Masses and Mixing

We now progress to a more complete discussion of sparticle masses and mixing.

Sfermions : Each flavour of charged lepton or quark has both left- and right-handed components $f_{L,R}$, and these have separate spin-0 boson superpartners $\tilde{f}_{L,R}$. These have different isospins $I = \frac{1}{2}, 0$, but may mix as soon as the electroweak gauge symmetry is broken. Thus, for each flavour we should consider a 2×2 mixing matrix for the $\tilde{f}_{L,R}$, which takes the following general form:

$$M_{\tilde{f}}^2 \equiv \begin{pmatrix} m_{\tilde{f}LL}^2 & m_{\tilde{f}LR}^2 \\ m_{\tilde{f}LR}^2 & m_{\tilde{f}RR}^2 \end{pmatrix} \quad (111)$$

The diagonal terms may be written in the form

$$m_{\tilde{f}LL,RR}^2 = m_{\tilde{f}L,R}^2 + m_{\tilde{f}L,R}^{D^2} + m_f^2 \quad (112)$$

where m_f is the mass of the corresponding fermion, $\tilde{m}_{\tilde{f}L,R}^2$ is the soft supersymmetry-breaking mass discussed in the previous section, and $m_{\tilde{f}L,R}^{D^2}$ is a contribution due to the quartic D terms in the effective potential:

$$m_{\tilde{f}L,R}^{D^2} = m_Z^2 \cos 2\beta (I_3 + \sin^2 \theta_W Q_{em}) \quad (113)$$

where the term $\propto I_3$ is non-zero only for the $\tilde{f}_L$. Finally, the off-diagonal mixing term takes the general form

$$m_{\tilde{f}_{L,R}}^2 = m_f \left(A_f + \mu \frac{\tan \beta}{\cot \beta} \right) \quad \text{for } f = \begin{smallmatrix} e, \mu, \tau, d, s, b \\ u, c, t \end{smallmatrix} \quad (114)$$

It is clear that $\tilde{f}_{L,R}$ mixing is likely to be important for the $\tilde{t}$, and it may also be important for the $\tilde{b}_{L,R}$ and $\tilde{\tau}_{L,R}$ if $\tan \beta$ is large.

We also see from (112) that the diagonal entries for the $\tilde{t}_{L,R}$ would be different from those of the $\tilde{u}_{L,R}$ and $\tilde{c}_{L,R}$, even if their soft supersymmetry-breaking masses were universal, because of the m_f^2 contribution. In fact, we also expect non-universal renormalization of $m_{\tilde{t}_{LL,RR}}^2$ (and also $m_{\tilde{b}_{LL,RR}}^2$ and $m_{\tilde{\tau}_{LL,RR}}^2$ if $\tan \beta$ is large), because of Yukawa effects analogous to those discussed in the previous section for the renormalization of the soft Higgs masses. For these reasons, the $\tilde{t}_{L,R}$ are not usually assumed to be degenerate with the other squark flavours. Indeed, one of the $\tilde{t}$ could well be the lightest squark, perhaps even lighter than the t quark itself [38].

Charginos: These are the supersymmetric partners of the $W^\pm$ and $H^\pm$, which mix through a 2×2 matrix

$$-\frac{1}{2} (\tilde{W}^-, \tilde{H}^-) M_C \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} + \text{herm.conj.} \quad (115)$$

where

$$M_C \equiv \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix} \quad (116)$$

Here M_2 is the unmixed $SU(2)$ gaugino mass and μ is the Higgs mixing parameter introduced in (74). Fig. 2 displays (among other lines to be discussed later) the contour $m_{\chi^\pm} = 91$ GeV for the lighter of the two chargino mass eigenstates [39].

Neutralinos: These are characterized by a 4×4 mass mixing matrix [40], which takes the following form in the $(\tilde{W}^3, \tilde{B}, \tilde{H}_2^0, \tilde{H}_1^0)$ basis :

$$m_N = \begin{pmatrix} M_2 & 0 & \frac{-g_2 v_2}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} \\ 0 & M_1 & \frac{g' v_2}{\sqrt{2}} & \frac{-g' v_1}{\sqrt{2}} \\ \frac{-g_2 v_2}{\sqrt{2}} & \frac{g' v_2}{\sqrt{2}} & 0 & \mu \\ \frac{g_2 v_1}{\sqrt{2}} & \frac{-g' v_1}{\sqrt{2}} & \mu & 0 \end{pmatrix} \quad (117)$$

Figure 2. The (μ, M_2) plane characterizing charginos and neutralinos, for (a) $\mu < 0$ and (b) $\mu > 0$, including contours of m_χ and $m_{\chi^\pm}$, and of neutralino purity [39].

Note that this has a structure similar to M_C (116), but with its entries replaced by 2×2 submatrices. As has already been mentioned, one conventionally assumes that the $SU(2)$ and $U(1)$ gaugino masses $M_{1,2}$ are universal at the GUT or supergravity scale, so that

$$M_1 \simeq M_2 \frac{\alpha_1}{\alpha_2} \quad (118)$$

so the relevant parameters of (117) are generally taken to be $M_2 = (\alpha_2/\alpha_{GUT})m_{1/2}$, μ and $\tan\beta$.

Fig. 2 also displays contours of the mass of the lightest neutralino χ , as well as contours of its gaugino and Higgsino contents [39]. In the limit $M_2 \rightarrow 0$, χ would be approximately a photino and it would be approximately a Higgsino in the limit $\mu \rightarrow 0$. Unfortunately, these idealized limits are excluded by unsuccessful LEP and other searches for neutralinos and charginos, as discussed in more detail below.

3.5 The Lightest Supersymmetric Particle

This is expected to be stable in the MSSM, and hence should be present in the Universe today as a cosmological relic from the Big Bang [41, 40]. Its stability arises because there is a multiplicatively-conserved quantum number called R parity, that takes the values $+1$ for all conventional particles and -1 for all sparticles [20]. The conservation of R parity can be related to that of baryon number B and lepton number L ,

since

$$R = (-1)^{3B+L+2S} \quad (119)$$

where S is the spin. Note that R parity could be violated either spontaneously if $\langle 0|\tilde{\nu}|0\rangle \neq 0$ or explicitly if one of the supplementary couplings (77) is present. There could also be a coupling HL , but this can be defined away by choosing a field basis such that $\tilde{H}$ is defined as the superfield with a bilinear coupling to H . Note that R parity is not violated by the simplest models for neutrino masses, which have $\Delta L = 0, \pm 2$, nor by simple GUTs, which violate combinations of B and L that leave R invariant. There are three important consequences of R conservation:

- 1 sparticles are always produced in pairs, e.g., $\bar{p}p \rightarrow \tilde{q}\tilde{g}X$, $e^+e^- \rightarrow \tilde{\mu}^+ + \tilde{\mu}^-$,
- 2 heavier sparticles decay to lighter ones, e.g., $\tilde{q} \rightarrow q\tilde{g}$, $\tilde{\mu} \rightarrow \mu\tilde{\gamma}$, and
- 3 the lightest sparticle is stable,

because it has no legal decay mode.

This feature constrains strongly the possible nature of the lightest supersymmetric sparticle. If it had either electric charge or strong interactions, it would surely have dissipated its energy and condensed into galactic disks along with conventional matter. There it would surely have bound electromagnetically or via the strong interactions to conventional nuclei, forming anomalous heavy isotopes that should have been detected. There are upper limits on the possible abundances of such bound relics, as compared to conventional nucleons:

$$\frac{n(\text{relic})}{n(p)} \lesssim 10^{-15} \text{ to } 10^{-29} \quad (120)$$

for $1 \text{ GeV} \lesssim m_{\text{relic}} \lesssim 1 \text{ TeV}$. These are far below the calculated abundances of such stable relics:

$$\frac{n(\text{relic})}{n(p)} \gtrsim 10^{-6} \text{ } (10^{-10}) \quad (121)$$

for relic particles with electromagnetic (strong) interactions. We may conclude [40] that any supersymmetric relic is probably electromagnetically neutral with only weak interactions, and could in particular not be a gluino. Whether the lightest hadron containing a gluino is charged or neutral, it would surely bind to some nuclei. Even if one pleads for some level of fractionation, it is difficult to see how such gluino nuclei

could avoid the stringent bounds established for anomalous isotopes of many species.

