Monte Carlo analysis of MEGA microlensing events towards M31

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Abstract. We perform an analytical study and a Monte Carlo (MC) analysis of the main features for microlensing events in pixel lensing observations towards M31. Our main aim is to investigate the lens nature and location of the 14 candidate events found by the MEGA collaboration. Assuming a reference model for the mass distribution in M31 and the standard model for our galaxy, we estimate the MACHO-to-self lensing probability and the event time duration towards M31. Reproducing the MEGA observing conditions, as a result we get the MC event number density distribution as a function of the event full-width half-maximum duration $t_{1/2}$ and the magnitude at maximum $R_{\text{max}}$. For a MACHO mass of $0.5 \, M_\odot$ we find typical values of $t_{1/2}$ $\approx$ 20 day and $R_{\text{max}}$ $\approx$ 22, for both MACHO-lensing and self-lensing events occurring beyond about 10 arcminutes from the M31 center. A comparison of the observed features ($t_{1/2}$ and $R_{\text{max}}$) with our MC results shows that for a MACHO mass $>0.1 \, M_\odot$ the four innermost MEGA events are most likely self-lensing events, whereas the six outermost events must be genuine MACHO-lensing events.

Key words. Gravitational Lensing; Galaxy: halo; Galaxies: individuals: M31

1. Introduction

Gravitational microlensing has become since about a decade a robust tool for analyzing the galactic structure and for gaining information about the dark mass component in our galaxy (Alcock et al., 1993; Aubourg et al., 1993). Several hundreds of microlensing events have been detected so far towards the galactic bulge, the spiral arms, and the Magellanic Clouds (Alcock et al., 2000; Jetzer et al., 2004). Recently, pixel lensing observations towards M31 and even M87 have been undertaken and some microlensing events have been found. The MEGA collaboration (de Jong et al., 2004) has reported the detection of 14 candidate events towards M31 by using Isaac Newton Telescope (INT) data. Two of these events have been previously reported by the POINT-AGAPE collaboration (Paulin-Henriksson et al., 2003), which recently has presented a high-threshold analysis of the full 3 years data set (Calchi Novati et al., 2007). This analysis shows that the observed signal is much larger than expect from self-lensing alone and that some fraction of the halo mass must be in form of MACHOs. Other collaborations have also reported preliminary results for pixel lensing towards M31 (Calchi Novati et al., 2002, 2003; Uglesich et al., 2004; Riffeser et al., 2003; Joshi et al., 2004).

In this paper we consider in particular the MEGA candidates. A map of the MEGA events is reported by de Jong et al. (2004). A preliminary analysis by the MEGA collaboration indicates that the events located in the outer part of M31 are consistent with being due to halo lens objects, whereas the innermost ones are most likely due to self-lensing. The aim of this paper is to perform a more complete analysis by using a Monte Carlo (MC) program with the purpose of using all available information by the MEGA collaboration, which are summarized in Tab. II to characterize the nature of the lenses.

The paper is organized as follows: in Section 2 we present the calculation of the microlensing rate towards M31 and we remind the pixel lensing basics. In Section 3 we give some details on the adopted M31 and Milky Way (MW) mass distribution models as well as the stellar and mass functions. The analytic and MC results and a comparison between the two are given in Sections 4-6 and conclusions are presented in Section 7.

2. Microlensing rate

The differential number of expected microlensing events is (De Rújula et al., 1991; Griest, 1991)

$$dN_{\text{ev}} = N_* t_{\text{obs}} d\Gamma,$$

where $N_*$ is the total number of monitored stars during the observation time $t_{\text{obs}}$. The differential rate $d\Gamma$ at which
Table 1. For the 14 MEGA events we give the position, the full-width half-maximum duration $t_{1/2}$ and the magnitude at maximum $R_{\text{max}}$. The coordinate system we adopt has origin in the M31 center and the X axis is oriented along the M31 disk major axis (see also Fig. 1).

<table>
<thead>
<tr>
<th>MEGA</th>
<th>X arcmin</th>
<th>Y arcmin</th>
<th>$t_{1/2}$ obs day</th>
<th>$R_{\text{max}}$ obs magn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.367</td>
<td>-2.814</td>
<td>4.20 ± 1.30</td>
<td>22.2 ± 1.1</td>
</tr>
<tr>
<td>2</td>
<td>-4.478</td>
<td>-3.065</td>
<td>4.60 ± 0.60</td>
<td>21.6 ± 0.3</td>
</tr>
<tr>
<td>3</td>
<td>-7.379</td>
<td>-1.659</td>
<td>2.60 ± 2.20</td>
<td>21.8 ± 0.8</td>
</tr>
<tr>
<td>4</td>
<td>-10.219</td>
<td>3.420</td>
<td>29.10 ± 1.00</td>
<td>22.8 ± 0.2</td>
</tr>
<tr>
<td>5</td>
<td>-19.989</td>
<td>-13.955</td>
<td>9.40 ± 4.10</td>
<td>22.9 ± 0.8</td>
</tr>
<tr>
<td>6</td>
<td>-21.564</td>
<td>-13.169</td>
<td>22.90 ± 0.70</td>
<td>22.6 ± 0.2</td>
</tr>
<tr>
<td>7</td>
<td>-21.163</td>
<td>-6.230</td>
<td>21.60 ± 0.70</td>
<td>19.3 ± 0.2</td>
</tr>
<tr>
<td>8</td>
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<td>7.670</td>
<td>27.40 ± 0.90</td>
<td>22.7 ± 0.2</td>
</tr>
<tr>
<td>9</td>
<td>-33.834</td>
<td>-2.251</td>
<td>3.80 ± 1.60</td>
<td>21.8 ± 0.8</td>
</tr>
<tr>
<td>10</td>
<td>-3.933</td>
<td>-13.846</td>
<td>46.80 ± 4.40</td>
<td>22.2 ± 0.3</td>
</tr>
<tr>
<td>11</td>
<td>19.192</td>
<td>-11.833</td>
<td>2.00 ± 0.30</td>
<td>20.5 ± 0.2</td>
</tr>
<tr>
<td>12</td>
<td>29.781</td>
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<td>131.00 ± 9.40</td>
<td>32.2 ± 0.3</td>
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<tr>
<td>13</td>
<td>22.072</td>
<td>-22.022</td>
<td>22.80 ± 3.80</td>
<td>23.3 ± 0.3</td>
</tr>
<tr>
<td>14</td>
<td>19.348</td>
<td>-29.560</td>
<td>28.10 ± 1.40</td>
<td>22.5 ± 0.2</td>
</tr>
</tbody>
</table>

where $D_{\text{os}}$ and $D_{\text{ol}}$ are the source and lens distances from the observer, respectively.

As for the source stars we split the lens transverse velocity into a random component and an ordered rotation component. Moreover, by taking into account that the microlensing tube moves with velocity $v_{\perp}$, it follows that $v_{\perp} = v_{\perp, \text{ran}} + v_{\perp, \text{rot}} - v_{\perp}$. Accordingly, assuming for $v_{\perp, \text{ran}}$ a Maxwellian distribution with 2-dimensional velocity dispersion $\sigma_{v}$, we obtain using also the definition for $g(v_{\perp})$ given in eq. (3)

$$f(v_{\perp})v_{\perp}dv_{\perp}\,d\beta = \frac{1}{(\pi\sigma_{v})^{2}}\int_{0}^{2\pi}d\varphi \times \int_{0}^{\infty}e^{-\frac{(v_{\perp}^{2}-(v_{\perp, \text{rot}}^{2})}{\sigma_{v}^{2}}}v_{\perp}dv_{\perp}e^{-\frac{(v_{\perp}^{2}-(v_{\perp, \text{rot}}^{2})}{\sigma_{v}^{2}}}v_{\perp}dv_{\perp}d\beta,$$ (5)

where $v_{\perp} = v_{\perp, \text{rot}} - v_{\perp}$ and $\beta$ is the angle between $v_{\perp}$ and $v_{\perp}$.

We can then write the volume element, $d^{3}x$, as

$$d^{3}x = (v_{\perp} \cdot \hat{n})\,dt\,ds = v_{\perp} \cos \theta\,dt\,dl\,dD_{\text{ol}},$$ (6)

where $\theta$ is the angle between $v_{\perp}$ and the normal, $\hat{n}$, to the lateral superficial element, $ds = dD_{\text{ol}},$ of the microlensing tube, with $dl = R_{\text{e}}d\mu_{\text{th}}d\alpha$ being the cylindrical segment of the tube ($\mu_{\text{th}}$ is the threshold value for the impact parameter). Note that, if $\alpha$ is taken to be the angle between $\hat{n}$ and $v_{\perp}$, it follows that $\theta = \alpha + \beta$, so that for a constant value of $\beta$, $d\alpha = d\theta$. Therefore, the microlensing differential event rate becomes

$$d\Gamma = n_{1}(x, \mu)\,dt\,d\psi_{0}(\mu) \frac{v_{\perp}^{2}}{\cos \theta R_{\text{e}}d\mu_{\text{th}}d\alpha}dD_{\text{ol}},$$ (7)

where $n_{1}(x, \mu)$ is the number density of the lens in the Galaxy (factorization hypothesis). So, the lens number density $n_{1}(x, \mu)$ can be written as (Jetzer et al., 2002)

$$n_{1}(x, \mu) = \frac{\rho_{0}(x)}{\rho_{0}}\,\psi_{0}(\mu),$$ (8)

where $\rho_{0}$ is the local mass density in the Galaxy or the central density in M31, $\psi_{0}(\mu)$ the corresponding lens number density per unit of mass and the normalization is

$$\int_{\mu_{\text{min}}}^{\mu_{\text{up}}}\psi_{0}(\mu)\,d\mu = \frac{\rho_{0}}{M_{\odot}}.$$ (9)

