Recent Theoretical Progress on Higgs Production at Hadron Colliders

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1 Higgs mass limits and bounds

Let us begin by reviewing the current limits and bounds on the Higgs boson mass. On the experimental side, the negative search results for the Higgs boson at LEP lead to a lower bound of $M_H \geq 114.4 \text{ GeV}$ at 95% CL. A global fit to EW precision data gives a fairly narrow probability range of $M_H = 81^{+52}_{-33} \text{ GeV}$, and an upper bound of $M_H \leq 193 \text{ GeV}$ at 95% CL [1].

From the theoretical side, the bounds on the Higgs boson mass depend strongly on the underlying model. In the Standard Model (SM), the Higgs mass is *a priori* a free parameter. However, certain limitations arise from the requirement of a non-trivial Higgs potential with a (meta-)stable vacuum. New physics effects, appearing at a scale Λ , may change the SM shape of the Higgs potential. A detailed investigation of these requirements on the allowed values for the Higgs mass has been performed in Ref. [2]. An interesting outcome is that if the SM should obey both of these requirements up to the GUT scale, the Higgs boson mass must be between 100 and 200 GeV. Other values of M_H require new physics effects at a lower scale.

The limits on the Higgs mass values become generally much more stringent in supersymmetric theories. The Minimal Supersymmetric Standard Model (MSSM), for example, with its five physical Higgs particles, allows the derivation of an upper bound on the lightest Higgs mass M_h that depends only on SM parameters. At tree level, it is $M_h < M_Z$; radiative corrections raise this bound to about $M_h \lesssim 130 \text{ GeV}$ (for a review see Ref. [3]).

In summary, from both the experimental and the theoretical viewpoint, the interval between 100 and 200 GeV seems to be a reasonable mass range for a Higgs boson.

2 Higgs production modes

With the shutdown of LEP, Higgs searches have focused almost exclusively on hadron colliders. The current Run II of the Tevatron with an energy of 2 TeV in the $p\bar{p}$ c.m.s. has an expected reach of discovering a Higgs in the mass range of about 130 to 180 GeV at the 5σ level, if an integrated luminosity of 30 fb^{-1} can be achieved.

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Scheduled for 2007, the Large Hadron Collider (LHC), a proton–proton collider with 14 TeV c.m.s. energy, should be able to discover a Higgs boson in the full mass range between the current experimental lower bound and about $M_H = 1$ TeV.

The Higgs boson couplings are proportional to the mass of the particle that couples to it. But the constituents of (anti-)protons at Tevatron and LHC (predominantly gluons, in fact) are essentially massless. This means that Higgs production has to be mediated by “third-party particles” that couple the Higgs to the light quarks and gluons. The four most important production modes are as follows:¹

1. gluon fusion, l.h.s. of Fig. 6,
2. associated Higgs production with weak gauge bosons (Higgs Strahlung), Fig. 1 (a),
3. weak gauge boson fusion (WBF), Fig. 1 (b),
4. associated Higgs production with a top quark pair ($t\bar{t}H$), Fig. 4.

It turns out that gluon fusion is the process with the largest cross section at both the Tevatron and the LHC, for all relevant Higgs boson masses, see Fig. 2. In fact, for Higgs masses larger than about 140 GeV, gluon fusion provides several options to discover the Higgs at the 5σ level at the LHC through its subsequent decay into W or Z boson pairs.

Nevertheless, there is a sharp decline in the achievable significance towards Higgs masses $\lesssim 140$ GeV. This comes from the fact that the Higgs decays to almost 100% into a $b\bar{b}$ pair, and it seems impossible to separate the signal for $gg \rightarrow H \rightarrow b\bar{b}$ from the overwhelming background of QCD bottom quark production. Instead, one has to fall back on the rare decay into photon pairs with a branching ratio of around 10^{-3} . The LHC detectors should allow us to measure the $\gamma\gamma$ background, on top of which the Higgs signal should emerge as a small peak.

But it is clear that one has to include other modes in order to increase the chance of identifying a Higgs in the low-mass region. In particular at the Tevatron, where luminosity will be too low to make the $\gamma\gamma$ mode a viable option, the $gg \rightarrow H$ channel probably has to be completely discarded for small Higgs masses. Only when the decay into W pairs is kinematically allowed may one hope to make use of the gluon fusion channel again.