Plausible scandidates of different spins are the sneutrinos $\tilde{\nu}$ of spin 0, the lightest neutralino χ of spin 1/2, and the gravitino $\tilde{G}$ of spin 3/2. The sneutrinos have been ruled out by the combination of LEP experiments and direct searches for cosmological relics. The gravitino cannot be ruled out, but we concentrate on the neutralino possibility for the rest of these Lectures.

A very attractive feature of the neutralino candidature for the lightest supersymmetric particle is that it has a relic density of interest to astrophysicists and cosmologists: $\Omega_\chi h^2 = \mathcal{O}(0.1)$ over generic domains of the MSSM parameter space [40], as discussed in the next Lecture. In these domains, the lightest neutralino χ could constitute the cold dark matter favoured by theories of cosmological structure formation.

3.6 Supersymmetric Higgs Bosons

As was discussed in Lecture 2, one expects two complex Higgs doublets $H_2 \equiv (H_2^+, H_2^0)$, $H_1 \equiv (H_1^+, H_1^0)$ in the MSSM, with a total of 8 real degrees of freedom. Of these, 3 are eaten via the Higgs mechanism to become the longitudinal polarization states of the $W^\pm$ and Z^0 , leaving 5 physical Higgs bosons to be discovered by experiment. Three of these are neutral: the lighter CP-even neutral h , the heavier CP-even neutral H , the CP-odd neutral A , and charged bosons $H^\pm$. The quartic potential is completely determined by the D terms

$$V_4 = \frac{g^2 + g'^2}{8} (|H_1^0|^2 - |H_2^0|^2)^2 \quad (122)$$

for the neutral components, whilst the quadratic terms may be parametrized at the tree level by

$$m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + (m_3^2 H_1 H_2 + \text{herm.conj.}) \quad (123)$$

where $m_3^2 = B_\mu \mu$. One combination of the three parameters $(m_{H_1}^2, m_{H_2}^2, m_3^2)$ is fixed by the Higgs vacuum expectation $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV, and the other two combinations may be rephrased as $(m_A, \tan \beta)$. These characterize all Higgs masses and couplings in the MSSM at the tree level. Looking back at (122), we see that the gauge coupling strength of the quartic interactions suggests a relatively low mass for at least the lightest MSSM Higgs boson h , and this is indeed the case, with $m_h \leq m_Z$ at the tree level:

$$m_h^2 = m_Z^2 \cos^2 2\beta \quad (124)$$

This raised considerable hope that the lightest MSSM Higgs boson could be discovered at LEP, with its prospective reach to $m_H \sim 100$ GeV.

However, radiative corrections to the Higgs masses are calculable in a supersymmetric model (this was, in some sense, the whole point of introducing supersymmetry!), and they turn out to be non-negligible for $m_t \sim 175$ GeV [22]. Indeed, the leading one-loop corrections to m_h^2 depend quartically on m_t :

$$\Delta m_h^2 = \frac{3m_t^4}{4\pi^2 v^2} \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + \frac{3m_t^4 \hat{A}_t^2}{8\pi^2 v^2} \left[2h(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \hat{A}_t^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] + \dots \quad (125)$$

where $m_{\tilde{t}_{1,2}}$ are the physical masses of the two stop squarks $\tilde{t}_{1,2}$ to be discussed in more detail shortly, $\hat{A}_t \equiv A_t - \mu \cot \beta$, and

$$h(a, b) \equiv \frac{1}{a-b} \ln \left(\frac{a}{b} \right), \quad f(a, b) = \frac{1}{(a-b)^2} \left[2 - \frac{a+b}{a-b} \ln \left(\frac{a}{b} \right) \right] \quad (126)$$

Non-leading one-loop corrections to the MSSM Higgs masses are also known, as are corrections to coupling vertices, two-loop corrections and renormalization-group resummations [42]. For $m_{\tilde{t}_{1,2}} \lesssim 1$ TeV and a plausible range of A_t , one finds

$$m_h \lesssim 130 \text{ GeV} \quad (127)$$

as seen in Fig. 14. There we see the sensitivity of m_h to $(m_A, \tan \beta)$, and we also see how m_A, m_H and $m_{H^\pm}$ approach each other for large m_A .

4. PHENOMENOLOGY

4.1 Constraints on the MSSM

Important experimental constraints on the MSSM parameter space are provided by direct searches at LEP and the Tevatron collider, as compiled in the $(m_{1/2}, m_0)$ planes for different values of $\tan \beta$ and the sign of μ in Fig. 4. One of these is the limit $m_{\chi^\pm} \gtrsim 103.5$ GeV provided by chargino searches at LEP [43], where the fourth significant figure depends on other CMSSM parameters. LEP has also provided lower limits on slepton masses, of which the strongest is $m_{\tilde{e}} \gtrsim 99$ GeV [44], again depending only slightly on the other CMSSM parameters, as long as $m_{\tilde{e}} - m_{\chi} \gtrsim 10$ GeV. The most important constraints on the u, d, s, c, b squarks and gluinos are provided by the FNAL Tevatron collider: for equal masses $m_{\tilde{q}} = m_{\tilde{g}} \gtrsim 300$ GeV. In the case of the $\tilde{t}$, LEP provides the most stringent limit when $m_{\tilde{t}} - m_{\chi}$ is small, and the Tevatron for larger $m_{\tilde{t}} - m_{\chi}$ [43].

Figure 3. The lightest Higgs boson mass in the MSSM, for different values of $\tan\beta$ and the CP-odd Higgs boson mass M_A .

Another important constraint is provided by the LEP lower limit on the Higgs mass: $m_H > 114.4$ GeV [46]. This holds in the Standard Model, for the lightest Higgs boson h in the general MSSM for $\tan\beta \lesssim 8$, and almost always in the CMSSM for all $\tan\beta$, at least as long as CP is conserved². Since m_h is sensitive to sparticle masses, particularly $m_{\tilde{t}}$, via loop corrections:

$$\delta m_h^2 \propto \frac{m_t^4}{m_W^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \dots \quad (128)$$

the Higgs limit also imposes important constraints on the soft supersymmetry-breaking CMSSM parameters, principally $m_{1/2}$ [50] as seen in Fig. 4. The constraints are here evaluated using `FeynHiggs` [42], which is estimated to have a residual uncertainty of a couple of GeV in m_h .

Also shown in Fig. 4 is the constraint imposed by measurements of $b \rightarrow s\gamma$ [47]. These agree with the Standard Model, and therefore provide bounds on MSSM particles, such as the chargino and charged Higgs masses, in particular. Typically, the $b \rightarrow s\gamma$ constraint is more important

²The lower bound on the lightest MSSM Higgs boson may be relaxed significantly if CP violation feeds into the MSSM Higgs sector [49].

Figure 4. Compilations of phenomenological constraints on the CMSSM for (a) $\tan \beta = 10, \mu < 0$, (b) $\tan \beta = 10, \mu > 0$, (c) $\tan \beta = 35, \mu < 0$ and (d) $\tan \beta = 50, \mu > 0$, assuming $A_0 = 0, m_t = 175$ GeV and $m_b(m_b)_{\overline{MS}} = 4.25$ GeV [45]. The near-vertical lines are the LEP limits $m_{\chi^\pm} = 103.5$ GeV (dashed and black) [43], shown in (b) only, and $m_h = 114$ GeV (dotted and red) [46]. Also, in the lower left corner of (b), we show the $m_{\tilde{e}} = 99$ GeV contour [44]. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_1$, so this region is excluded. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_\chi h^2 \leq 0.3$ [45]. The medium (dark green) shaded regions that are most prominent in panels (a) and (c) are excluded by $b \rightarrow s\gamma$ [47]. The shaded (pink) regions in the upper right regions show the $\pm 2\sigma$ ranges of $g_\mu - 2$. For $\mu > 0$, the $\pm 2(1)\sigma$ contours are also shown as solid (dashed) black lines [48].

for $\mu < 0$, as seen in Fig. 4a and c, but it is also relevant for $\mu > 0$, particularly when $\tan \beta$ is large as seen in Fig. 4d.