Here $\mu_{\text{min}}$ and $\mu_{\text{up}}$ are the lower and the upper limits for the lens masses (see Subsect. 3.3). Accordingly, the microlensing event rate is given by:

$$\Gamma(D_{\text{os}}) = \sqrt{\frac{4GM_{\odot}}{c^{2}}}\int_{\mu_{\text{min}}}^{\mu_{\text{up}}}d\mu\,\mu^{1/2}\psi_{0}(\mu)\mu^{3/2}\int_{0}^{\pi/2}du_{\text{th}} \times \int_{0}^{D_{\text{os}}}dD_{\text{ol}}\frac{D_{\text{ol}}(D_{\text{os}} - D_{\text{ol}})}{D_{\text{os}}}\frac{\rho_{1}(D_{\text{ol}})}{\rho_{0}},$$

$$\int_{0}^{2\pi}d\beta\int_{0}^{\infty}dD_{\perp}f(v_{\perp})v_{\perp}^{2}d\beta dD_{\perp}$$ (10)
where the integration on $\theta$ is performed between $-\pi/2$ and $+\pi/2$, since only lenses entering the microlensing tube are considered. After integrations on $\theta$ and $\beta$ we finally get

\[
\Gamma(D_{\text{os}}) = 2\sigma_T \sqrt{4GM_\odot c^2} \int_{\mu_{\text{min}}}^{\mu_{\text{sup}}} d\mu \mu^{1/2} \psi_0(\mu) \times \int_0^{D_{\text{os}}} dD_{\text{ol}} \sqrt{\frac{D_a(D_{\text{os}} - D_{\text{ol}}) \rho_l(D_{\text{ol}})}{D_{\text{os}} \rho_0}} \int_0^{\infty} P(z) dz ,
\]

where $z = v_{\perp}/\sigma_l$ and the function $P(z)$ is given by

\[
P(z) = \frac{2e^{-a^2/2}}{\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} ye^{-\left(y^2 - 2a\cos\varphi + \eta^2\right)/2} dy \times z e^{-z^2} I_0(2\eta z) .
\]

Here $a(D_{\text{os}}) = v_{\perp, \text{rot}}/\sigma_s$, $y = v_{\perp}/\sigma_s$, $\eta(D_{\text{os}}, y, \varphi) = w_{\perp}/\sigma_l$ and $I_0(2\eta z)$ is the zero-order modified Bessel function of the argument $2\eta z$.

Note that $\chi(z) = z^{-1} P(z)$ is the dimensionless form of eq. (5) and that it is properly normalized as it can be easily verified by using twice the relation

\[
\int_0^{\infty} dt \exp(-t^2) t I_0(2gt) = \exp(g^2)/2.
\]

To take into account the source distribution in the M31 bulge and disk, eq. (11) has to be integrated not only over the distance of the lenses but also over the distance of the sources. Accordingly, the microlensing rate becomes

\[
\Gamma(x, y) = \frac{\int_0^{\infty} \rho_s(D_{\text{os}}) \Gamma(D_{\text{os}}) dD_{\text{os}}}{\int_0^{\infty} \rho_s(D_{\text{os}}) dD_{\text{os}}} ,
\]

where $x$ and $y$ are coordinates in the plane orthogonal to line of sight, $\rho_s$ is the source mass density (which is the sum of the sources in the M31 bulge and disk).

Moreover, we compute the average Einstein time (which depends on the line of sight position given by the coordinates $x$ and $y$) as

\[
\langle t_E \rangle = \frac{\int_0^\Gamma t_E d\Gamma}{\Gamma} .
\]

### 2.1. Pixel lensing basics

Pixel lensing technique is based on the observation of the flux variations of every element (pixel) of an image (Ansari et al., 1997). Looking towards M31 a large number of stars contribute at the same time to the flux received by each pixel so that only highly magnified events can be detected.

To be detectable a microlensing event must give rise to a substantial flux variation with respect to the background $N_{\text{bl}} = N_{\text{gal}} + N_{\text{sky}}$, which is the sum of the M31 surface brightness and the sky contribution. The excess photon count per pixel due to an ongoing microlensing event is

\[
\Delta N_{\text{pix}} = N_{\text{bl}}[A_{\text{pix}} - 1] = f_{\text{see}} N_s[A(t) - 1]
\]

where $N_s$ is the source photon count in the absence of lensing, $A(t)$ is the source magnification factor due to lensing (see e.g. Griest, 1991) and $f_{\text{see}}$ the fraction of the seeing disk contained in a pixel. Therefore, the expected number of photons in a pixel will be $N_{\text{pix}} = N_{\text{bl}} + \Delta N_{\text{pix}}$.

Of course, a pixel lensing event is detectable if the excess pixel photon count is greater than the threshold pixel noise $\sigma_T$. Accordingly, by requiring the signal to be at least $3\sigma_T$ level above the baseline count, one obtains a threshold value for the amplification (Kerins et al., 2001)

\[
A_T = 1 + \frac{3\sigma_T}{f_{\text{see}} N_s} ,
\]

which corresponds to a threshold value $u_T$ for the impact parameter, via the well known relation between the lens impact parameter and the amplification factor.

One can estimate $\sigma_T$ as the maximum between the statistical error $\propto \sqrt{N_{\text{bl}}}$ and $\approx 3 \times 10^{-3} N_{\text{bl}}$ that is determined by the pixel flux stability. Accordingly, $u_T$ depends on both the line of sight to M31 and the source magnitude $M$.

Hence, by averaging on the source luminosity function $\phi(M)$, we can evaluate the average threshold impact parameter for any direction towards the M31 galaxy, so that we get

\[
\langle u_T(x, y) \rangle_\phi = \frac{\int u_T(x, y; M) \phi(M) dM}{\int \phi(M) dM} ,
\]

where the coordinates $x$ and $y$ span the sky plane towards M31.

By using the threshold impact parameter defined in eq. (17), one obtains the pixel lensing rate as follows (Kerins et al., 2001, 2003; De Paolis et al., 2003)

\[
\Gamma_p(x, y) = \langle u_T(x, y) \rangle_\phi \Gamma(x, y) .
\]
M31 surface brightness and rotation curve profiles, can be considered as an acceptable model for the mass distribution in the M31 galaxy. Accordingly, the mass density of the M31 disk stars is described by a sech-squared profile

$$\rho_D(R, z) = \rho_D(0) \exp(-R/H) \text{sech}^2(z/H),$$

where \(H = 0.3 \, \text{kpc}, h = 6.4 \, \text{kpc}\) and \(\rho_D(0) = 0.35 \times 10^9 \, \text{M}_\odot \, \text{pc}^{-3}\) are, respectively, the scale height and scale lengths of the disk and the disk central mass density. \(R\) is the distance on the M31 disk plane (described by the coordinates \(x\) and \(y\)) and \(z\) is the distance from it. The M31 disk is assumed to be inclined by an angle \(i = 77^0\) and the azimuthal angle relative to the near minor axis is \(\phi = -38.6^0\).

The M31 bulge is parameterized by a flattened power law of the form

$$\rho_B(R, z) = \rho_B(0) \left[ 1 + \left( \frac{R}{a} \right)^2 + q^{-2} \left( \frac{z}{a} \right)^2 \right]^{-s/2},$$

where \(\rho_B(0) \approx 4.5 \times 10^9 \, \text{M}_\odot \, \text{kpc}^{-3}, q \approx 0.6\) is the ratio of the minor to major axis, \(a \approx 1 \, \text{kpc}\) and \(s \approx 3.8\). Both the M31 disk and bulge are truncated at a distance \(R = 40 \, \text{kpc}\).

We remark that the twisting of the optical isophotes in the inner M31 regions indicates that the bulge major axis is offset by \(\approx 15^0\) from the disk major axis (Stark and Binney, 1994). The consideration of this effect by Kerins et al. (2005) leads to the evaluation of pixel lensing rates that show spatial distributions tilted of the same amount inside 5 arcmin from the M31 center. The twisting effect vanishes at larger distances due to the increasing contribution to microlensing by M31 disk and halo. Clearly, our results in Figs. 1-3 do not show the above mentioned effect since here we are considering only a flattened bulge without twist. However, we expect that the consideration of the isophote twisting does not substantially modify our results about both MACHO-to-self lensing probability and event time scale ratios (given in Tabs. 2-5), particularly for events at large distance from the M31 center.

The dark matter in the M31 halo is assumed to follow an isothermal profile

$$\rho_H(r) = \rho_H(0) \frac{a^2}{a^2 + r^2},$$

with core radius \(a = 4 \, \text{kpc}\) and central dark matter density \(\rho_H(0) = 6.5 \times 10^7 \, \text{M}_\odot \, \text{pc}^{-3}\). The M31 halo is truncated at 100 kpc with asymptotic rotational velocity \(v_{\text{rot}} \approx 235 \, \text{km s}^{-1}\).

We do not consider other dark matter distribution models, as King-Michie (Binney and Tremaine, 1987) or NFW (Navarro et al., 1997) suggested by N body simulations. The effect of using such models, which have a more concentrated dark mass distribution, is both to decrease the spatial distribution of MACHOs at large distance from the M31 center, where the rotation curve is poorly determined, and to increase it in the innermost region, where

\[\text{Fig. 2. MACHO-to-self lensing probability ratio} \ (P_h/P_s)_{\text{An}} \ \text{map projected onto the sky plane. Here and in the following figures we assume a fraction} \ f_h = 20\% \ \text{of dark matter and a MACHO mass} \ \mu_h = 0.5, \ \text{both in M31 and MW halos. From the inner to the outer part} \ (P_h/P_s)_{\text{An}} \ \text{increases with lines referring to the values} \ 0.5, 1, 2, 3, 4 \ \text{and} \ 5, \ \text{respectively.}\]

\[\text{Fig. 3. The MACHO-to-self lensing Einstein time ratio} \ <t_{E_h}> / <t_{E_s}> \ \text{map is given for} \ \mu_h = 0.5. \ \text{Going from the inner to the outer part the ratio decreases with lines corresponding to values in between} \ 2 \ \text{to} \ 1.25 \ \text{with a step of} \ 0.25.\]
the MACHO contribution is relatively unimportant with respect to that of bulge and disk. Regarding the former aspect, we also note that in the MEGA experiment the typical MACHO lens distance (about 20 kpc) is too small to appreciate the effect of this choice. Overall, the current data do not allow one to perform a meaningfully fine tuning of the dark matter parameters.

As usual, the mass density profile for the MW disk is described with a double exponential profile

$$\rho_D(R, z) = \rho_D(R_0) \exp\left(-\frac{(R-R_0)}{h}\right) \exp\left(-\frac{|z|}{H}\right),$$  

(22)

with Earth position from the Galactic center at $R_0 \approx 8.5$ kpc, scale height $H \approx 0.3$ kpc, scale length $h \approx 3.5$ kpc and local mass density $\rho_D(R_0) \approx 1.67 \times 10^8 M_\odot$ kpc$^{-3}$.

The dark halo in our Galaxy is also assumed to follow an isothermal profile with core radius $a \approx 5.6$ kpc and local dark matter density $\rho_H(R_0) \approx 1.09 \times 10^7 M_\odot$ kpc$^{-3}$. The corresponding asymptotic rotational velocity is $v_{rot} \approx 220$ km s$^{-1}$. The MW halo is truncated at $R \approx 100$ kpc.