Let us therefore briefly discuss the four production modes, with emphasis on the status of the theoretical predictions for the signal cross sections. From this point of view, the largest progress has been made for the associated $t\bar{t}H$ and the gluon fusion mode. This is why we will be more explicit on these two cases, and rather brief on Higgs Strahlung and WBF in the following discussion.

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¹ Not included in this list is the diffractive Higgs production, for which we refer the interested reader to the contribution by J. Forshaw [4].

Fig. 1. Leading order contributions to (a) Higgs Strahlung and (b) Weak Boson Fusion (WBF).

Fig. 2. Higgs production cross section at (a) the Tevatron and (b) the LHC, including known higher orders and theoretical uncertainties due to scale variation (from [5]).

2.1 Higgs Strahlung

The leading order diagram for the Higgs Strahlung process at hadron colliders is shown in Fig. 1 (a). The Higgs boson is produced in association with a weak gauge boson. This is the most important mode at the Tevatron in the low mass region. The QCD corrections can be derived from the Drell-Yan process and give a 30% increase with respect to the LO prediction [7]. The cross section is almost one order of magnitude smaller than for gluon fusion (see Fig. 2). However, the benefits in this mode are that the b -quarks from Higgs decay can be distinguished from the background through the tagging of the W - and Z -decay products.

The most important background processes to the associated WH and ZH production channels, with subsequent decay $H \rightarrow b\bar{b}$, are under fairly good control from the theoretical point of view. They have been studied at NLO in the framework of the Monte Carlo generator MCFM [8].

Fig. 3. Signal significance at the LHC (from [6]).

2.2 Weak Boson Fusion (WBF)

As can be seen in Fig. 2 (b), this channel has the second largest rate at the LHC. It has been pointed out [9] that WBF is indeed extremely useful in the low-mass region, where the gluon fusion channel becomes problematic. In fact, as demonstrated in Fig. 3, the subsequent decays of the Higgs into $\tau^+\tau^-$ and into $WW^{(*)}$ are particularly important close to the current lower limit of LEP.

QCD corrections to WBF are known up to NLO [10] and are moderate, so that the theoretical prediction for the cross section is under fairly good control.

It has been shown that this mode is also very useful for Higgs measurements that go beyond discovery and mass determination. For example, using the azimuthal distribution of the two forward jets that result from the scattering quarks, one can determine the CP properties of the Higgs boson [11]. Or, combining WBF with gluon fusion and associated $t\bar{t}H$ production, one can determine various ratios of Higgs couplings, and, within certain limitations, also the total Higgs decay width [12,13].

2.3 $t\bar{t}H$

Even though the cross section for the production of a Higgs with a $t\bar{t}$ pair is rather small, this mode is very attractive because of its remarkable signature:

Fig. 4. Leading order contributions to associated $t\bar{t}H$ production.

four b quarks in association with two W bosons. In addition, this process is directly sensitive to the top-Yukawa coupling. The leading order Feynman diagrams are shown in Fig. 4. At the Tevatron, the cross section is dominated to about 95% by the $q\bar{q}$ initial state, Fig. 4 (a). At the LHC, the gluon-gluon channel is dominant.

The hope was that QCD corrections might increase the cross section, as it is usually observed in QCD processes. The calculation of the NLO corrections is rather challenging. For example, one of the obstacles are massive pentagon diagrams whose treatment is at the edge of current technology. Thus, the first NLO result was obtained in the limit where the Higgs mass is much smaller than the top quark mass. In this case, the amplitudes can be written as the convolution of the amplitude for $t\bar{t}$ production and the radiation of a Higgs boson off a top quark. Similarly to the gluon fusion process, as will be discussed in more detail below, one keeps the full top mass dependence at leading order, and evaluates only the K -factor in the heavy-top limit. In this way, the NLO K -factor was determined for the LHC to be $K \approx 1.3$ [14].

Only recently was the full result at NLO made available [15,16]. From the theoretical point of view, the NLO terms have a very satisfactory effect. The variation of the cross section prediction with the renormalization and factorization scale reduces dramatically, as one can see in Fig. 5. From the experimental point of view, however, for the Tevatron, the outcome is rather disappointing. The NLO corrections are *negative*, reducing the cross section with respect to the leading order prediction by about 20% at a scale of $\mu_R = \mu_F = M_t + M_H/2$. At the LHC, the full calculation gives a result that is very close to the one derived in the heavy-top limit, mentioned before.