The final experimental constraint we consider is that due to the measurement of the anomalous magnetic moment of the muon. Following its first result last year [51], the BNL E821 experiment has recently reported a new measurement [52] of $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$, which deviates by 3.0 standard deviations from the best available Standard Model predictions based on low-energy $e^+e^- \rightarrow \text{hadrons}$ data [53]. On the other hand, the discrepancy is more like 1.6 standard deviations if one uses $\tau \rightarrow \text{hadrons}$ data to calculate the Standard Model prediction. Faced with this confusion, and remembering the chequered history of previous theoretical calculations [54], it is reasonable to defer judgement whether there is a significant discrepancy with the Standard Model. However, either way, the measurement of a_μ is a significant constraint on the CMSSM, favouring $\mu > 0$ in general, and a specific region of the $(m_{1/2}, m_0)$ plane if one accepts the theoretical prediction based on $e^+e^- \rightarrow \text{hadrons}$ data [55]. The regions preferred by the current $g - 2$ experimental data and the $e^+e^- \rightarrow \text{hadrons}$ data are shown in Fig. 4.

Fig. 4 also displays the regions where the supersymmetric relic density $\rho_\chi = \Omega_\chi \rho_{\text{critical}}$ falls within the preferred range

$$0.1 < \Omega_\chi h^2 < 0.3 \quad (129)$$

The upper limit on the relic density is rigorous, since astrophysics and cosmology tell us that the total matter density $\Omega_m \lesssim 0.4$ [56], and the Hubble expansion rate $h \sim 1/\sqrt{2}$ to within about 10 % (in units of 100 km/s/Mpc). On the other hand, the lower limit in (129) is optional, since there could be other important contributions to the overall matter density. Smaller values of $\Omega_\chi h^2$ correspond to smaller values of $(m_{1/2}, m_0)$, in general.

As is seen in Fig. 4, there are generic regions of the CMSSM parameter space where the relic density falls within the preferred range (129). What goes into the calculation of the relic density? It is controlled by the annihilation cross section [40]:

$$\rho_\chi = m_\chi n_\chi, \quad n_\chi \sim \frac{1}{\sigma_{\text{ann}}(\chi\chi \rightarrow \dots)}, \quad (130)$$

where the typical annihilation cross section $\sigma_{\text{ann}} \sim 1/m_\chi^2$. For this reason, the relic density typically increases with the relic mass, and this combined with the upper bound in (129) then leads to the common expectation that $m_\chi \lesssim \mathcal{O}(1)$ GeV.

However, there are various ways in which the generic upper bound on m_χ can be increased along filaments in the $(m_{1/2}, m_0)$ plane. For ex-

ample, if the next-to-lightest sparticle (NLSP) is not much heavier than χ : $\Delta m/m_\chi \lesssim 0.1$, the relic density may be suppressed by coannihilation: $\sigma(\chi + \text{NLSP} \rightarrow \dots)$ [57]. In this way, the allowed CMSSM region may acquire a ‘tail’ extending to larger sparticle masses. An example of this possibility is the case where the NLSP is the lighter stau: $\tilde{\tau}_1$ and $m_{\tilde{\tau}_1} \sim m_\chi$, as seen in Fig. 4 [58].

Another mechanism for extending the allowed CMSSM region to large m_χ is rapid annihilation via a direct-channel pole when $m_\chi \sim \frac{1}{2}m_{Higgs,Z}$ [59, 45]. This may yield a ‘funnel’ extending to large $m_{1/2}$ and m_0 at large $\tan\beta$, as seen in panels (c) and (d) of Fig. 4 [45]. Yet another allowed region at large $m_{1/2}$ and m_0 is the ‘focus-point’ region [60], which is adjacent to the boundary of the region where electroweak symmetry breaking is possible. The lightest supersymmetric particle is relatively light in this region.

4.2 Benchmark Supersymmetric Scenarios

As seen in Fig. 4, all the experimental, cosmological and theoretical constraints on the MSSM are mutually compatible. As an aid to understanding better the physics capabilities of the LHC, various e^+e^- linear collider designs and non-accelerator experiments, a set of benchmark supersymmetric scenarios have been proposed [62]. Their distribution in the $(m_{1/2}, m_0)$ plane is sketched in Fig. 5. These benchmark scenarios are compatible with all the accelerator constraints mentioned above, including the LEP searches and $b \rightarrow s\gamma$, and yield relic densities of LSPs in the range suggested by cosmology and astrophysics. The benchmarks are not intended to sample ‘fairly’ the allowed parameter space, but rather to illustrate the range of possibilities currently allowed.

In addition to a number of benchmark points falling in the ‘bulk’ region of parameter space at relatively low values of the supersymmetric particle masses, as seen in Fig. 5, we also proposed [62] some points out along the ‘tails’ of parameter space extending out to larger masses. These clearly require some degree of fine-tuning to obtain the required relic density and/or the correct $W^\pm$ mass, and some are also disfavoured by the supersymmetric interpretation of the $g_\mu - 2$ anomaly, but all are logically consistent possibilities.

4.3 Prospects for Discovering Supersymmetry

In the CMSSM discussed here, there are just a few prospects for discovering supersymmetry at the FNAL *Tevatron collider* [62], but these could be increased in other supersymmetric models [63]. Fig. 6 shows the physics reach for observing pairs of supersymmetric particles at the

Figure 5. The locations of the benchmark points proposed in [62] in the region of the $(m_{1/2}, m_0)$ plane where $\Omega_\chi h^2$ falls within the range preferred by cosmology (shaded blue). Note that the filaments of the allowed parameter space extending to large $m_{1/2}$ and/or m_0 are sampled.

Figure 6. The regions of the $(m_0, m_{1/2})$ plane that can be explored by the LHC with various integrated luminosities [66], using the missing energy + jets signature [65].

LHC. The signature for supersymmetry - multiple jets (and/or leptons) with a large amount of missing energy - is quite distinctive, as seen in Fig. 7 [64, 65]. Therefore, the detection of the supersymmetric partners of quarks and gluons at the LHC is expected to be quite easy if they weigh less than about 2.5 TeV [66]. Moreover, in many scenarios one should be able to observe their cascade decays into lighter supersymmetric particles, as seen in Fig. 8 [67]. As seen in Fig. 9, large fractions of the supersymmetric spectrum should be seen in most of the benchmark scenarios, although there are a couple where only the lightest supersymmetric Higgs boson would be seen [62], as seen in Fig. 9.

Electron-positron colliders provide very clean experimental environments, with egalitarian production of all the new particles that are kinematically accessible, including those that have only weak interactions.

Figure 7. The distribution expected at the LHC in the variable M_{eff} that combines the jet energies with the missing energy [?, 64, 65].

Figure 8. The dilepton mass distributions expected at the LHC due to sparticle decays in two different supersymmetric scenarios [?, 66, 65].

CMSSM Benchmarks

Figure 9. The numbers of different sparticles expected to be observable at the LHC and/or linear e^+e^- colliders with various energies, in each of the proposed benchmark scenarios [62], ordered by their difference from the present central experimental value of $g_\mu - 2$.

Moreover, polarized beams provide a useful analysis tool, and $e\gamma$, $\gamma\gamma$ and e^-e^- colliders are readily available at relatively low marginal costs.