For both M31 and MW halos, the fraction of dark matter in form of MACHOs is assumed to be $f_k \approx 0.2$ (Alcock et al., 2000). However, most of our results can easily be rescaled to get the corresponding figures for other values of $f_{\text{MACHO}}$.

Moreover, we assume that the random velocities of stars and MACHOs follow Maxwellian distributions with one-dimensional velocity dispersion $\sigma = 30, 100, 166$ km s$^{-1}$ and $30, 156$ km s$^{-1}$ for the M31 disk, bulge, halo and MW disk and halo, respectively. In addition, a M31 bulge rotational velocity of $30$ km s$^{-1}$ has been taken into account (Kerins et al., 2001; An et al., 2004).

### 3.2. Stellar luminosity function

Pixel lensing event detection by the MEGA collaboration is performed in the red band and, thus, red giants are the most luminous stars in this band. Therefore, we may safely assume that the overwhelming majority of the pixel lensing event sources are red giants.

Moreover, in the lack of precise information about the stellar luminosity function in M31, we adopt the luminosity function derived from the stars in the Galaxy and assume that it also holds for M31.

Accordingly, following (Mamon and Soneira, 1982) we assume that the stellar luminosity function does not depend on the position and in the magnitude range $-6 \leq M \leq 16$ it is proportional to the expression

$$\phi_*(M) \propto \frac{10^{\delta(M-M^*)}}{[1 + 10^{-(\alpha-\beta)\delta(M-M^*)}]^{1/\delta}},$$  

(23)

where, in the red band, $M^* = 1.4$, $\alpha \approx 0.74$, $\beta = 0.045$ and $\delta = 1/3$. Moreover, the fraction of red giants (over the

### 3.3. Mass functions

As concerns the lens mass function $\psi_0(\mu)$ in eqs. 8 - 11, for lenses belonging to the bulge and disk star populations, the lens mass is assumed to follow a broken power law (Gould et al., 1997)

$$\psi_0(\mu) = K \mu^{-0.56} \frac{\mu}{\mu_{\text{up}}}$$

for $0.1 \leq \mu \leq 0.59$

$$= K \mu^{-2.20} \frac{\mu}{\mu_{\text{up}}}$$

for $0.59 \leq \mu \leq \mu_{\text{up}}$

(26)

where the lower limit $\mu_{\text{min}} = 0.1$ and the upper limit $\mu_{\text{up}} = 1$ for M31 bulge stars and 10 for M31 and MW disk stars (see also Kerins et al., 2001). $K$ is fixed according to the normalization given by eq. 9. The resulting mean mass for lenses in the bulges and disks are $\langle m_h \rangle \sim 0.37 M_\odot$ and $\langle m_d \rangle \sim 0.69 M_\odot$, respectively.

For the lens mass in the M31 and MW halos we assume the $\delta$-function approximation

$$\psi_0(\mu) = \frac{\rho_0}{M_\odot \mu_h} \delta(\mu - \mu_h)$$  

(27)

and we take a MACHO mass, in solar units, $\mu_h = 10^{-2}, 10^{-1}, 0.5, 1$.

### 4. Analytical results

In the present analysis we adopt the parameters for the INT and the Sloan-r filter on the WFC (Wide-Field Camera) used by the MEGA collaboration (de Jong et al., 2001). The Telescope diameter, the pixel field of view and the image exposition time are 2.5 m, 0.33 arcsec and $t_{\text{exp}} = 760$ s, respectively. We also use a gain or conversion factor of 2.8 e$^{-}$/ADU, and a loss factor $\approx 3$, for both atmospheric and instrumental. The zero-point with the Sloan-r WFC turns out to be $\sim 24.3$ mag arcsec$^{-2}$ (Belokurov et al., 2007). Moreover, we adopt a value $\approx 1.5$ arcsec for the average seeing conditions, a sky background $n_{\text{sky}} \approx 20.9$ mag arcsec$^{-2}$ (corresponding to a Moon eclipse) and a minimum noise level of $\sim 2.5 \times 10^{-3} N_{bl}$. $N_{bl}$ is the baseline photon count which is the sum of the total star number as a function of $M$ is approximated as

$$f_{\text{RG}}(M) = 1 - C \exp[\alpha(M + \beta)^\gamma]$$

for $-6 \leq M \leq 3$

$$= 0$$

for $M \geq 3$,  

(24)

where, in the red band, $C \approx 0.31$, $\alpha \approx 6.5 \times 10^{-4}$, $\beta = 7.5$ and $\gamma \approx 3.2$.

Therefore, the red giant luminosity function will be $\phi_{\text{RG}}(M) \propto \phi_*(M) \times f_{\text{RG}}(M)$ and the fraction of red giants averaged on the magnitude is given by

$$\langle f_{\text{RG}} \rangle = \frac{\int_{-6}^{10} \phi_{\text{RG}}(M) dM}{\int_{-6}^{10} \phi_*(M) dM} \approx 5.3 \times 10^{-3},$$  

(25)

from which it follows that the local number density of red giants will be $n_{\text{RG}} \approx 5.3 \times 10^{-3} n_*$, where $n_*$ is the local stellar number density.
M31 surface brightness given by Kent (1989) and the sky contribution.

Maps of optical depth, expected event number and event timescale in pixel lensing experiments have been presented in a previous paper (De Paolis et al., 2005) together with the study of the dependence on microlensing quantities with the assumed M31 mass distribution model (see also Kerins, 2004). Our results are also in good agreement with previous analytical estimates for the rate, the timescale distribution and the optical depth (Baltz et al., 2003; Gyuk and Crotts, 2000).

In Fig. 1 the map of \( u_T(x,y) \) shows that in the central M31 regions the lens impact parameter on average is \( \leq 0.04 \) implying that only high magnified events with \( A_T \geq 25 \) are in principle detectable. The asymmetrical shape is due to the internal extinction of the M31 disk for which we use the value 0.74 mag given by Kent (1989). Indeed, due to the inclination of the M31 disk, along a line of sight towards the southern region there exists a larger number of source stars (with respect to the corresponding northern field) which are not absorbed by the M31 disk dust (and therefore appear with a smaller magnitude), lying at larger averaged values for \( u_T \).

![Plot of the source absolute magnitude distribution](image)

**Fig. 4.** Plot of the source absolute magnitude distribution for the generated events (solid line) and for the revealed events (dashed line) towards the MEGA 7 direction.

In Fig. 2 assuming a MACHO mass \( \mu_h = 0.5 \) and a halo MACHO fraction \( f_h = 0.2 \), we show the map of the MACHO-to-self lensing probability ratio \( (P_h/P_s)_{\text{An}} \). We find that in the M31 central regions microlensing is dominated by self-lensing contributions while MACHO-lensing becomes important at distances > 10 arcmin from the center. In the figure it is also evident the well known near-far disk asymmetry due to the inclination of the M31 disk (Crotts, 1992; Baillon et al., 1993; Jetzer, 1994).

![Map of optical depth](image)

In T.able 2, we show how the MACHO-to-self lensing probability ratio \( (P_h/P_s)_{\text{An}} \) towards the 14 MEGA events are given for different MACHO mass values.

<table>
<thead>
<tr>
<th>MEGA</th>
<th>( \mu_h = 0.01 )</th>
<th>0.1</th>
<th>0.5</th>
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<td>35.47</td>
<td>17.07</td>
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</table>

![Map of the MACHO-to-self lensing Einstein time ratio](image)

Table 3. MACHO-to-self lensing Einstein time ratio \( \langle t_{E_h} / t_{E_s} \rangle_{\text{An}} \) towards the 14 MEGA events are given for different MACHO mass values.

<table>
<thead>
<tr>
<th>MEGA</th>
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</tr>
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<td>0.29</td>
<td>0.94</td>
<td>2.02</td>
<td>2.91</td>
</tr>
<tr>
<td>4</td>
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<td>0.65</td>
<td>1.47</td>
<td>2.12</td>
</tr>
<tr>
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<td>1.10</td>
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</tr>
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<td>1.58</td>
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<td>1.35</td>
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<tr>
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<td>0.97</td>
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<td>10</td>
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<td>0.54</td>
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<td>1.51</td>
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<td>14</td>
<td>0.15</td>
<td>0.48</td>
<td>1.03</td>
<td>1.52</td>
</tr>
</tbody>
</table>

In Tab. 2 we show how the MACHO-to-self lensing probability ratio \( (P_h/P_s)_{\text{An}} \) depends on the MACHO mass \( \mu_h \) towards the 14 MEGA events. The general trend is that \( (P_h/P_s)_{\text{An}} \) increases as \( \mu_h \) decreases as a consequence of the increase of the MACHO number density. Moreover, for a given value of \( \mu_h \), \( (P_h/P_s)_{\text{An}} \) increases with the distance from the M31 center.

In Fig. 3 the map of \( \langle t_{E_h} / t_{E_s} \rangle_{\text{An}} \) is given for a MACHO mass of 0.5 \( M_\odot \). In the region inside \( \approx 5 \) arcmin from the M31 center microlensing events due to MACHOs have twice as long a duration as compared to self-lensing events, while in the regions far away from the M31 center all events have roughly the same duration. Obviously, the MACHO-to-self lensing Einstein time ratio depends on the assumed MACHO mass. This fact is clearly seen in Tab. 3 where the ratio \( \langle t_{E_h} / t_{E_s} \rangle_{\text{An}} \) towards the...
14 MEGA events is given for different MACHO mass values. \( \mu_h = 0.5 \) corresponds to the MACHO mass value for which the 10 outer MEGA events (5-14) are characterized by having a MACHO-to-self lensing Einstein time ratio equal to unity, i.e., events due to either MACHO-lensing or self-lensing have roughly the same duration. Thus, these different types of events are indistinguishable on the basis of their timescale alone. The situation is instead much more favorable for MACHO masses smaller or larger than \( 0.5 M_\odot \).