3 Gluon fusion

Let us now turn to the gluon fusion channel, which is the dominant Higgs production mode at both the Tevatron and the LHC. Its importance for the Higgs discovery has already been discussed in Sect. 2.

Fig. 5. Variation of the inclusive cross section for $pp \rightarrow t\bar{t}H$ at (a) the Tevatron and (b) the LHC with the renormalization/factorization scale $\mu = \mu_R = \mu_F$ (from Ref. [15]; see also Ref. [16]).

In both the signal and the background calculations, considerable progress has been made over the last two to three years. NLO effects on the $\gamma\gamma$ as well as the $\pi\gamma$ and $\pi\pi$ background processes have been studied [17], with the pions decaying into collinear photon pairs, misidentified as a single photon in the detector.

Furthermore, the loop-induced process $gg \rightarrow \gamma\gamma$ has been investigated at NLO in Ref. [18]. The rather positive conclusion in this case is that the K -factor for this process turns out to be smaller than the corresponding K -factor for the signal.

Let us now turn to the signal process. The coupling between the gluons and the Higgs boson is mediated by a top quark loop, see the l.h.s. of Fig. 6. The high gluon luminosity at the Tevatron and the LHC, as well as the large top–Yukawa coupling, as predicted by the SM, compensate for the loop suppression and let this mode dominate over all the other channels for Higgs production. The leading order prediction for the cross section suffers from large uncertainties, though. This is because it is proportional to $\alpha_s^2(\mu)$, leading to strong variations with the renormalization scale. Also, as is known from the Drell–Yan process, QCD corrections are enhanced, since the leading order process is pure color singlet production, which is no longer true at higher orders, where real gluon and quark radiation has to be included. This is even more pronounced in the case of Higgs production, because the initial state particles are in the octet rather than the triplet representation of QCD. In other words, the relevant coupling for Higgs production is $C_A\alpha_s = 3\alpha_s$, compared with $C_F\alpha_s = 4\alpha_s/3$ for Drell–Yan.

The fact that the leading order contribution to the gluon fusion mechanism is given by massive one-loop triangle diagrams makes the evaluation of QCD corrections very challenging. Fortunately, the problem can be con-

siderably simplified by restricting it to a limiting case, which turns out to be extendable to the general case almost without any loss in accuracy. This approximation is given by the heavy-top limit, meaning that the top quark is taken to be much heavier than the Higgs boson. In view of the fact that $M_t \approx 175 \text{ GeV}$ and $M_H \gtrsim 114 \text{ GeV}$, this does not seem to be a very good approximation. However, first recall that the relevant parameter is not really M_H/M_t , but rather $M_H^2/(4M_t^2)$. This means that an *expansion* in terms of M_H^2/M_t^2 presumably converges within $M_H < 2M_t$. Here, however, we are going to consider only the leading term in this expansion, so one may expect this approximation to work well only for Higgs masses well below $2M_t$.

But an interesting observation made at NLO is the following [19,20]. If one keeps the full top mass dependence at leading order and evaluates only the QCD corrections in the heavy-top limit, the full NLO result is approximated to better than 2% up to the threshold $M_H = 2M_t$. Rather surprisingly, even above threshold the approximation works extremely well up to very large Higgs masses, and deviates from the exact result by only 10% at $M_H = 1 \text{ TeV}$.

Fig. 6. The heavy-top limit in diagrammatical form at leading order.

This observation makes it plausible to apply the heavy-top approximation also at NNLO. The Feynman diagrams simplify considerably in this case. The effect is shown schematically for the leading order diagram in Fig. 6: the original diagrams factorize into a term that contains all the top quark lines, but has vanishing external momenta; the second term depends on the kinematics, but has no massive internal lines. The second factor can be viewed as a direct Higgs–gluon coupling, where the “coupling constant” is given by the first factor. Higher order QCD corrections affect both the massive and the massless factor. For the latter, they are known up to four loops [21,22], *i.e.* even one order of perturbation theory higher than is needed for a NNLO prediction of the production rate.

While the first factor in the r.h.s. of Fig. 6 is universal for any process with a ggH coupling (Higgs production, hadronic decay, ...), the second factor depends on the specific kinematics under consideration. In the following, we will discuss recent developments for various Higgs production parameters.