The $e^+e^- \rightarrow t\bar{t}$ threshold is known to be at $E_{\text{CM}} \sim 350$ GeV. Moreover, if the Higgs boson indeed weighs less than 200 GeV, as suggested by the precision electroweak data, its production and study would also be easy at an e^+e^- collider with $E_{\text{CM}} \sim 500$ GeV. With a luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ or more, many decay modes of the Higgs boson could be measured very accurately, and one might be able to find a hint whether its properties were modified by supersymmetry [68, 69].

However, the direct production of supersymmetric particles at such a collider cannot be guaranteed [70]. We do not yet know what the supersymmetric threshold energy may be (or even if there is one!). We may well not know before the operation of the LHC, although $g_\mu - 2$ might provide an indication [55], if the uncertainties in the Standard Model calculation can be reduced.

If an e^+e^- collider is above the supersymmetric threshold, it will be able to measure very accurately the sparticle masses. By comparing their masses with those of different sparticles produced at the LHC, one would be able to make interesting tests of string and GUT models of supersymmetry breaking, as seen in Fig. 10 [71]. However, independently from the particular benchmark scenarios proposed, a linear e^+e^- collider with $E_{\text{CM}} < 1$ TeV would not cover all the supersymmetric parameter space allowed by cosmology [70, 62].

Nevertheless, there are compelling physics arguments for such a linear e^+e^- collider, which would be very complementary to the LHC in terms of its exploratory power and precision [68]. It is to be hoped that the world community will converge on a single project with the widest possible energy range.

CERN and collaborating institutes are studying the possible following step in linear e^+e^- colliders, a multi-TeV machine called CLIC [72, 73]. This would use a double-beam technique to attain accelerating gradients as high as 150 MV/m, and the viability of accelerating structures capable of achieving this field has been demonstrated in the CLIC test facility [74]. Parameter sets have been calculated for CLIC designs with $E_{\text{CM}} = 3, 5$ TeV and luminosities of $10^{35} \text{ cm}^{-2}\text{s}^{-1}$ or more [72].

In many of the proposed benchmark supersymmetric scenarios, CLIC would be able to complete the supersymmetric spectrum and/or measure in much more detail heavy sparticles found previously at the LHC, as seen in Fig. 9 [62]. CLIC produces more beamstrahlung than lower-energy linear e^+e^- colliders, but the supersymmetric missing-energy signature would still be easy to distinguish, and accurate measurements of masses and decay modes could still be made, as seen in Fig. 11 [75].

Figure 10. Measurements of sparticle masses at the LHC and a linear e^+e^- linear collider will enable one to check their universality at some input GUT scale, and check possible models of supersymmetry breaking [71]. Both axes are labelled in GeV units.

Figure 11. Like lower-energy e^+e^- colliders, CLIC enables very accurate measurements of sparticle masses to be made, in this case the supersymmetric partner of the muon and the lightest neutralino χ^0 [75].

4.4 Searches for Dark Matter Particles

In the above discussion, we have paid particular attention to the region of parameter space where the lightest supersymmetric particle could constitute the cold dark matter in the Universe [40]. How easy would this be to detect? Fig. 12 shows rates for the elastic spin-independent scattering of supersymmetric relics [76], including the projected sensitivities for CDMS II [77] and CRESST [78] (solid) and GENIUS [79] (dashed). Also shown are the cross sections calculated in the proposed benchmark scenarios discussed in the previous section, which are considerably below the DAMA [80] range ($10^{-5} - 10^{-6}$ pb), but may be within reach of future projects. The prospects for detecting elastic spin-independent scattering are less bright, as also shown in Fig. 12. Indirect searches for supersymmetric dark matter via the products of annihilations in the galactic halo or inside the Sun also have prospects in some of the benchmark scenarios [76], as seen in Fig. 13.

Figure 12. Left panel: elastic spin-independent scattering of supersymmetric relics on protons calculated in benchmark scenarios [76], compared with the projected sensitivities for CDMS II [77] and CRESST [78] (solid) and GENIUS [79] (dashed). The predictions of the SSARD code (blue crosses) and Neutdriver[81] (red circles) for neutralino-nucleon scattering are compared. The labels A, B, ..., L correspond to the benchmark points as shown in Fig. 5. Right panel: prospects for detecting elastic spin-independent scattering in the benchmark scenarios, which are less bright.

Figure 13. Left panel: prospects for detecting photons with energies above 1 GeV from annihilations in the centre of the galaxy, assuming a moderate enhancement there of the overall halo density, and right panel: prospects for detecting muons from energetic solar neutrinos produced by relic annihilations in the Sun, as calculated [76] in the benchmark scenarios using Neutdriver[81].

5. LEPTON FLAVOUR VIOLATION

5.1 Why not?

There is no good reason why either the total lepton number L or the individual lepton flavours $L_{e,\mu,\tau}$ should be conserved. We have learnt that the only conserved quantum numbers are those associated with exact gauge symmetries, just as the conservation of electromagnetic charge is associated with $U(1)$ gauge invariance. On the other hand, there is no exact gauge symmetry associated with any of the lepton numbers.

Moreover, neutrinos have been seen to oscillate between their different flavours [82, 83], showing that the separate lepton flavours $L_{e,\mu,\tau}$ are indeed not conserved, though the conservation of total lepton number L is still an open question. The observation of such oscillations strongly suggests that the neutrinos have different masses. Again, massless particles are generally associated with exact gauge symmetries, e.g., the photon with the $U(1)$ symmetry of the Standard Model, and the gluons with its $SU(3)$ symmetry. In the absence of any leptonic gauge symmetry, non-zero lepton masses are to be expected, in general.

The conservation of lepton number is an accidental symmetry of the renormalizable terms in the Standard Model lagrangian. However, one could easily add to the Standard Model non-renormalizable terms that would generate neutrino masses, even without introducing a ‘right-handed’ neutrino field. For example, a non-renormalizable term of the form [84]

$$\frac{1}{M} \nu^H \cdot \nu H, \quad (131)$$

where M is some large mass beyond the scale of the Standard Model, would generate a neutrino mass term:

$$m_\nu \nu \cdot \nu : m_\nu = \frac{\langle 0|H|0\rangle^2}{M}. \quad (132)$$

Of course, a non-renormalizable interaction such as (131) seems unlikely to be fundamental, and one should like to understand the origin of the large mass scale M .

The minimal renormalizable model of neutrino masses requires the introduction of weak-singlet ‘right-handed’ neutrinos N . These will in general couple to the conventional weak-doublet left-handed neutrinos via Yukawa couplings Y_ν that yield Dirac masses $m_D \sim m_W$. In addition, these ‘right-handed’ neutrinos N can couple to themselves via Majorana masses M that may be $\gg m_W$, since they do not require electroweak symmetry breaking. Combining the two types of mass term, one obtains

the seesaw mass matrix [85]:

$$(\nu_L, N) \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}, \quad (133)$$

where each of the entries should be understood as a matrix in generation space.

In order to provide the two measured differences in neutrino masses-squared, there must be at least two non-zero masses, and hence at least two heavy singlet neutrinos N_i [86, 87]. Presumably, all three light neutrino masses are non-zero, in which case there must be at least three N_i . This is indeed what happens in simple GUT models such as SO(10), but some models [88] have more singlet neutrinos [89]. In this Lecture, for simplicity we consider just three N_i .

As we discuss in the next Section, this seesaw model can accommodate the neutrino mixing seen experimentally, and naturally explains the small differences in the masses-squared of the light neutrinos. By itself, it would lead to unobservably small transitions between the different charged-lepton flavours. However, supersymmetry may enhance greatly the rates for processes violating the different charged-lepton flavours, rendering them potentially observable, as we discuss in subsequent Sections.