We would like to emphasize that in pixel lensing experiments \( t_E \) is not a directly observable quantity, since the relevant time scale is the full-width half-maximum event duration \( t_{1/2} \), which depends on \( t_E \) and the impact parameter \( u_0 \) (Gondolo, 1999). However, since the probability for a given \( u_0 \) value is practically the same for self-lensing and MACHO-lensing events (as we have verified by using the MC code) the ratio \( \langle t_{E_h} \rangle / \langle t_{E_s} \rangle \) is equivalent to the ratio \( \langle t_{1/2} \rangle / \langle t_{1/2} \rangle \) at least for the MC generated events. Of course, as it will be more clear in the following Sections (in particular from Tab. 4 and Fig. 10), it does not mean that the same conclusion holds for the MC revealed events, since the (normalized) event number which pass the MEGA selection criteria turns out to depend on \( t_{1/2} \) and \( R_{\max} \) (see Fig. 6).

5. Monte Carlo simulation

Once the event location (one of the 14 MEGA events towards M31) has been selected, for any lens and source population present along the line of sight we have to make as next the choice over the following five parameters: source distance \( D_{os} \), lens distance \( D_{ol} \), lens effective transverse velocity \( v_{\perp} \), lens mass \( \mu \) and source magnitude \( M \). We shall denote these parameters by \( x_i \), with \( i = 1, \ldots, 5 \) in the order just listed (e.g. \( x_1 = D_{os} \), etc.). The probability with which we select the events according to one of the parameters is then given by

\[
P(x_i)dx_i = \frac{1}{N_{ev}} \frac{\partial N_{ev}}{\partial x_i}dx_i
\]

where \( \frac{\partial N_{ev}}{\partial x_i} \) is defined as being the integrand of the event number \( N_{ev} \) integrated over all variables \( x_j \) without, however, the considered variable \( x_i \). Clearly when integrating \( \frac{\partial N_{ev}}{\partial x_i} \) over \( dx_i \) we obtain again \( N_{ev} \).

This way, for instance, for self-lensing events the probability of extracting a lens with mass \( \mu \) turns out to be

\[
P(x_4 = \mu) d\mu \propto \mu^{1/2} \psi_0(\mu) d\mu,
\]

where \( \psi_0(\mu) \) is the lens number density distribution defined in eq. (20).

The probability of a source \(^2\) at distance \( D_{os} \) and a lens at distance \( D_{ol} \) from the observer, respectively, is given by

\[
P(D_{os})dD_{os} \propto \rho_s(D_{os})D_{os}^{3/2} \left( \int_{0}^{D_{os}} P(D_{ol})dD_{ol} \right) dD_{os}
\]

\[
P(D_{ol})dD_{ol} \propto \rho_l[D_{ol}(D_{os} - D_{ol})]^{1/2}(D_{ol}) dD_{ol},
\]

where \( \rho_s(D_{os}) \) and \( \rho_l(D_{os}) \) are, respectively, the lens and the source mass densities.

---

\(^2\) Here we take into account that the source number inside the pixel solid angle increases with the distance as \( D_{os}^2 \).
Once all the parameters have been fixed, for each event we build the corresponding lightcurve using the same time sampling of the MEGA campaign and the same observing and instrumental conditions of the considered experiment. We add a gaussian noise modulated by the Moon phase and use a Paczyński fit (Paczyński, 1986) to evaluate the microlensing parameters.

The selection of the MC generated events is based on the same criteria adopted by the MEGA collaboration. By using the results of the Paczyński fit we filter the lightcurves with the following selection criteria: peak sampling, peak significance, peak width, baseline flatness and goodness of fit (for more details see Appendix A.1 in de Jong et al. 2004). The criteria that more severely cut the MC generated events are the peak sampling and the peak significance, which depends through the evaluation of the statistical errors on the the Moon phase. The former criterium leads to select events with well sampled lightcurves (within the MEGA campaign), the latter to events with high signal-to-noise ratios. The fraction of MC selected events also depends on the event location through the threshold value of $R_{\text{max}}$. This effect is further discussed in the following section.

### 6. Monte Carlo results

MC results allow to estimate the features of the revealed events which pass the adopted selection criteria. In Fig. 6 assuming a MACHO mass $\mu_0 = 0.5$ and a halo MACHO fraction $f_h = 0.2$ we give the contour plot, in the $(t_{1/2}, R_{\text{max}})$ parameter space, of the event number density

$$N_{\text{ev},\text{rev}}^{\text{rev}} = \frac{d^2 N_{\text{ev},\text{rev}}}{dt_{1/2} dR_{\text{max}}},$$

averaged on all the 14 MEGA directions, for both self-lensing and MACHO-lensing events. We also give in the same parameter space the position of the 14 observed MEGA events. $N_{\text{ev},\text{rev}}^{\text{rev}}$ is maximum in the region $t_{1/2} \simeq 20$ day and $R_{\text{max}} \simeq 22$. Moreover, $N_{\text{ev},\text{rev}}^{\text{rev}}$ rapidly decreases for $R_{\text{max}} > 23.5$ (due to the lack of revealed events with low signal-to-noise ratio) and $t_{1/2} < 5$ day (due to the peak sampling) and in the region of high amplification and long duration events (upper-left region of the figure), due to the absence of MC generated events. Actually, the last cutoff depends on the adopted MACHO mass value and shifts towards smaller $t_{1/2}$ values decreasing $\mu_0$. Indeed, in Fig. 6 the crucial parameter determining the event distribution is the lens mass value and it turns out that the region where $N_{\text{ev},\text{rev}}^{\text{rev}}$ is maximum scales with $\mu_0$ in the same way as $t_{1/2}^{\text{median}}$ and $R_{\text{max}}^{\text{median}}$ (see Tab. 4) where we give the median values of the distributions of the MC revealed events as a function of the lens star mass $\mu_0$, $t_{1/2}$ and $R_{\text{max}}$.

From Tab. 4 for $\mu_0 = 0.5$ one can infer that the $N_{\text{ev},\text{rev}}^{\text{rev}}$ plot does not vary substantially for MACHO-lensing and self-lensing events occurring away from the M31 center, since the lens stellar mass is of the same order of the...
Table 4. For the MC revealed events (which pass the MEGA selection criteria), the median values $\mu_{\text{median}}$ (column 2) of the lens star mass distribution for self-lensing events, $t_{1/2}$ (columns 3-7) and $R_{\text{max}}$ (columns 8-12) for self-lensing and MACHO-lensing are given towards the 14 MEGA events.

<table>
<thead>
<tr>
<th>MEGA</th>
<th>$\mu_{\text{median}}$ self</th>
<th>$\mu_{\text{median}}$ MACHO</th>
<th>$t_{1/2}$ median $\mu_h = 0.01$</th>
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<th>0.5</th>
<th>1</th>
<th>$R_{\text{max}}$ median $\mu_h = 0.01$</th>
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<td>22.13</td>
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<td>22.34</td>
<td>22.03</td>
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</table>

Table 5. MACHO-to-self lensing probability ratios ($P_{\text{h}}/P_{\text{f}}$)$_{\text{MC}}$ towards the 14 observed MEGA events are given for different MACHO mass values. Probabilities are now calculated from eq. (6) by considering microlensing rates, $t_{1/2}$ and $R_{\text{max}}$ distributions for the MC revealed events. The results are in the table scale with the MACHO fraction value as $f_h/0.2$.

<table>
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<th>MEGA</th>
<th>$\mu_h = 0.01$</th>
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<td>0.08</td>
</tr>
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<td>1.06</td>
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<td>0.14</td>
</tr>
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<td>0.56</td>
<td>0.33</td>
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<td>3.64</td>
<td>1.98</td>
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<td>14</td>
<td>26.99</td>
<td>29.88</td>
<td>18.50</td>
<td>12.24</td>
</tr>
</tbody>
</table>

MACHO mass. Moreover, for any value of $\mu_h$, the $N_{\text{ev}}$ rev distribution is also weakly dependent on the selected direction towards M31. The situation is completely different for $\mu_h \neq 0.5$, and this means that self-lensing and MACHO-lensing events lie on different regions in the ($t_{1/2}, R_{\text{max}}$) parameter space of the corresponding $N_{\text{ev}}$ rev plot.

The dependence of $t_{1/2}$ median and $R_{\text{max}}$ median on the lens mass and the MEGA direction is also clear from Tab. 4. For self-lensing events, $t_{1/2}$ median increases from the inner to the outer part of the M31 galaxy, and in this region $t_{1/2}$ median also increases for increasing values of $\mu_{\text{median}}$, following the dependence of the Einstein radius with the lens mass $^4$. Moreover, for MACHO-lensing events with a fixed $\mu_h$ value, $t_{1/2}$ median is almost the same for any MEGA direction, while it increases, as expected, with the lens mass.

In Tab. 4 is also evident the decrease, moving towards the M31 center, of the $R_{\text{max}}$ median value that follows the surface brightness variation along the field. In particular, we find a shift of about 1 mag going from the inner to the outer regions. In Fig. 8 we give the (revealed) event distributions as a function of $R_{\text{max}}$ towards the MEGA 1, 7 and 14 directions. This clearly shows how the event location affects the fraction of the revealed events.

The event distributions as a function of $t_{1/2}$, for both self-lensing (solid line) and MACHO-lensing (dashed line) revealed events are shown in Fig. 5 for $\mu_h = 0.5$ and for the 1 and 7 MEGA directions towards M31 (representative of inner and outer events, respectively). Only in the case of the MEGA 1 direction the obtained distributions are markedly different for self-lensing and MACHO-lensing, since self-lensing events are shorter than MACHO-lensing events. For the MEGA 7 direction (and the other outer directions) self-lensing and MACHO lensing with the same lens mean mass have roughly the same $t_{1/2}$ distributions.

Our MC results can be used to estimate the lens nature and location of the 14 MEGA candidate events, by weighting the microlensing rate - giving the analytical estimates of the MACHO-to-self lensing probability ratio ($P_{\text{h}}/P_{\text{f}}$)$_{\text{AN}}$ shown in Tab. 4 - with the revealed event number density distribution and taking also into account the observed features of the MEGA events. In Tab. 5 assuming a halo MACHO fraction $f_h = 0.2$ and different MACHO mass

$^4$ For self-lensing events, the lens median mass value changes with the MEGA direction, since the disk lens mass is on average higher than the bulge lens mass and the disk-to-bulge probability ratio depends on the MEGA direction.
values, we give the MC MACHO-to-self lensing probability ratio
\[
\left(\frac{P_h}{P_s}\right)_{\text{MC}} = \left(\frac{P_h}{P_s}\right)_{\text{An}} \frac{N_{\text{ev}}^{\text{rev}}(t_{1/2}^{\text{obs}})}{N_{\text{ev}}^{\text{rev}}(t_{1/2}^{\text{obs}}) + N_{\text{ev}}^{\text{rev}}(R_{\text{max}}^{\text{obs}})}
\]
where we indicate with $N_{\text{ev}}^{\text{rev}}(t_{1/2}^{\text{obs}})$ and $N_{\text{ev}}^{\text{rev}}(R_{\text{max}}^{\text{obs}})$ the number of events (either MACHO or self-lensing) with duration and magnitude at maximum, respectively, within 2 standard deviations around the observed values. Here, we remark that each $N_{\text{ev}}$ distribution is normalized to the total number of revealed events. A comparison between the analytical and MC results given, respectively, in Tabs. 2 and 3 is presented in Figs. 9a and 9b, where we plot

**Fig. 8.** The $t_{1/2}$ distributions are shown for MEGA 1 and 7 events and for $\mu_h = 0.5$. In each panel, the solid curve refers to self-lensing events and the dashed curve to MACHO-lensing events.