3.1 Total rate at NNLO

For the total Higgs production rate through gluon fusion at NNLO, one has to include the virtual two-loop corrections in the second factor on the r.h.s. of Fig. 6, as well as real radiation of one quark or gluon up to the one-loop level, and of two quarks or gluons up to tree level. The purely virtual terms were obtained in Ref. [23]. The real radiation was first computed in the soft limit $M_H^2/\hat{s} \rightarrow 1$, where $\hat{s}$ is the c.m.s. energy [24,25]. Ambiguities in the definition of the soft limit revealed non-negligible uncertainties due to hard gluon radiation, however. These uncertainties were eliminated by systematically including hard gluon radiation through an expansion of the partonic cross section around the soft limit. Inclusion of 16 powers of $(1 - M_H^2/\hat{s})$ was more than sufficient to obtain the hadronic cross section with NNLO precision [26]. This result was later confirmed in [27,28], where the partonic cross section was evaluated in closed analytic form.

Bottom quark contributions to the gluon-Higgs coupling are parametrically suppressed by a factor M_b/M_t in the SM. Nevertheless, in these notes we will always keep the b -quark effects at leading order in QCD. At NLO, they amount to less than 2% at the LHC, and less than 5% at the Tevatron, for all relevant values of the Higgs mass.

The results for the total cross section are shown in Fig. 7 for (a) Run II at the Tevatron, and (b) the LHC. The NNLO curves reveal a nicely converging perturbative series, and a clear reduction of the remaining scale dependence. Using this variation as an indicator for the theoretical uncertainty, we arrive at values of around $\pm 15\%$.

Fig. 7. Total cross section for Higgs production in gluon fusion at (a) the Tevatron Run II, and (b) the LHC at leading (dotted), next-to-leading (dashed), and next-to-next-to leading order (solid). The upper (lower) curve of each pair corresponds to a choice of the renormalization and factorization scale of $\mu_R = \mu_F = M_H/2$ ($\mu_R = \mu_F = 2 M_H$). Leading order effects of the bottom quark to the gluon-Higgs coupling are included.

Strictly speaking, however, the curves in Fig. 7 are not fully consistent NNLO results yet. This is because, to this aim, the NNLO partonic cross section, as computed in Refs. [26,27], has to be convoluted with NNLO parton densities. The latter, in turn, require NNLO splitting functions, which are not yet fully available. Using a finite number of moments of these splitting functions [29] allows the derivation of interpolations [30] that seem to approximate the full expressions very well, down to momentum fractions of around $x \gtrsim 10^{-4}$. Based on these approximations, there are approximate NNLO parton densities, MRSTNNLO [31]. The choice among three different NNLO sets (“fast”, “average”, “slow” evolution) results in variations of less than 2% in the hadronic cross section. This should give an estimate of the error induced by the lack of exact NNLO splitting functions. The curves in Fig. 7 are produced with the “average” set.

The fast convergence of the expansion of Ref. [26] shows that a large portion of the cross section is contained in the soft and the collinear terms. This suggests the resummation of the soft gluon contribution to all orders [32]. The resummed result increases the cross section by only about 10%, in accordance with the apparent perturbative stability of the NNLO terms.

3.2 Distributions

Other interesting quantities in Higgs production apart from the total rate are kinematic distributions of the Higgs boson in its transverse momentum p_T and its rapidity y . In order to receive a non-trivial p_T , the Higgs has to recoil against a radiated gluon, so that the LO effect is already of order α_s^3 . The p_T distribution is also interesting in the light of trying to reduce the experimental background for a Higgs signal by tagging a hard jet in the reaction $gg \rightarrow H + \text{jet(s)}$. The NLO contributions have been computed in the heavy-top limit in Ref. [33] by integrating the phase space numerically, and analytically in Refs. [34,35]. The K -factor for the y and p_T distributions is very similar to the NLO K -factor for the total production rate, fairly independent of the values of p_T and y . This means that the distributions should be well approximated by the leading order expressions, multiplied by the K -factor of the total rate. For a pseudo-scalar Higgs boson, the NLO p_T and y distributions were obtained in [36].

The Monte-Carlo approach of Ref. [33] was later on used to investigate the effects of jet vetoing in Higgs searches [37], where events with high- p_T jets are excluded from the experimental analysis. The study was based on the soft/virtual/collinear approximations of Refs. [24,25], since the full result of Refs. [26,27] was not yet available.