5.2 Neutrino Masses and Mixing in the Seesaw Model

The effective mass matrix for light neutrinos in the seesaw model may be written as:

$$\mathcal{M}_\nu = Y_\nu^T \frac{1}{M} Y_\nu v^2 [\sin^2 \beta] \quad (134)$$

where we have used the relation $m_D = Y_\nu v [\sin \beta]$ with $v \equiv \langle 0|H|0 \rangle$, and the factors of $\sin \beta$ appear in the supersymmetric version of the seesaw model. It is convenient to work in the field basis where the charged-lepton masses $m_{l\pm}$ and the heavy singlet-neutrino masses M are real and diagonal. The seesaw neutrino mass matrix $\mathcal{M}_\nu$ (134) may then be diagonalized by a unitary transformation U :

$$U^T \mathcal{M}_\nu U = \mathcal{M}_\nu^d. \quad (135)$$

This diagonalization is reminiscent of that required for the quark mass matrices in the Standard Model. In that case, it is well known that one can redefine the phases of the quark fields [90] so that the mixing matrix U_{CKM} has just one CP-violating phase [91]. However, in the neutrino

case, there are fewer independent field phases, and one is left with three physical CP-violating parameters:

$$U = \tilde{P}_2 V P_0 : P_0 \equiv \text{Diag} (e^{i\phi_1}, e^{i\phi_2}, 1). \quad (136)$$

Here $\tilde{P}_2 = \text{Diag} (e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ contains three phases that can be removed by phase rotations and are unobservable in light-neutrino physics, V is the light-neutrino mixing matrix first considered by Maki, Nakagawa and Sakata (MNS) [92], and P_0 contains 2 observable CP-violating phases $\phi_{1,2}$. The MNS matrix describes neutrino oscillations

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13}e^{-i\delta} \end{pmatrix} \quad (137)$$

The Majorana phases $\phi_{1,2}$ are in principle observable in neutrinoless double- β decay, whose matrix element is proportional to

$$\langle m_\nu \rangle_{ee} \equiv \sum_i U_{ei}^* m_{\nu_i} U_{ie}^\dagger. \quad (138)$$

Later we discuss how other observable quantities might be sensitive indirectly to the Majorana phases.

The first matrix factor in (137) is measurable in solar neutrino experiments. As seen in Fig. 14, the recent data from SNO [83] and Super-Kamiokande [93] prefer quite strongly the large-mixing-angle (LMA) solution to the solar neutrino problem with $\Delta m_{12}^2 \sim 6 \times 10^{-5} \text{ eV}^2$, though the LOW solution with lower δm^2 cannot yet be ruled out. The data favour large but non-maximal mixing: $\theta_{12} \sim 30^\circ$. The second matrix factor in (137) is measurable in atmospheric neutrino experiments. As seen in Fig. 15, the data from Super-Kamiokande in particular [82] favour maximal mixing of atmospheric neutrinos: $\theta_{23} \sim 45^\circ$ and $\Delta m_{23}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$. The third matrix factor in (137) is basically unknown, with experiments such as Chooz [94] and Super-Kamiokande only establishing upper limits on θ_{13} , and *a fortiori* no information on the CP-violating phase δ .

The phase δ could in principle be measured by comparing the oscillation probabilities for neutrinos and antineutrinos and computing the CP-violating asymmetry [95]:

$$P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = 16s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta \quad (139)$$

$$\sin\left(\frac{\Delta m_{12}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{13}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{23}^2 L}{4E}\right),$$

as seen in Fig. 16 [96, 97]. This is possible only if Δm_{12}^2 and s_{12} are large enough - as now suggested by the success of the LMA solution to

Figure 14. A global fit to solar neutrino data, following the SNO measurements of the total neutral-current reaction rate, the energy spectrum and the day-night asymmetry, favours large mixing and $\Delta m^2 \sim 6 \times 10^{-5} \text{ eV}^2$ [83].

Figure 15. A fit to the Super-Kamiokande data on atmospheric neutrinos [82] indicates near-maximal $\nu_\mu - \nu_\tau$ mixing with $\Delta m^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$.

Figure 16. Correlations in a simultaneous fit of θ_{13} and δ , using a neutrino energy threshold of about 10 GeV. Using a single baseline correlations are very strong, but can be largely reduced by combining information from different baselines and detector techniques [96], enabling the CP-violating phase δ to be extracted.

the solar neutrino problem, and if s_{13} is large enough - which remains an open question.

We have seen above that the effective low-energy mass matrix for the light neutrinos contains 9 parameters, 3 mass eigenvalues, 3 real mixing angles and 3 CP-violating phases. However, these are not all the parameters in the minimal seesaw model. As shown in Fig. 17, this model has a total of 18 parameters [98, 99]. Most of the rest of this Lecture is devoted to understanding better the origins and possible manifestations of the remaining parameters, many of which may have controlled the generation of matter in the Universe via leptogenesis [100] and may be observable via renormalization in supersymmetric models [101, 99, 102, 103].

To see how the extra 9 parameters appear [99], we reconsider the full lepton sector, assuming that we have diagonalized the charged-lepton mass matrix:

$$(Y_\ell)_{ij} = Y_{\ell_i}^d \delta_{ij}, \quad (140)$$

as well as that of the heavy singlet neutrinos:

$$M_{ij} = M_i^d \delta_{ij}. \quad (141)$$

Figure 17. Roadmap for the physical observables derived from Y_ν and N_i [104].

We can then parametrize the neutrino Dirac coupling matrix Y_ν in terms of its real and diagonal eigenvalues and unitary rotation matrices:

$$Y_\nu = Z^* Y_{\nu_k}^d X^\dagger, \quad (142)$$

where X has 3 mixing angles and one CP-violating phase, just like the CKM matrix, and we can write Z in the form

$$Z = P_1 \bar{Z} P_2, \quad (143)$$

where $\bar{Z}$ also resembles the CKM matrix, with 3 mixing angles and one CP-violating phase, and the diagonal matrices $P_{1,2}$ each have two CP-violating phases:

$$P_{1,2} = \text{Diag} \left(e^{i\theta_{1,3}}, e^{i\theta_{2,4}}, 1 \right). \quad (144)$$

In this parametrization, we see explicitly that the neutrino sector has 18 parameters: the 3 heavy-neutrino mass eigenvalues M_i^d , the 3 real

eigenvalues of $Y_{\nu_i}^D$, the $6 = 3 + 3$ real mixing angles in X and $\bar{Z}$, and the $6 = 1 + 5$ CP-violating phases in X and $\bar{Z}$ [99].

As we discuss later in more detail, leptogenesis [100] is proportional to the product

$$Y_\nu Y_\nu^\dagger = P_1^* \bar{Z}^* (Y_\nu^d)^2 \bar{Z}^T P_1, \quad (145)$$

which depends on 13 of the real parameters and 3 CP-violating phases, whilst the leading renormalization of soft supersymmetry-breaking masses depends on the combination

$$Y_\nu^\dagger Y_\nu = X (Y_\nu^d)^2 X^\dagger, \quad (146)$$

which depends on just 1 CP-violating phase, with two more phases appearing in higher orders, when one allows the heavy singlet neutrinos to be non-degenerate [102].

In order to see how the low-energy sector is embedded in this full parametrization, we first recall that the 3 phases in $\tilde{P}_2$ (136) become observable when one also considers high-energy quantities. Next, we introduce a complex orthogonal matrix

$$R \equiv \sqrt{M^d}^{-1} Y_\nu U \sqrt{M^d}^{-1} [v \sin \beta], \quad (147)$$

which has 3 real mixing angles and 3 phases: $R^T R = 1$. These 6 additional parameters may be used to characterize Y_ν , by inverting (147):

$$Y_\nu = \frac{\sqrt{M^d} R \sqrt{M^d} U^\dagger}{[v \sin \beta]}, \quad (148)$$

giving us the same grand total of $18 = 9 + 3 + 6$ parameters [99]. The leptogenesis observable (145) may now be written in the form

$$Y_\nu Y_\nu^\dagger = \frac{\sqrt{M^d} R \mathcal{M}_\nu^d R^\dagger \sqrt{M^d}}{[v^2 \sin^2 \beta]}, \quad (149)$$

which depends on the 3 phases in R , but *not* the 3 low-energy phases $\delta, \phi_{1,2}$, *nor* the 3 real MNS mixing angles [99]! Conversely, the leading renormalization observable (146) may be written in the form

$$Y_\nu^\dagger Y_\nu = U \frac{\sqrt{\mathcal{M}_\nu^d} R^\dagger M^d R \sqrt{\mathcal{M}_\nu^d}}{[v^2 \sin^2 \beta]} U^\dagger, \quad (150)$$

which depends explicitly on the MNS matrix, including the CP-violating phases δ and $\phi_{1,2}$, but only one of the three phases in $\tilde{P}_2$ [99].