**Fig. 9.** Plot of $(P_h/P_s)_{\text{An}}$ versus $(P_h/P_s)_{\text{MC}}$ for all the 14 MEGA events. In panel a) we consider a MACHO mass $\mu_h = 0.01$ and we use the symbol plus; in panel b) we take $\mu_h = 0.1, 0.5, 1$ and use symbols time, circle, asterisk, respectively.
Also for the time scale ratios between MACHO-lensing and self-lensing events analytical and MC results give different estimates. A comparison of the analytical results in Tab. 3 and MC results in Tab. 4 is presented in Figs. 10a and 10b, where we show \((t_{1/2} h/t_{1/2} s)_{\text{An}}\) versus \((t_{1/2} h/t_{1/2} s)_{\text{MC}}\) for MEGA 1-4 and 5-14 events. The difference is particularly important for the events 1-4.

It emerges, therefore, clearly that the MC analysis is essential for determining the lens nature and location of microlensing events, at least if the MACHO mass differs substantially from the average self-lensing mass. From Tab. 3 we find that MEGA events 1-4 are most likely self-lensing events for MACHO masses greater than 0.1 solar masses, while events 5, 6, 9, 11, 13, 14 are likely MACHO-lensing. For the other MEGA directions the lens nature is more uncertain. As a final comment on Fig. 8 we note that the event 12, lying in a region of low (revealed) event number density, hardly can be considered a reliable microlensing event, unless the MACHO mass is considerably greater than 1 \(M_\odot\).

7. Conclusions

We have studied the main features of the expected microlensing events in pixel lensing observations towards M31, by using both analytical estimates (from the microlensing rate) as well as results by a MC code where we reproduce the observing and instrumental conditions of the MEGA experiment.

First of all, we derive in Section 2 the microlensing rate, and assuming a specific mass distribution model for M31 and the Galaxy, we calculate the MACHO-to-self lensing probability and the MACHO-to-self lensing event time scale ratios. For \(\mu_h > 0.1\), we find that self-lensing dominates in the M31 central regions. Moreover, for \(\mu_s \geq 0.5\), towards the innermost MEGA directions, MACHOs events have duration twice as long as self-lensing events, while outer events have roughly the same duration.

We then generate a large number of MC microlensing events by choosing relevant source and lens parameters as outlined in Section 5. We study the observability of the MC events, by considering the capabilities of the INT Telescope, typical observing conditions and the event selection criteria adopted by the MEGA Collaboration.

MC results can be used to evaluate, for the 14 MEGA candidate events, the MACHO-to-self lensing probability and the event time scale ratios \((P_h/P_s)_{\text{MC}}\) and \((t_{1/2} h/t_{1/2} s)_{\text{MC}}\) (given in Tabs. 4 and 5), by taking into account not only the analytical expectations from the microlensing rate (as already done in Tabs. 2 and 3) but also the features of the MC (revealed) events and the observed values of \(t_{1/2}\) and \(R_{\text{max}}\). MC results and analytical expectations are compared in Figs. 9 and 10, where one can see that in determining the lens nature and location of the MEGA candidate events, the MC analysis is particularly important for \(\mu_s \neq 0.5\). Accordingly, we find that event 12, lying in a region of low event number density,
hardly can be considered a reliable microlensing event, unless the MACHO mass is considerably larger than 1 \( M_\odot \). Moreover, for a MACHO mass greater than 0.1 \( M_\odot \), the innermost MEGA events 1, 2, 3, 4 are most likely self-lensing events, while 5, 6, 9, 11, 13, 14 are MACHO-lensing events. For the other MEGA directions the lens nature is more uncertain.

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Natural Sciences Citations and References
(Author–Year and Numerical Schemes)

Patrick W. Daly

This paper describes package natbib
version 7.1 from 2003/06/06.

Abstract

The natbib package is a reimplementation of the \LaTeX \cite command, to work with both author–year and numerical citations. It is compatible with the standard bibliographic style files, such as plain.bst, as well as with those for harvard, apalike, chicago, astron, authordate, and of course natbib.

In contrast to the packages listed above, the natbib package supports not only the various author–year bibliography styles, but also those for standard numerical citations. In fact, it can also produce numerical citations even with an author–year bibliographic style, something that permits easy switching between the two citation modes. To this end, replacements for the standard \LaTeX .bst files are also provided.

It is possible to define the citation style (type of brackets and punctuation between citations) and even to associate it with the name of the bibliographic style so that it is automatically activated. Citation styles can be defined for local .bst files by means of a configuration file natbib.cfg.

It is compatible with the packages: babel, index, showkeys, chapterbib, hyperref, koma and with the classes amsbook and amsart. It can also emulate the sorting and compressing functions of the cite package (with which it is otherwise incompatible).

The natbib package therefore acts as a single, flexible interface for most of the available bibliographic styles.
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1 Introduction

The first problem of using author–year literature citations with standard \LaTeX{} is that the two forms of citations are not supported. These are:

- textual: ... as shown by Jones et al. (1990) ...
- parenthetical: It has been shown (Jones et al., 1990) that ...

There is only one \cite{} command to do both jobs.

A second problem is that the \texttt{thebibliography} environment for listing the references insists on including the \texttt{labels} in the list. These labels are normally the numbers, needed for referencing. In the author–year system, they are superfluous and should be left off. Thus, if one were to make up a bibliography with the author–year as label, as

\begin{verbatim}
\begin{thebibliography}{...}
\bibitem[Jones et al., 1990]{jon90}
Jones, P. K., . . .
\end{thebibliography}
\end{verbatim}

then \cite{jon90} produces the parenthetical citation [Jones et al., 1990], but there is no way to get the textual citation. Furthermore, the citation text will also be included in the list of references.

The final problem is to find a Bib\TeX{} bibliography style that will be suitable.

2 Previous Solutions

This section may not be of interest to all users. To find out how to use \texttt{natbib} without reading about the historical background, go to Section 4.

Although the author–year citation mode is not supported by standard \LaTeX{}, there are a number of contributed packages that try to solve this problem. The various bibliographic styles (.bst files) that exist are usually tailored to be used with a particular \LaTeX{} package.

I have found a large number of .bst files on file servers that may act as indicators of the various systems available.

2.1 The \texttt{natsci.bst} Style

What gave me my first inspiration was Stephen Gildea’s \texttt{natsci.bst} for use with his \texttt{agujgr.sty} file. This showed me that the problem was solvable. However, Gildea’s style formats \texttt{\bibitem} just as I illustrated above: with an optional label consisting of abbreviated authors and year. Thus only parenthetical citations can be accommodated. The list of references, however, is fixed up in his style files.

2.2 The \texttt{apalike.bst} Style

Oren Patashnik, the originator of Bib\TeX{} and the standard .bst files, has also worked on an author–year style, called \texttt{apalike.bst} with a corresponding \texttt{apalike.sty} to support it. Again, only the parenthetical citation is provided.
Except for the fact that his style works with version 0.99 of Bib\TeX, its functionality is identical to that of the natsci files.

Patashnik does not like author–year citations. He makes this very clear in his Bib\TeX manuals and in the header to apalike.bst. Nevertheless, one should respect his work in this area, simply because he should be the best expert on matters of Bib\TeX. Thus apalike.bst could be the basis for other styles.

The form of the \texttt{thebibliography} entries in this system is

\begin{verbatim}
\bibitem[Jones et al., 1990]{jon90}...
\end{verbatim}

the same as I illustrated earlier. This is the most minimal form that can be given. I name it the apalike variant, after Patashnik’s apalike.bst and apalike.sty. However, there could be many independent .bst files that follow this line.

The bibliography style files belonging to this group include:

\texttt{apalike, apalike2, cea, cell, jmb, phapalik, phppcf, phrmp}

\section{The newapa Style}

A major improvement has been achieved with newapa.bst and the accompanying newapa.sty files by Stephen N. Spencer and Young U. Ryu. Under their system, three separate items of information are included in the \texttt{\bibitem} label, to be used as required. These are: the full author list, the abbreviated list, and the year. This is accomplished by means of a \texttt{\citeauthoryear} command included in the label, as

\begin{verbatim}
\bibitem[\protect\citeauthoryear{Jones, Barker, and Williams}{Jones et al.}{1990}]{jon90}...
\end{verbatim}

Actually, this only illustrates the basic structure of \texttt{\citeauthoryear}; the newapa files go even further to replace some words and punctuation with commands. For example, the word ‘and’ above is really \texttt{betweenauthors}, something that must be defined in the .sty file. Of course, \texttt{\citeauthoryear} is also defined in that file. A number of different \texttt{\cite} commands are available to print out the citation with complete author list, with the short list, with or without the date, the textual or parenthetical form.

Thus the \texttt{\citeauthoryear} entry in \texttt{\bibitem} is very flexible, permitting the style file to generate every citation form that one might want. It is used by a number of other styles, with corresponding .sty files. They all appear to have been inspired by newapa.bst, although they lack the extra punctuation commands.

Bibliographic style files belonging to the newapa group include

\texttt{newapa, chicago, chicagoa, jas99, named}

Note: the last of these, named.bst, uses \texttt{\citeauthoryear} in a slightly different manner, with only two arguments: the short list and year.

\section{The Harvard Family}

The same effect is achieved by a different approach in the Harvard family of bibliographic styles. Here a substitute for \texttt{\bibitem} is used, as

\begin{verbatim}
\harvarditem[Jones et al.]{Jones, Baker, and Williams}{1990}{jon90}...
\end{verbatim}
The accompanying interface package file is called \texttt{harvard.sty} and is written by Peter Williams and Thorsten Schnier. It defines \texttt{harvarditem} as well as the citation commands \texttt{cite}, for parenthetical, and \texttt{citeasnoun}, for textual citations. The first citation uses the long author list, following ones the shorter list, if it has been given in the optional argument to \texttt{harvarditem}.