In a rather different context, the process $gg \rightarrow H + 2\text{jets}$ turns out to be important, namely as a “background” for the WBF production mode (see Sect. 2.2). As discussed above for the total cross section, the validity of the heavy-top limit is rather restricted at LO, whereas it describes the higher order corrections extremely well. Thus, the LO expression for $gg \rightarrow H + 2\text{jets}$

had to be evaluated keeping the full M_t dependence [38]. Since the LO is already of $\mathcal{O}(\alpha_s^4)$, the result depends strongly on the renormalization scale. Nevertheless, it was found that the contamination of the WBF signal through this contribution can be efficiently reduced by appropriate kinematical cuts.

So far, we have considered the double differential distributions in p_T and y , where the LO process always has a quark or a gluon in the final state. But one may also consider the single differential cross section $d\sigma/dy$, where y is the rapidity of the Higgs. In this case, the triangle graph on the l.h.s. of Fig. 6 already gives a non-trivial contribution due to the boost of the partonic system in the hadronic c.m.s. frame. The NLO terms for this quantity have been presented in Ref. [39], demonstrating the flexibility of the method of Ref. [27]. In principle, this method should be applicable even at NNLO. Similarly to the double differential distributions, the effect of the NLO terms was found to be well described by scaling the LO prediction with the NLO K -factor of the total rate.

3.3 Higgs pair production

As was mentioned in Sect. 2.2, the combination of various production and decay modes of the Higgs boson at the LHC should allow the determination of various Higgs properties, in particular its couplings to fermions and gauge bosons. The measurement of the Higgs self-coupling λ , however, requires the investigation of Higgs pair production. At the LHC, the dominant Higgs pair production mechanism is through gluon fusion, $gg \rightarrow HH$ [40]. It has been evaluated up to NLO in the heavy-top limit [41].

The possibility of measuring λ was first discussed in the framework of an upgraded LHC with 6000 fb^{-1} integrated luminosity [13]; it was found that λ could be measured with a statistical uncertainty of 19% for $m_H = 170 \text{ GeV}$ (25% for $m_H = 200 \text{ GeV}$).

Recently it has been pointed out that λ should be accessible also at the “standard” LHC with an integrated luminosity of 300 fb^{-1} [42], through the Higgs decays into same-sign leptons via W bosons. It was shown that the ratio $\lambda/\lambda_{\text{SM}}$ can be restricted to the range between 0 and 3.8 at 95% CL, for Higgs masses between 150 and 200 GeV.

4 MSSM

The Minimal Supersymmetric Standard Model (MSSM) predicts five physical Higgs bosons: two neutral scalars (h , H , with $m_h \leq M_H$), one neutral pseudo-scalar (A), and two charged scalars ($H^\pm$). Many of the above results can be directly transferred from the SM to the neutral scalars in the MSSM. There are some important differences, however. Let us look at the gluon–Higgs coupling, for example: particles other than the top quark may contribute in the loop. These are, for example, top squarks, as long as they are not too

heavy; but also bottom quark effects may become much more important, as their Yukawa coupling grows linear with $\tan\beta$. A true NNLO calculation of Higgs production should therefore include bottom quarks as well. This is beyond current technology, however. The NLO corrections for squark loops were evaluated in Ref. [43], and for bottom quarks in Ref. [20].

An enhanced bottom–Yukawa coupling would also bring in a new production mechanism, namely associated production of a Higgs and a $b\bar{b}$ pair. At first glance, this looks similar to the previously discussed $t\bar{t}H$ mode (see Sect. 2.3), which is known to NLO. However, since the bottom mass is much smaller than the Higgs mass, the fixed order calculation leads to large logarithms of the form $\ln(M_b^2/M_h^2)$, which should be resummed to all orders in perturbation theory. This can be done by treating the bottom quark as a constituent of the proton. In this approach, the NLO corrections are known [44]. However, there is still no consensus in the literature on the extent to which this approach, in which the bottom quark mass and large transverse bottom momenta are neglected, is valid at Tevatron or LHC energies. A recent discussion of this issue has been given in Ref. [45], where the production rate for a charged Higgs is association with a top quark has been evaluated up to NLO.