5.3 Renormalization of Soft Supersymmetry-Breaking Parameters

Let us now discuss the renormalization of soft supersymmetry-breaking parameters m_0^2 and A in more detail, assuming that the input values at the GUT scale are flavour-independent. If they are not, there will be additional sources of flavour-changing processes, beyond those discussed in this and subsequent sections [36, 105]. In the leading-logarithmic approximation, and assuming degenerate heavy singlet neutrinos, one finds the following radiative corrections to the soft supersymmetry-breaking terms for sleptons:

$$\begin{aligned} (\delta m_{\tilde{L}}^2)_{ij} &= -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y_\nu^\dagger Y_\nu)_{ij} \text{Ln} \left(\frac{M_{GUT}}{M} \right), \\ (\delta A_\ell)_{ij} &= -\frac{1}{8\pi^2} A_0 Y_{\ell i} (Y_\nu^\dagger Y_\nu)_{ij} \text{Ln} \left(\frac{M_{GUT}}{M} \right), \end{aligned} \quad (151)$$

where we have initially assumed that the heavy singlet neutrinos are approximately degenerate with $M \ll M_{GUT}$. In this case, there is a single analogue of the Jarlskog invariant of the Standard Model [106]:

$$J_{\tilde{L}} \equiv \text{Im} \left[(m_{\tilde{L}}^2)_{12} (m_{\tilde{L}}^2)_{23} (m_{\tilde{L}}^2)_{31} \right], \quad (152)$$

which depends on the single phase that is observable in this approximation. There are other Jarlskog invariants defined analogously in terms of various combinations with the A_ℓ , but these are all proportional [99].

There are additional contributions if the heavy singlet neutrinos are not degenerate:

$$(\tilde{\delta} m_{\tilde{L}}^2)_{ij} = -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y_\nu^\dagger L Y_\nu)_{ij} : L \equiv \text{Ln} \left(\frac{\bar{M}}{\bar{M}_i} \right) \delta_{ij}, \quad (153)$$

where $\bar{M} \equiv \sqrt[3]{M_1 M_2 M_3}$, with $(\tilde{\delta} A_\ell)_{ij}$ being defined analogously. These new contributions contain the matrix factor

$$Y^\dagger L Y = X Y^d P_2 \bar{Z}^T L \bar{Z}^* P_2^* y^d X^\dagger, \quad (154)$$

which introduces dependences on the phases in $\bar{Z} P_2$, though not P_1 . In this way, the renormalization of the soft supersymmetry-breaking parameters becomes sensitive to a total of 3 CP-violating phases [102].

5.4 Exploration of Parameter Space

Now that we have seen how the 18 parameters in the minimal supersymmetric seesaw model might in principle be observable, we would like

to explore the range of possibilities in this parameter space. This requires confronting two issues: the unwieldy large dimensionality of the parameter space, and the inclusion of the experimental information already obtained (or obtainable) from low-energy studies of neutrinos. Of the 9 parameters accessible to these experiments: $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, \theta_{12}, \theta_{23}, \theta_{31}, \delta, \phi_1$ and ϕ_2 , we have measurements of 4 combinations: $\Delta m_{12}^2, \Delta m_{23}^2, \theta_{12}$ and θ_{23} , and upper limits on the overall light-neutrino mass scale, θ_{13} and the double- β decay observable (138).

The remaining 9 parameters not measurable in low-energy neutrino physics may be characterized by an auxiliary Hermitean matrix of the following form [101, 103]:

$$H \equiv Y_\nu^\dagger D Y_\nu, \quad (155)$$

where D is an arbitrary real and diagonal matrix. Possible choices for D include $\text{Diag}(\pm 1, \pm 1, \pm 1)$ and the logarithmic matrix L defined in (153). Once one specifies the 9 parameters in H , either in a statistical survey or in some definite model, one can calculate

$$H' \equiv \sqrt{\mathcal{M}_\nu^d} U^\dagger H U \sqrt{\mathcal{M}_\nu^d}, \quad (156)$$

which can then be diagonalized by a complex orthogonal matrix R' :

$$H' = R'^\dagger \mathcal{M}'^d R' : R'^T R' = 1. \quad (157)$$

In this way, we can calculate all the remaining physical parameters:

$$(\mathcal{M}_\nu, H) \rightarrow (\mathcal{M}_\nu, \mathcal{M}'^d, R') \rightarrow (Y_\nu, M_i) \quad (158)$$

and then go on to calculate leptogenesis, charged-lepton violation, etc [101, 103].

A freely chosen model will in general violate the experimental upper limit on $\mu \rightarrow e\gamma$ [107]. It is easy to avoid this problem using the parametrization (155) [103]. If one chooses $D = L$ and requires the entry $H_{12} = 0$, the leading contribution to $\mu \rightarrow e\gamma$ from renormalization of the soft supersymmetry-breaking masses will be suppressed. To suppress $\mu \rightarrow e\gamma$ still further, one may impose the constraint $H_{13}H_{23} = 0$. This condition evidently has two solutions: either $H_{13} = 0$, in which case $\tau \rightarrow e\gamma$ is suppressed but not $\tau \rightarrow \mu\gamma$, or alternatively $H_{23} = 0$, which favours $\tau \rightarrow e\gamma$ over $\tau \rightarrow \mu\gamma$. Thus we may define two generic textures H^1 and H^2 :

$$H^1 \equiv \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & d^\dagger & c \end{pmatrix}, \quad H^2 \equiv \begin{pmatrix} a & 0 & d \\ 0 & b & 0 \\ d^\dagger & 0 & c \end{pmatrix}. \quad (159)$$

We use these as guides in the following, whilst recalling that they represent extremes, and the truth may not favour one $\tau \rightarrow \ell\gamma$ decay mode so strongly over the other.

5.5 Leptogenesis

In addition to the low-energy neutrino constraints, we frequently employ the constraint that the model parameters be compatible with the leptogenesis scenario for creating the baryon asymmetry of the Universe [100]. We recall that the baryon-to-entropy ratio Y_B in the Universe today is found to be in the range $10^{-11} < Y_B < 3 \times 10^{-10}$. This is believed to have evolved from a similar asymmetry in the relative abundances of quarks and antiquarks before they became confined inside hadrons when the temperature of the Universe was about 100 MeV. In the leptogenesis scenario [100], non-perturbative electroweak interactions caused this small asymmetry to evolve out of a similar small asymmetry in the relative abundances of leptons and antileptons that had been generated by CP violation in the decays of heavy singlet neutrinos.

The total decay rate of such a heavy neutrino N_i may be written in the form

$$\Gamma_i = \frac{1}{8\pi} \left(Y_\nu Y_\nu^\dagger \right)_{ii} M_i. \quad (160)$$

One-loop CP-violating diagrams involving the exchange of heavy neutrino N_j would generate an asymmetry in N_i decay of the form:

$$\epsilon_{ij} = \frac{1}{8\pi} \frac{1}{\left(Y_\nu Y_\nu^\dagger \right)_{ii}} \text{Im} \left(\left(Y_\nu Y_\nu^\dagger \right)_{ij} \right)^2 f \left(\frac{M_j}{M_i} \right), \quad (161)$$

where $f(M_j/M_i)$ is a known kinematic function.