Bibliography styles belonging to the Harvard family are

\texttt{agsm, dcu, kluwer}

This package has been updated for \LaTeX\ 2\epsilon, with many additions to add flexibility. The result is a powerful interface that should meet most citation needs. (It does not suppress repeated authors, though, as \texttt{natbib} does.)

2.5 The Astronomy Style

Apparently realizing the limitations of his \texttt{apalike} system, Oren Patashnik went on to develop a ‘true’ \texttt{apa} bibliographic style, making use of the method already employed by an astronomy journal. This is actually very similar to the \texttt{newapa} label but with only the short list of authors:

\begin{verbatim}
\bibitem{\protect\astroncite{Jones et al.}{1990}}{jon90}
...
\end{verbatim}

It requires the package file \texttt{astron.sty} or any other style that defines \texttt{astroncite} appropriately.

Bibliographic styles belonging to the astronomy group are

\texttt{apa, astron, bbs, cbe, humanbio, humannat, jtb}

This is as good as the \texttt{citeauthoryear} command, although not as flexible since the full list of authors is missing.

2.6 The authordate Style

Finally, I have also found some packages making use of a label command called \texttt{citename} in the form

\begin{verbatim}
\bibitem{\protect\citename{Jones et al.,}{1990}}{jon90}
...
\end{verbatim}

This is not a good system since the author list and date are not cleanly separated as individual arguments, and since the punctuation is included in the label text. It is better to keep the punctuation fully removed, as part of the definitions in the \texttt{.sty} file, for complete flexibility.

Bibliographic styles belonging to this group are

\texttt{authordate1, authordate2, authordate3, authordate4, aaai-named}

with accompanying style file \texttt{authordate1-4.sty}. 

3 The natbib System

The form of the \bibitem entry that I have used for all my bibliographic styles is only slightly more complicated than the minimal one, but allows a clean separation between authors and date:

\bibitem{Jones et al.(1990)}{jon90}...

or alternatively

\bibitem{Jones et al.(1990) Jones, Baker, and Williams}{jon90}...

(One weakness of the natbib format is that it fails if the author list itself contains parentheses! This may be fixed up if the author list is grouped in curly braces.)

I wanted to name the system something like ‘natural sciences bibliography’, intending it to be a variant of natsci.sty. Since that name was already taken, I resorted to the rather cryptic, and definitely ugly, natbib.

The natbib.sty package\footnote{Formerly called a style file in the older \LaTeX{} 2.09 terminology.} supports not only my own \bibitem format, but also all the others described here, plus numerical citation modes. The additional questions of citation style (type of brackets, commas or semi-colons between citations) can be defined once and for all for each .bst file and need never be specified explicitly in the source text. The \cite commands and syntax are always those of natbib, even when used with a .bst file such as chicago.bst that would normally have a different set of commands (defined in chicago.sty). The result is a single \LaTeX{} package to handle all the bibliographic styles in a uniform manner.

All the author–year bibliographic style files can also be used for numerical citations, by simply selecting the mode in one of the ways described in Sections 4.8 and 7. It is not possible to employ author–year citations with pure numerical .bst files, and never will be. See Section 7 for more information.

4 Using this Package

In this paper, I distinguish between the citation mode (author–year or numerical) and citation style (the type of punctuation used for citations). The citation style is something that is independent of the bibliography style and is not programmed in the .bst files.

4.1 New Bibliography Styles

I provide three new .bst files to replace the standard \LaTeX{} numerical ones:

plainnat.bst    abbrvnat.bst    unsrtnat.bst

These produce reference lists in the same style as the corresponding standard .bst file, but work with natbib. The advantage is that they can be used in both numerical and author–year mode.

These .bst files are not meant to be exhaustive by any means. Other style files conforming to the natbib format exist, or may be generated with my custom-bib (also known as makebst) program.
4.2 Basic Citation Commands

\cite{jon90} \Rightarrow Jones et al. (1990)
\cite[chap. 2]{jon90} \Rightarrow Jones et al. (1990, chap. 2)
\citet{jon90} \Rightarrow (Jones et al., 1990)
\citet[see][]{jon90} \Rightarrow (see Jones et al., 1990, chap. 2)
\citet*{jon90} \Rightarrow Jones, Baker, and Williams (1990)
\citet*{jon90} \Rightarrow (Jones, Baker, and Williams, 1990)

The starred versions can only list the full authors if the .bst file supports this feature; otherwise, the abbreviated list is printed.

In standard \LaTeX, the \cite command can only take a single optional text for a note after the citation; here, a single optional text is a post-note, while two are the pre- and post-notes. To have only a pre-note, it is necessary to provide an empty post-note text, as shown above.

More complex mixtures of text and citations can be generated with the all-purpose \citetext command in Section 4.3.

Multiple citations may be made by including more than one citation key in the \cite command argument. If adjacent citations have the same author designation but different years, then the author names are not reprinted.

\cite{jon90,jam91} \Rightarrow Jones et al. (1990); James et al. (1991)
\citep{jon90,jam91} \Rightarrow (Jones et al., 1990; James et al. 1991)
\citep{jon90,jon91} \Rightarrow (Jones et al., 1990, 1991)
\citep{jon90a,jon90b} \Rightarrow (Jones et al., 1990a,b)

These examples are for author–year citation mode. In numerical mode, the results are different.

\cite{jon90} \Rightarrow Jones et al. [21]
\cite[chap. 2]{jon90} \Rightarrow Jones et al. [21, chap. 2]
\citep{jon90} \Rightarrow [21]
\citep[see][]{jon90} \Rightarrow [see 21]
\citep[see][]{jon90} \Rightarrow [see 21, chap. 2]
\citep{jon90a,jon90b} \Rightarrow [21, 32]

The authors can only be listed if the .bst file supports author–year citations. The standard .bst files, such as plain.bst are numerical only and transfer no author–year information to \LaTeX. In this case, \citet prints “(author?) [21].”

In the original versions of natbib, the traditional \cite command was used for both textual and parenthetical citations. The presence of an empty optional text in square brackets signalled parenthetical. This syntax has been retained for compatibility, but is no longer encouraged.
This means that `\cite` (without notes) is the same as `\citet` in author–year mode, whereas in numerical mode, it is the same as `\citep`. The starred version, as well as the one or two optional notes, may also be used.

It is possible to have multiple citations sorted into the same sequence as they appear in the list of references, regardless of their order as arguments to the `\cite` commands. The option `sort` is required for this feature. See Section 4.14.

Some publishers require that the first citation of any given reference be given with the full author list, but that all subsequent ones with the abbreviated list. Include the option `longnamesfirst` to enable this for `natbib`. See Section 4.15.

### 4.3 Extended Citation Commands

\citealt, `\citet` is the same as `\citep` but **without parentheses**. Similarly, `\citealp` is `\citep` without parentheses. Multiple references, notes, and the starred variants also exist.

\begin{verbatim}
\citealt{jon90} ⇒ Jones et al. 1990
\citealt*{jon90} ⇒ Jones, Baker, and Williams 1990
\citealp{jon90} ⇒ Jones et al., 1990
\citealp*{jon90} ⇒ Jones, Baker, and Williams, 1990
\citealp{jon90,jam91} ⇒ Jones et al., 1990; James et al., 1991
\citealp[pg.~32]{jon90} ⇒ Jones et al., 1990, pg. 32
\citealp[priv.\ comm.]{jon90} ⇒ (priv. comm.)
\end{verbatim}

The `\citealp` command allows arbitrary text to be placed in the current citation parentheses. This may be used in combination with `\citealp`. For example,

\begin{verbatim}
\citealp[see \citealp{jon90},
or even better \citealp{jam91}]
\end{verbatim}

to produce (see Jones et al., 1990, or even better James et al., 1991).

In author–year schemes, it is sometimes desirable to be able to refer to the authors without the year, or vice versa. This is provided with the extra commands

\begin{verbatim}
\citeauthor{jon90} ⇒ Jones et al.
\citeauthor*{jon90} ⇒ Jones, Baker, and Williams
\citeyear{jon90} ⇒ 1990
\citeyearpar{jon90} ⇒ (1990)
\end{verbatim}

There also exists a command `\citefullauthor` which is equivalent to `\citeauthor*`.

If the full author information is missing, then `\citeauthor*` is the same as `\citeauthor`, printing only the abbreviated list. This also applies to the starred versions of `\citet` and `\citep`.

If the author or year information is missing (as is the case with the standard `\LaTeX` `bst` files), these commands issue a warning.

**Note:** these commands may also be used with numerical citations, provided an author–year `bst` file is being employed.

**Note:** all `\cite` commands have the same syntax, allowing multiple citations and up to two notes (there is, however, no starred `\citeyear` variant). It does not really make much sense to add notes to `\citeyear` and `\citeauthor`, especially with multiple citations; however, this can be done, there will be no error message,
but the results are sometimes strange. For example, in numerical mode, the notes are fully ignored, while in author–year mode, only the post-note is accepted. Multiple citations in \cite are also not recommended (nor are they in my opinion meaningful), but if they are used with notes, the pre-note will appear before each year, and the post-note only after the last year. These are admittedly bugs, but the effort to remove them is not justified by the questionable usefulness of these features.

In summary, notes are only intended for \citep but they may also be used with \cite in author–year mode, with single citations. In any other situation, the results are unpredictable.

4.4 Forcing Upper Cased Name

If the first author’s name contains a von part, such as “della Robbia”, then \cite{dRob98} produces “della Robbia (1998)”, even at the beginning of a sentence. One can force the first letter to be in upper case with the command \Citet instead. Other upper case commands also exist.

\begin{verbatim}
when \cite{dRob98} ⇒ della Robbia (1998)
then \Citet{dRob98} ⇒ Della Robbia (1998)
\citep{dRob98} ⇒ (Della Robbia, 1998)
\citealt{dRob98} ⇒ Della Robbia 1998
\citealp{dRob98} ⇒ Della Robbia, 1998
\citeauthor{dRob98} ⇒ Della Robbia
\end{verbatim}

These commands also exist in starred versions for full author names.

\textbf{Note:} the coding for the upper casing commands is tricky and likely buggy. It operates on the names that are stored in the \bibitem entry, and works even if old style font commands are used; however, NFSS commands will cause it to crash. Thus \bibitem[\textit{della Robbia}(1998)]{dRob98} is okay, but \bibitem[\textit{della Robbia}(1998)]{dRob98} crashes. I hope to improve this situation in future.