Concerning the production of a pseudo-scalar Higgs boson, A , the NNLO corrections have been evaluated in analogy to the scalar Higgs case [46]. The corresponding process-independent coefficient functions, analogous to the massive triangle on the r.h.s. of Fig. 6, had been calculated in Ref. [47]. The NNLO corrections in the pseudo-scalar Higgs production turn out to be very similar to the scalar Higgs case [46]. The main conclusions about their size and the remaining theoretical uncertainty are thus the same. One should note, however, that the uncertainty due to the possibility of a large value for $\tan\beta$ is even larger in the pseudo-scalar case than in the scalar case. This is because the ratio of the bottom to the top couplings is quadratic in $\tan\beta$ for the pseudo-scalar, rather than linear as for the scalar Higgs.

5 Conclusions

We have presented a review of recent developments in the evaluation of Higgs production cross sections at hadron colliders. Emphasis has been put on the theoretically most challenging modes, namely associated production of a Higgs with a $t\bar{t}$ pair, and gluon fusion. The rapid progress within the last three years has resulted in a significant reduction of the theoretical uncertainties in both cases.

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References

1. <http://lepewwg.web.cern.ch/LEPEWWG>.
2. G. Isidori, G. Ridolfi, A. Strumia, *Nucl. Phys. B* **609** (2001) 387; T. Hambye, K. Riesselmann, *Phys. Rev. D* **55** (1997) 7255.
3. M. Carena, H.E. Haber, arXiv:hep-ph/0208209.
4. J.R. Forshaw, arXiv:hep-ph/0212154, these proceedings.
5. S. Dawson, L.H. Orr, L. Reina, D. Wackeroth, arXiv:hep-ph/0211373, *Proceedings of RADCOR / Loops and Legs 2002*, Sep. 8-13, 2002, Kloster Banz, Germany.
6. K. Jakobs, private communication.
7. T. Han, S. Willenbrock, *Phys. Lett. B* **273** (1991) 167; A. Djouadi, M. Spira, *Phys. Rev. D* **62** (2000) 014004.
8. R.K. Ellis, S. Veseli, *Phys. Rev. D* **60** (1999) 011501; J. Campbell, R.K. Ellis, *Phys. Rev. D* **65** (2002) 113007.
9. D. Rainwater, D. Zeppenfeld, *JHEP* **9712** (1997) 005; D. Rainwater, D. Zeppenfeld, K. Hagiwara, *Phys. Rev. D* **59** (1999) 014037; N. Kauer *et al.*, *Phys. Lett. B* **503** (2001) 113.
10. T. Han, G. Valencia, S. Willenbrock, *Phys. Rev. Lett.* **69** (1992) 3274.
11. T. Plehn, D. Rainwater, D. Zeppenfeld, *Phys. Rev. Lett.* **88** (2002) 051801.
12. D. Zeppenfeld *et al.*, *Phys. Rev. D* **62** (2000) 013009; D. Zeppenfeld, arXiv:hep-ph/0203123, *Proceedings of Snowmass 2001*; A. Belyaev, L. Reina, *JHEP* **0208** (2002) 041.
13. M.L. Mangano *et al.*, arXiv:hep-ph/0210261.
14. S. Dawson, L. Reina, *Phys. Rev. D* **57** (1998) 5851.
15. W. Beenakker, S. Dittmaier, M. Krämer, B. Plümper, M. Spira, P.M. Zerwas, *Phys. Rev. Lett.* **87** (2001) 201805 and arXiv:hep-ph/0211352.
16. L. Reina, S. Dawson, *Phys. Rev. Lett.* **87** (2001) 201804; L. Reina, S. Dawson, D. Wackeroth, *Phys. Rev. D* **65** (2002) 053017. S. Dawson, L.H. Orr, L. Reina, D. Wackeroth, arXiv:hep-ph/0211438.
17. T. Binoth, J.P. Guillet, E. Pilon, M. Werlen, arXiv:hep-ph/0203064 and *Eur. Phys. J. C* **16** (2000) 311.
18. Z. Bern, L. Dixon, C. Schmidt, *Phys. Rev. D* **66** (2002) 074018.
19. S. Dawson, *Nucl. Phys. B* **359** (1991) 283; S. Dawson, R.P. Kauffman, *Phys. Rev. D* **49** (1994) 2298; A. Djouadi, M. Spira, P.M. Zerwas, *Phys. Lett. B* **264** (1991) 440.
20. M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas, *Nucl. Phys. B* **453** (1995) 17 and *Phys. Lett. B* **318** (1993) 347. D. Graudenz, M. Spira, P.M. Zerwas, *Phys. Rev. Lett.* **70** (1993) 1372; M. Spira, *Fortschr. Phys.* **46** (1998) 203.
21. K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, *Phys. Rev. Lett.* **79** (1997) 353 and *Nucl. Phys. B* **510** (1998) 61.
22. M. Krämer, E. Laenen, M. Spira, *Nucl. Phys. B* **511** (1998) 523.
23. R.V. Harlander, *Phys. Lett. B* **492** (2000) 74.
24. R.V. Harlander, W.B. Kilgore, *Phys. Rev. D* **64** (2001) 013015.
25. S. Catani, D. de Florian, M. Grazzini, *JHEP* **0105** (2001) 025.
26. R.V. Harlander, W.B. Kilgore, *Phys. Rev. Lett.* **88** (2002) 201801.
27. C. Anastasiou, K. Melnikov, *Nucl. Phys. B* **646** (2002) 220.
28. W.B. Kilgore, arXiv:hep-ph/0208143, *Proceedings of ICHEP'02*.