As already remarked, the relevant combination

$$\left(Y_\nu Y_\nu^\dagger \right) = \sqrt{M^d} R \mathcal{M}^d R^\dagger \sqrt{M^d} \quad (162)$$

is independent of U and hence of the light neutrino mixing angles and CP-violating phases. The basic reason for this is that one makes a unitary sum over all the light lepton species in evaluating the asymmetry ϵ_{ij} . It is easy to derive a compact expression for ϵ_{ij} in terms of the heavy neutrino masses and the complex orthogonal matrix R :

$$\epsilon_{ij} = \frac{1}{8\pi} M_j f \left(\frac{M_j}{M_i} \right) \frac{\text{Im} \left(\left(R \mathcal{M}_\nu^d R^\dagger \right)_{ij} \right)^2}{\left(R \mathcal{M}_\nu^d R^\dagger \right)_{ii}}. \quad (163)$$

This depends explicitly on the extra phases in R : how can we measure them?

The basic principle of a strategy to do this is the following [99, 102, 103]. The renormalization of soft supersymmetry-breaking parameters, and hence flavour-changing interactions and CP violation in the lepton sector, depend on the leptogenesis parameters as well as the low-energy neutrino parameters $\delta, \phi_{1,2}$. If one measures the latter in neutrino experiments, and the discrepancy in the soft supersymmetry-breaking determines the leptogenesis parameters.

An example how this could work is provided by the two-generation version of the supersymmetric seesaw model [99]. In this case, we have $\mathcal{M}_\nu^d = \text{Diag}(m_{\nu_1}, m_{\nu_1})$ and $M^d = \text{Diag}(M_1, M_2)$, and we may parameterize

$$R = \begin{pmatrix} \cos(\theta_r + i\theta_i) & \sin(\theta_r + i\theta_i) \\ -\sin(\theta_r + i\theta_i) & \cos(\theta_r + i\theta_i) \end{pmatrix}. \quad (164)$$

In this case, the leptogenesis decay asymmetry is proportional to

$$\text{Im} \left((Y_\nu Y_\nu^\dagger)^{21} \right)^2 = \frac{(m_{\nu_1}^2 - m_{\nu_2}^2) M_1 M_2}{2v^4 \sin^4 \beta} \sinh 2\theta_i \sin 2\theta_r. \quad (165)$$

We see that this is related explicitly to the CP-violating phase and mixing angle in R (164), and is independent of the low-energy neutrino parameters. Turning now to the renormalization of the soft supersymmetry-breaking parameters, assuming for simplicity maximal mixing in the MNS matrix V and setting the diagonal Majorana phase matrix $P_0 = \text{Diag}(e^{-i\phi}, 1)$, we find that

$$\begin{aligned} \text{Re} \left[(Y_\nu^\dagger Y_\nu)^{12} \right] &= -\frac{(m_{\nu_2} - m_{\nu_1})}{4v^2 \sin^2 \beta} (M_1 + M_2) \cosh 2\theta_i + \dots, \\ \text{Im} \left[(Y_\nu^\dagger Y_\nu)^{12} \right] &= \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{2v^2 \sin^2 \beta} (M_1 + M_2) \sinh 2\theta_i \cos \phi + \dots \end{aligned} \quad (166)$$

In this case, the strategy for relating leptogenesis to low-energy observables would be: (i) use double- β decay to determine ϕ , (ii) use low-energy observables sensitive to $\text{Re}, \text{Im} \left[(Y_\nu^\dagger Y_\nu)^{12} \right]$ to determine θ_r and θ_i (166), which then (iii) determine the leptogenesis asymmetry (165) in this two-generation model.

In general, one may formulate the following strategy for calculating leptogenesis in terms of laboratory observables:

- Measure the neutrino oscillation phase δ and the Majorana phases $\phi_{1,2}$,

- Measure observables related to the renormalization of soft supersymmetry-breaking parameters, that are functions of $\delta, \phi_{1,2}$ and the leptogenesis phases,
- Extract the effects of the known values of δ and $\phi_{1,2}$, and isolate the leptogenesis parameters.

In the absence of complete information on the first two steps above, we are currently at the stage of preliminary explorations of the multi-dimensional parameter space. As seen in Fig. 18, the amount of the leptogenesis asymmetry is explicitly independent of δ [104]. An important observation is that there is a non-trivial lower bound on the mass of the lightest heavy singlet neutrino N :

$$M_{N_1} \gtrsim 10^{10} \text{ GeV} \quad (167)$$

if the light neutrinos have the conventional hierarchy of masses, and

$$M_{N_1} \gtrsim 10^{11} \text{ GeV} \quad (168)$$

if they have an inverted hierarchy of masses [104]. This observation is potentially important for the cosmological abundance of gravitinos, which would be problematic if the cosmological temperature was once high enough for leptogenesis by thermally-produced singlet neutrinos weighing as much as (167, 168) [108]. However, these bounds could be relaxed if the two lightest N_i were near-degenerate, as seen in Fig. 19 [109]. Striking aspects of this scenario include the suppression of $\mu \rightarrow e\gamma$, the relatively large value of $\tau \rightarrow \mu\gamma$, and a preferred value for the neutrinoless double- β decay observable:

$$\langle m \rangle_{ee} \sim \sqrt{\Delta m_{\text{solar}}^2} \sin^2 \theta_{12}. \quad (169)$$

5.6 Flavour-Violating Decays of Charged Leptons

Several such decays can be studied within this framework, including $\mu \rightarrow e\gamma, \tau \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \mu \rightarrow 3e$, and $\tau \rightarrow 3\mu/e$ [110].

The effective Lagrangian for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ can be written in the form [111, 99]:

$$\begin{aligned} \mathcal{L} = & -\frac{4G_F}{\sqrt{2}} \{ m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} \\ & + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R) (\bar{e}_L e_R) \\ & + g_3 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_R \gamma_\mu e_R) + g_4 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_L \gamma_\mu e_L) \\ & + g_5 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_L \gamma_\mu e_L) + g_6 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_R \gamma_\mu e_R) + h.c. \}. \end{aligned} \quad (170)$$

Figure 18. Heavy singlet neutrino decay may exhibit a CP-violating asymmetry, leading to leptogenesis and hence baryogenesis, even if the neutrino oscillation phase δ vanishes [104].

Figure 19. The lower limit on the mass of the lightest heavy singlet neutrino may be significantly reduced if the two lightest singlet neutrinos are almost degenerate [109].

The decay $\mu \rightarrow e\gamma$ is related directly to the coefficients $A_{L,R}$:

$$Br(\mu^+ \rightarrow e^+\gamma) = 384\pi^2 (|A_L|^2 + |A_R|^2), \quad (171)$$

and the branching ratio for $\mu \rightarrow 3e$ is given by

$$\begin{aligned} B(\mu \rightarrow e\gamma) &= 2(C_1 + C_2) + C_3 + C_4 + 32 \left(\ln \frac{m_\mu^2}{m_e^2} - \frac{11}{4} \right) (C_5 + C_6) \\ &+ 16(C_7 + C_8) + 8(C_9 + C_{10}), \end{aligned} \quad (172)$$

where

$$\begin{aligned} C_1 &= \frac{|g_1|^2}{16} + |g_3|^2, \quad C_2 = \frac{|g_2|^2}{16} + |g_4|^2, \\ C_3 &= |g_5|^2, \quad C_4 = |g_6|^2, \quad C_5 = |eA_R|^2, \quad C_6 = |eA_L|^2, \quad C_7 = \text{Re}(eA_R g_4^*), \\ C_8 &= \text{Re}(eA_L g_3^*), \quad C_9 = \text{Re}(eA_R g_6^*), \quad C_{10} = \text{Re}(eA_L g_5^*).. \end{aligned} \quad (173)$$

These coefficients may easily be calculated using the renormalization-group equations for soft supersymmetry-breaking parameters [99, 103].