4.5 Citation Aliasing

Sometimes one wants to refer to a reference with a special designation, rather than by the authors, i.e. as Paper I, Paper II. Such aliases can be defined and used, textual and/or parenthetical with:

\begin{verbatim}
\defcitealias{jon90}{Paper~I}
\cite{jon90} ⇒ Paper I
\citep{jon90} ⇒ (Paper I)
\end{verbatim}

These citation commands function much like \cite and \citep: they may take multiple keys in the argument, may contain notes, and are marked as hyperlinks.

A warning is issued if the alias is used before it is defined, or if an alias is redefined for a given citation. No warning is issued if an alias is defined for a citation key that does not exist; the warning comes when it is used!

See Section 4.6 for an alternative means of citing with a code name.
4.6 Authorless and Yearless References

What does one do about references that do not have authors? This has long bothered me but I do have a suggestion. Standard \texttt{BibTeX} styles make use of a \texttt{KEY} field in the entries to be used for alphabetizing when the authors or editors are missing. The author–year styles go even further and insert the \texttt{KEY} field in place of the authors. One can imagine giving a code designation for the work at this point. For example,

\begin{verbatim}
@MANUAL{handbk98,
  title = {Assembling Computers},
  year = 1998,
  organization = {MacroHard Inc.},
  key = "MH-MAN"
}
\end{verbatim}

With \texttt{plain}, the key text \texttt{MH-MAN} is used only to order the reference, but with \texttt{plainnat} and other author–year styles, it is used in place of the authors. One can then refer to it as \texttt{\citeauthor{handbk98}} to get \texttt{MH-MAN} or as \texttt{\citetext{\citeauthor{handbk98}}} for (MH-MAN), a parenthetical citation.

This can be greatly simplified if the bibliography style leaves the date blank in the \texttt{\bibitem} entry, as

\texttt{\bibitem[MH-MAN()]{handbk98}}

for then \texttt{natbib} suppresses the date, preceding punctuation, and the braces for \texttt{\citet}. This means that \texttt{\citet} and \texttt{\citep} behave automatically like the two examples above. The date still may appear in the text of the reference.

The \texttt{natbib} bibliography styles have been modified accordingly to omit the date from the \texttt{\bibitem} entry when missing authors and/or editors are replaced by key text.

Similarly, if the year is missing, it will be left blank in the \texttt{\bibitem} entry; thus citing such a work will only produce the authors’ names.

\textbf{Note:} there are many other possibilities with this feature. One can even produce citations like those of the \texttt{alpha} bibliography style, by placing the citation code in place of the authors in the \texttt{\bibitem} entry and leaving the year blank. A second code (or maybe even the authors themselves) could be placed where the full author list normally appears, to be printed with the starred version of the \texttt{\cite} commands. For example,

\texttt{\bibitem[MH-MAN()MacroHard Inc.]{handbk98}}

4.7 Extra Features in the plainnat Family

The special \texttt{.bst} files for \texttt{natbib} mentioned in Section 4.1 have a number of extra fields compared to the original files:

\begin{itemize}
  \item \texttt{ISBN} for the ISBN number in books,
  \item \texttt{ISSN} for the ISSN number in periodicals,
  \item \texttt{URL} for the Internet address of on-line documents.
\end{itemize}

The URL address is set in a typewriter font and often leads to line-breaking problems. It is advisable to load the \texttt{url} package of Donald Arseneau, which allows typewriter text to be broken at punctuation marks. The URL addresses are set with the \texttt{\url} command.
in this package, but if it is not loaded, then \url is defined to be \texttt, with no line breaks.

As pointed out in Section 4.6, the KEY field is treated differently by plainnat than in plain. Whereas the latter uses this field only to alphabetize entries without authors, plainnat actually inserts it in place of the author, both in the reference text and in the citation label (\bibitem entries). Furthermore, the year is left empty in \bibitem so that \citet prints only the “author” text, which is now the KEY. This should be some code designation for the work.

4.8 Selecting Citation Punctuation

The above examples have been printed with the default citation style. It is possible to change this, as well as to select numerical or author–year mode, by means of the \bibpunct command, which takes one optional and 6 mandatory arguments. The mandatory ones are:

1. the opening bracket symbol, default = (  
2. the closing bracket symbol, default = )  
3. the punctuation between multiple citations, default = ;  
4. the letter ‘n’ for numerical style, or ‘s’ for numerical superscript style, any other letter for author–year, default = author–year; note, it is not necessary to specify which author–year interface is being used, for all will be recognized;  
5. the punctuation that comes between the author names and the year (parenthetical case only), default = ,  
6. the punctuation that comes between years or numbers when common author lists are suppressed (default = ,); if both authors and years are common, the citation is printed as ‘1994a,b’, but if a space is wanted between the extra letters, then include the space in the argument, as {,~}.  

For numerical mode, \citet{jon90,jon91} produces ‘Jones et al. [21, 22]’ with this punctuation between the numbers. A space is automatically included for numbers, but not for superscripts.

The optional argument is the character preceding a post-note, default is a comma plus space. In redefining this character, one must include a space if one is wanted.

The \bibpunct command must be issued in the preamble, that is, before \begin{document}.

Example 1, \bibpunct{[}{]}{,}{a}{}{;} changes the output of

\citet{jon90,jon91,jam92}


Example 2, \bibpunct{; }{{}{}{,}{a}{}{;}} changes the output of

\citet[and references therein]{jon90}
Usually the citation style is determined by the journal for which one is writing, and is as much a part of the bibliography style as everything else. The \texttt{natbib} package allows punctuation definitions to be directly coupled to the \texttt{bibliographystyle} command that must always be present when \texttt{BibTi\TeX} is used. It is this command that selects the \texttt{.bst} file; by adding such a coupling to \texttt{natbib} for every \texttt{.bst} file that one might want to use, it is not necessary to add \texttt{bibpunct} explicitly in the document itself, unless of course one wishes to override the preset values.

Such a coupling is achieved by defining a command \texttt{\bibstyle@bst}, where \texttt{bst} stands for the name of the \texttt{.bst} file. For example, the American Geophysical Union (AGU) demands in its publications that citations be made with square brackets and separated by semi-colons. I have an \texttt{agu.bst} file to accomplish most of the formatting, but such punctuations are not included in it. Instead, \texttt{natbib} has the definition

\begin{verbatim}
\newcommand{\bibstyle@agu}{\bibpunct{[}{]}{;}{a}{,}{,~}}
\end{verbatim}

These style defining commands may contain more than just \texttt{\bibpunct}. Some numerical citation scheme require even more changes. For example, the journal \textit{Nature} not only uses superscripted numbers for citations, it also prints the numbers in the list of references without the normal square brackets. To accommodate this, \texttt{natbib} contains the style definition

\begin{verbatim}
\newcommand{\bibstyle@nature}{\bibpunct{}{}{,}{s}{}{\textsuperscript{,}}
\gdef\NAT@biblabelnum##1{##1.}}
\end{verbatim}

The redefined \texttt{\NAT@biblabelnum} command specifies how the reference numbers are to be formatted in the list of references itself. The redefinition must be made with \texttt{\gdef}, not \texttt{\def} or \texttt{\renewcommand}.

The selected punctuation style and other redefinitions will not be in effect on the first \texttt{BibTi\TeX} run, for they are stored to the auxiliary file for the subsequent run.

The user may add more such definitions of his own, to accommodate those journals and \texttt{.bst} files that he has. He may either add them to his local copy of \texttt{natbib.sty}, or better put them into a file named \texttt{natbib.cfg}. This file will be read in if it exists, adding any local configurations. Thus such configurations can survive future updates of the package. (This is for \texttt{BibTi\TeX 2\varepsilon} only.)

\textbf{Note:} any explicit call to \texttt{\bibpunct} has priority over the predefined citation styles.

A preprogrammed citation style is normally invoked by the command \texttt{\bibliographystyle}, as described above. However, it may be that one wants to apply a certain citation style to another bibliography style. This may be done with \texttt{\citestyle}, given \textit{before} \texttt{\begin{document}}. For example, to use the \texttt{plainnat} bibliography style (for the list of references) with the \textit{Nature} style of citations (superscripts),

\begin{verbatim}
\documentclass{article}
\usepackage{natbib}
\citestyle{plainnat}
\citestyle{nature}
\begin{document}
\end{verbatim}
\bibliographystyle{plainnat}

Note: all changes to the citation style, including punctuation, must be made before \begin{document}, which freezes the citation style.

4.9 Priority of Style Commands

The citation style (punctuation and mode) can be selected by means of the \bibpunct, \citestyle, and predefined \bibstyle@bst commands. They can also be selected by \LaTeX options (Section 4). What happens if there are several conflicting selections?

The lowest priority is assigned to the predefined \bibstyle@bst commands, since they are implicit and not obvious to the user. The \LaTeX options have the next priority. Finally, any selection by \bibpunct and/or \citestyle overrides those of the other methods.

4.10 Other Formatting Options

\bibsection

The list of references normally appears as a \section* or \chapter*, depending on the main class. If one wants to redesign one’s own heading, say as a numbered section with \section, then \bibsection may be redefined by the user accordingly.

\bibpreamble

A preamble appearing after the \bibsection heading may be inserted before the actual list of references by defining \bibpreamble. This will appear in the normal text font unless it contains font declarations. The \bibfont applies to the list of references, not to this preamble.

\bibfont

The list of references is normally printed in the same font size and style as the main body. However, it is possible to define \bibfont to be font commands that are in effect within the thebibliography environment after any preamble. For example,

\newcommand{\bibfont}{\small}

\citenumfont

Numerical citations may be printed in a different font. Define \citenumfont to be a font declaration like \itshape or even a command taking arguments like \textit.

\newcommand{\citenumfont}[1]{\textit{#1}}

The above is better than \itshape since it automatically adds italic correction.

\bibnumfmt

The format of the numerical listing in the reference list may also be changed from the default [32] by redefining \bibnumfmt, for example

\renewcommand{\bibnumfmt}[1]{\textbf{#1};}

To achieve 32; instead.

\bibhang

The list of references for author–year styles uses a hanging indentation format: the first line of each reference is flush left, the following lines are set with an indentation from the left margin. This indentation is 1 cm by default but may be changed by redefining (with \setlength) the length parameter \bibhang.