29. A. Rety, J.A.M. Vermaseren, *Nucl. Phys. B* **604** (2001) 281;
S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, *Nucl. Phys. B* **427** (1994) 41;
S.A. Larin, P. Nogueira, T. van Ritbergen, J.A.M. Vermaseren, *Nucl. Phys. B* **492** (1997) 338.
30. W.L. van Neerven, A. Vogt, *Phys. Lett. B* **490** (2000) 111.
31. A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, *Phys. Lett. B* **531** (2002) 216.
32. S. Catani, D. de Florian, M. Grazzini, P. Nason, in: W. Giele *et al.*, “The QCD/SM Working Group: Summary Report”, Les Houches 2001, arXiv:hep-ph/0204316.
33. D. de Florian, M. Grazzini, Z. Kunszt, *Phys. Rev. Lett.* **82** (1999) 5209.
34. C.J. Glosser, C.R. Schmidt, *JHEP* **0212** (2002) 016.
35. V. Ravindran, J. Smith, W.L. van Neerven, *Nucl. Phys. B* **634** (2002) 247.
36. B. Field, J. Smith, M.E. Tejeda-Yeomans, W.L. van Neerven, *Phys. Lett. B* **551** (2003) 137.
37. S. Catani, D. de Florian, M. Grazzini, *JHEP* **0201** (2002) 015.
38. V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt, D. Zeppenfeld, *Phys. Rev. Lett.* **87** (2001) 122001 and *Nucl. Phys. B* **616** (2001) 367.
39. C. Anastasiou, L. Dixon, K. Melnikov, arXiv:hep-ph/0211141, *Proceedings of RADCOR / Loops and Legs 2002*, Sep. 8-13, 2002, Kloster Banz, Germany.
40. D.A. Dicus, C. Kao, S. Willenbrock, *Phys. Lett. B* **203** (1988) 457; E.W. Glover, J.J. van der Bij, *Nucl. Phys. B* **309** (1988) 282; T. Plehn, M. Spira, P.M. Zerwas, *Nucl. Phys. B* **479** (1996) 46; erratum, *ibid.* **B 531** (1998) 655; A. Djouadi, W. Kilian, M. Mhlleitner, P.M. Zerwas, *Eur. Phys. J. C* **10** (1999) 45.
41. S. Dawson, S. Dittmaier, M. Spira, *Phys. Rev. D* **58** (1998) 115012.
42. U. Baur, T. Plehn, D. Rainwater, *Phys. Rev. Lett.* **89** (2002) 151801 and arXiv:hep-ph/0211224.
43. S. Dawson, A. Djouadi, M. Spira, *Phys. Rev. Lett.* **77** (1996) 16.
44. D. Dicus, T. Stelzer, Z. Sullivan, S. Willenbrock, *Phys. Rev. D* **59** (1999) 094016.
45. T. Plehn, arXiv:hep-ph/0206121.
46. R.V. Harlander, W.B. Kilgore, *JHEP* **0210** (2002) 017; C. Anastasiou, K. Melnikov, arXiv:hep-ph/0208115.
47. K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, W.A. Bardeen, *Nucl. Phys. B* **535** (1998) 3.