Fig. 20 displays a scatter plot of $B(\mu \rightarrow e\gamma)$ in the texture H^1 mentioned earlier, as a function of the singlet neutrino mass M_{N_3} . We see that $\mu \rightarrow e\gamma$ may well have a branching ratio close to the present experimental upper limit, particularly for larger M_{N_3} . Predictions for $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ decays are shown in Figs. 21 and 22 for the textures H^1 and H^2 , respectively. As advertized earlier, the H^1 texture favours $\tau \rightarrow \mu\gamma$ and the H^2 texture favours $\tau \rightarrow e\gamma$. We see that the branching ratios decrease with increasing sparticle masses, but that the range due to variations in the neutrino parameters is considerably larger than that due to the sparticle masses. The present experimental upper limits on $\tau \rightarrow \mu\gamma$, in particular, already exclude significant numbers of parameter choices.

The branching ratio for $\mu \rightarrow 3e$ is usually dominated by the photonic penguin diagram, which contributes the $C_{5,6}$ terms in (172), yielding an essentially constant ratio for $B(\mu \rightarrow 3e)/B(\mu \rightarrow e\gamma)$. However, if $\mu \rightarrow e\gamma$ decay is parametrically suppressed, as it may have to be in order to respect the experimental upper bound on this decay, then other diagrams may become important in $\mu \rightarrow 3e$ decay. In this case, the ratio $B(\mu \rightarrow 3e)/B(\mu \rightarrow e\gamma)$ may be enhanced, as seen in Fig. 23.

As a result, interference between the photonic penguin diagram and the other diagrams may in principle generate a measurable T-odd asymmetry in $\mu \rightarrow 3e$ decay. This is sensitive to the CP-violating parameters in the supersymmetric seesaw model, and is in principle observable in polarized $\mu^+ \rightarrow e^+e^-e^+$ decay:

$$A_T(\mu^+ \rightarrow e^+e^-e^+) = \frac{3}{2\mathcal{B}} (2.0C_{11} - 1.6C_{12}), \quad (174)$$

Figure 20. Scatter plot of the branching ratio for $\mu \rightarrow e\gamma$ in the supersymmetric seesaw model for various values of its unknown parameters [103].

Figure 21. Scatter plot of the branching ratio for $\tau \rightarrow \mu\gamma$ in one variant of the supersymmetric seesaw model for various values of its unknown parameters [103].

Figure 22. Scatter plot of the branching ratio for $\tau \rightarrow e\gamma$ in a variant the supersymmetric seesaw model for various values of its unknown parameters [103].

Figure 23. The branching ratio for $\mu \rightarrow e\gamma$ may be suppressed for some particular values of the model parameters, in which case the branching ratio for $\mu \rightarrow 3e$ gets significant contributions from other diagrams besides the photonic penguin diagram [99].

Figure 24. The T-violating asymmetry A_T in $\mu \rightarrow 3e$ decay is enhanced in the regions of parameter space shown in Fig. 23 where the branching ratio for $\mu \rightarrow e\gamma$ is suppressed, and different diagrams may interfere in the $\mu \rightarrow 3e$ decay amplitude [99].

where

$$C_{11} = \text{Im}(eA_R g_4^* + eA_L g_3^*), C_{12} = \text{Im}(eA_R g_6^* + eA_L g_5^*), \quad (175)$$

and $\mathcal{B}$ is the $\mu \rightarrow 3e$ branching ratio with an optimized cutoff for the more energetic positron:

$$\mathcal{B} = 1.8(C_1 + C_2) + 0.96(C_3 + C_4) + 88(C_5 + C_6) + 14(C_7 + C_8) + 8(C_9 + C_{10}). \quad (176)$$

As seen in Fig. 24, the T-odd asymmetry is enhanced in regions of parameter space where $\mathcal{B}(\mu \rightarrow e\gamma)$ is suppressed [99]. If/when $\mu \rightarrow e\gamma$ and/or $\mu \rightarrow 3e$ decays are observed, measuring A_T (174) may provide an interesting window on CP violation in the seesaw model.

5.7 Lepton Electric Dipole Moments

This CP violation may also be visible in electric dipole moments for the electron and muon d_e and d_μ [112]. It is usually thought that these are unobservably small in the minimal supersymmetric seesaw model, and that $|d_e/d_\mu| = m_e/m_\mu$. However, d_e and d_μ may be strongly enhanced if the heavy singlet neutrinos are not degenerate [102], and depend on new phases that contribute to leptogenesis³. The leading con-

³This effect makes lepton electric dipole moments possible even in a two-generation model.

tributions to d_e and d_μ in the presence of non-degenerate heavy-singlet neutrinos are produced by the following terms in the renormalization of soft supersymmetry-breaking parameters:

$$(\tilde{\delta}m_L^2)_{ij} = \frac{18}{(4\pi)^4} (m_0^2 + A_e^2) \{Y_\nu^\dagger L Y_\nu, Y_\nu^\dagger Y_\nu\}_{ij} \ln\left(\frac{M_{GUT}}{\bar{M}}\right), \quad (177)$$

and similarly for the trilinear terms, where the mean heavy-neutrino mass $\bar{M} \equiv \sqrt[3]{M_1 M_2 M_3}$ and the matrix $L \equiv \ln(\bar{M}/M_i)\delta_{ij}$ were introduced in (153).

It should be emphasized that non-degenerate heavy-singlet neutrinos are actually expected in most models of neutrino masses. Typical examples are texture models of the form

$$Y_\nu \sim Y_0 \begin{pmatrix} 0 & c\epsilon_\nu^3 & d\epsilon_\nu^3 \\ c\epsilon_\nu^3 & a\epsilon_\nu^2 & b\epsilon_\nu^2 \\ d\epsilon_\nu^3 & b\epsilon_\nu^2 & e^{i\psi} \end{pmatrix},$$

where Y_0 is an overall scale, ϵ_ν characterizes the hierarchy, a, b, c and d are $\mathcal{O}(1)$ complex numbers, and ψ is an arbitrary phase. For example, there is an SO(10) GUT model of this form with $d = 0$ and a flavour SU(3) model with $a = b$ and $c = d$. The hierarchy of heavy-neutrino masses in such a model is

$$M_1 : M_2 : M_3 = \epsilon_N^6 : \epsilon_N^4 : 1, \quad (178)$$

and indicative ranges of the hierarchy parameters are

$$\epsilon_\nu \sim \sqrt{\frac{\Delta m_{solar}^2}{\Delta m_{atmo}^2}}, \quad \epsilon_N \sim 0.1 \text{ to } 0.2. \quad (179)$$

Fig. 25 shows how much d_e and d_μ may be increased as soon as the degeneracy between the heavy neutrinos is broken: $\epsilon \neq 1$. We also see that $|d_\mu/d_e| \gg m_\mu/m_e$ when $\epsilon_N \sim 0.1$ to 0.2 . Scatter plots of d_e and d_μ are shown in Fig. 26, where we see that values as large as $d_\mu \sim 10^{-27}$ e.cm and $d_e \sim 3 \times 10^{-30}$ e.cm are possible. For comparison, the present experimental upper limits are $d_e < 1.6 \times 10^{-27}$ e.cm [113] and $d_\mu < 10^{-18}$ e.cm [51]. An ongoing series of experiments might be able to reach $d_e < 3 \times 10^{-30}$ e.cm, and a type of solid-state experiment that might be sensitive to $d_e \sim 10^{-33}$ e.cm has been proposed [114]. Also, $d_\mu \sim 10^{-24}$ e.cm might be accessible with the PRISM experiment proposed for the JHF [115], and $d_\mu \sim 5 \times 10^{-26}$ e.cm might be attainable at the front end of a neutrino factory [116]. It therefore seems that d_e might be measurable with foreseeable experiments, whilst d_μ would present more of a challenge.

Figure 25. The electric dipole moments of the electron and muon, d_e and d_μ , may be enhanced if the heavy singlet neutrinos are non-degenerate. The horizontal axis parameterizes the breaking of their degeneracy, and the vertical strip indicates a range favoured in certain models [102].

Figure 26. Scatter plots of d_e and d_μ in variants of the supersymmetric seesaw model, for different values of the unknown parameters [103].

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