\bibsep

The vertical spacing between references in the list, whether author–year or numerical, is controlled by the length \bibsep. If this is set to 0 pt, there is no
extra line spacing between references. The default spacing depends on the font size selected in \documentclass, and is almost a full blank line. Change this by redefining \bibsep with \setlength command.

4.11 Automatic Indexing of Citations

If one wishes to have the citations entered in the .idx indexing file, it is only necessary to issue \citeindextrue at any point in the document. All following \cite commands, of all variations, then insert the corresponding entry to that file. With \citeindexfalse, these entries will no longer be made.

The \bibitem commands in the thebibliography environment will also make index entries. If this is not desired, then issue \citeindexfalse before \bibliography or \begin{thebibliography}.

Of course, \makeindex must also be issued in the preamble to activate indexing, as usual. Otherwise, no indexing is done at all.

Make sure that the document has been processed at least twice after the last Bib\TeX run before running the makeindex program.

The form of the index entries is set by the internal \NAT@idxtxt, which can be redefined by hackers if wanted (in the natbib.cfg file please). By default, it prints the short author list plus date in the current parenthesis style.

The natbib package can also be used with the index package of David M. Jones. The order in which the packages are loaded is not important.

In that package, multiple index lists may be made by means of a \newindex command. For example, it may be desirable to put all the citation indexing into a separate list. First that list must be initiated with, e.g.,

\newindex{cite}{ctx}{cnd}{List of Citations}

and then the automatic citation indexing associated with this list with the natbib command

\renewcommand{\citeindextrue}{cite}

See the documentation for index.sty for details.

4.12 Hyper\TeX Compatibility

The natbib package is compatible with the hyperref package of Sebastian Rahtz, for use with \LaTeX \rightarrow HTML conversions, pdf\LaTeX, pdfmark. The compatibility is of a mutual nature: both packages contain coding that interact with that of the other.

There is a special option nonamebreak that can be used with the hyperref package; it keeps all the author names in a citation on one line, something that avoids certain problems with pdf\LaTeX. This is otherwise not recommended, since many overfull lines result.

4.13 Multiple Bibliographies in One Document

The natbib package is compatible with the chapterbib package of Donald Arsenneau and Niel Kempson,\footnote{I have used version 1.5 from 1995/10/09; cannot guarantee earlier versions.} which makes it possible to have several bibliographies.
in one document. The usual application is to have bibliographies in each chapter of a book, especially if they have been written by different authors.

The chapterbib package works in a very natural way for the author; only the editor who puts all the chapters together into one book has to do some extra work.

The package makes use of the \include command, and each \included file has its own bibliography. For large books, it makes very good sense to take advantage of this feature in any case.

To review the use of \include, recall that the main file

```latex
\documentclass{...}
\includeonly{ch2}
\begin{document}
\include{ch1}
\include{ch2}
\include{ch3}
\end{document}
```

will process only the file \texttt{ch2.tex} as though the files \texttt{ch1.tex} and \texttt{ch3.tex} were also present. That is, all counters, especially the page and section numbers, as well as cross-referencing definitions, will function as if the whole document were processed. The trick is that each \included file has its own .aux file containing these definitions, and they are all read in every time, even if the corresponding .tex file is not. The .aux files also contain the citation information for Bib\TeX, something that the chapterbib package exploits.

If \texttt{\usepackage{chapterbib}} has been given, the keys in each \cite and \bibitem command are associated with the current \included file and are distinguished from the identical key in a different file. Each of these files must contain its own \bibliography and \bibliographystyle commands. One processes Bib\TeX on each file separately before processing it under \LaTeX (at least twice).

4.13.1 Special Considerations for natbib and chapterbib

The order in which the chapterbib and natbib packages are loaded is unimportant.

The chapterbib package provides an option sectionbib that puts the bibliography in a \section* instead of \chapter*, something that makes sense if there is a bibliography in each chapter. This option will not work when natbib is also loaded; instead, add the option to natbib. (The sectionbib option can always be given, but it only has meaning for the \texttt{book} and \texttt{report} classes, or for classes derived from them.)

Every \included file must contain its own \bibliography command where the bibliography is to appear. The database files listed as arguments to this command can be different in each file, of course. However, what is not so obvious, is that each file must also contain a \bibliographystyle command, preferably with the same style argument. If different bibliography styles are specified for different files, then the preprogrammed citation style (punctuation and citation mode) will be that of the first bibliography style given. The preprogrammed citation styles can only be changed in the preamble (see Section 4.9), something that guarantees a uniform style for the entire document.\footnote{It would be relatively easy to allow changes in style anywhere in the document, but this strikes me as bad policy. However, it is provided for with the docstrip option \texttt{nopreonly}.}
4.14 Sorting and Compressing Numerical Citations

Another package by Donald Arseneau, cite.sty, reimplements the entire (numerical) citation system such that one can control the punctuation and citation format, all of which is done by natbib as well. However, it also can sort and compress numerical citations, something that is required by some journals.

What this means is that when multiple citations are given with a single \cite command, the normal order of the numbers is in the sequence given. This is usually a wild list of numbers, such as [4,2,8,3]. With the cite package, this list becomes [2–4,8].

It is impossible to make the cite and natbib packages compatible, since both reimplement \cite from scratch. Instead, I have taken over some of the coding from cite.sty, modifying it for natbib. This coding is activated by including one of the options sort or sort&compress in the \usepackage command.

For author–year citations, the option sort orders the citations in a single \citet or \citep command into the sequence in which they appear in the list of references. This is normally alphabetical first, year second. This should avoid citations of the type: “James et al. (1994b,a)”. For author–year mode, the sort&compress option is identical to sort.

4.15 Long Author List on First Citation

A feature that has often been requested by otherwise happy users of natbib is one that is found in the harvard package as standard: with the first citation of any reference, the full author list is printed, and afterwards only the abbreviated list. One can control this with \citet* for the first citation, and \citet or \citep thereafter. However, the automatic feature is very desired.

This can be activated with the option longnamesfirst. Some references have so many authors that you want to suppress the automatic long list only for them. In this case, issue

\shortcites{⟨key-list⟩}

before the first citations, and those included in key-list will have a short list on their first citation.

Full author lists can still be forced at any time with the starred variants.

5 Numerical Citations with Author–Year Styles

It is possible to produce numerical citations with any author-year .bst file, with minimal change to the text. The commands \citet and \citep will produce sensible results in both modes, without any special editing. Obviously, the opposite is not possible; a .bst file intended for numerical citation can never produce author–year citations, simply because the information is not transferred to the auxiliary file.

5.1 Selecting Numerical Mode

By default, natbib is in author–year mode. This can be changed by
1. selecting a numerical bibliography style with predefined citation style, defined either in the package or in the local configuration file;

2. giving options numbers or super to the \usepackage command;

3. issuing \bibpunct with the 4th mandatory argument set to n or s;

4. issuing \citestyle with the name of a predefined numerical bibliography style.

The methods are listed in order of increasing priority.

The \natbib package will automatically switch to numerical mode if any one of the \bibitem entries fails to conform to the possible author–year formats. There is no way to override this, since such an entry would cause trouble in the author–year mode.

There are certain special ‘numerical’ styles, like that of the standard alpha.bst, which include a non-numerical label in place of the number, in the form

\bibitem[ABC95]{able95}

As far as \natbib is concerned, this label does not conform to the author–year possibilities and is therefore considered to be numerical. The citation mode switches to numerical, and \cite{able95} prints [ABC95].

See however, the end of Section 4.6 for another possibility. The above result can be achieved with

\bibitem[ABC95()]{able95}

6 Local Configuration

For \LaTeX\, it is possible to add a local configuration file natbib.cfg, which is read in, if it exists, at the end of the package. It may thus contain coding to supersede that in the package, although its main purpose is to allow the user to add his own \texttt{\bibstyle@bst} definitions to couple citation punctuation with local bibliography styles.

7 Options with \LaTeX\, \vspace{1ex}

One of the new features of \LaTeX\, is options for the packages, in the same way as main styles (now called \texttt{classes}) can take options. This package is now installed with

\documentclass[...]{...}
\usepackage[\texttt{options}]{natbib}

The options available provide another means of specifying the punctuation for citations:

round (default) for round parentheses;

square for square brackets;

curly for curly braces;
angle for angle brackets;
colon (default) to separate multiple citations with colons;
comma to use commas as separators;
author–year (default) for author–year citations;
numbers for numerical citations;
super for superscripted numerical citations, as in Nature;
sort orders multiple citations into the sequence in which they appear in the list of references;
sort&compress as sort but in addition multiple numerical citations are compressed if possible (as 3–6, 15);
longnamesfirst makes the first citation of any reference the equivalent of the starred variant (full author list) and subsequent citations normal (abbreviated list);
sectionbib redefines \thebibliography to issue \section* instead of \chapter*; valid only for classes with a \chapter command; to be used with the chapterbib package;
nonamebreak keeps all the authors’ names in a citation on one line; causes overfull hboxes but helps with some hyperref problems.

If any of the formatting options are selected, the predefined citation styles in the commands \bibstyle@bst will be no longer be effective. If either \bibpunct or \citestyle is given in the preamble, the above punctuation options will no longer hold.

8 As Module to Journal-Specific Styles

Although natbib is meant to be an all-purpose bibliographic style package, it may also be incorporated as a module to other packages for specific journals. In this case, many of the general features may be left off. This is allowed for with docstrip options that not only leave off certain codelines, but also include extra ones. So far, options exist for

subpack produces a basic version with author–year only, fixed citation punctuation, no \bibpunct nor \citestyle nor predefined styles;
subpack,egs for journals of the European Geophysical Society, in particular Nonlinear Processes in Geophysics;
subpack,agu for American Geophysical Union journals.

The subpack option must always be used with package.

Previous options jgr and grl have become obsolete due to revisions in these journals; they have been replaced by the more general agu option.
9 Reference Sheet

A summarization of the main points on using natbib can be obtained by \LaTeX{}ing the file natnotes.tex, which is extracted from the main source file natbib.dtx with the docstrip option notes. This is intended to act as a handy reference sheet.

This file should be extracted automatically by the supplied installation file, natbib.ins.

10 Options with docstrip

The source .dtx file is meant to be processed with docstrip, for which a number of options are available:

all includes all of the other interfaces;
apalike allows interpretation of minimal apalike form of \cite{bibitem};
newapa allows \citeauthor\citeyear to be in the optional argument of \cite{bibitem}
along with the punctuation commands of newapa.sty;
chicago is the same as newapa;
harvard includes interpretation of \cite{harvarditem};
astron allows \cite{astroncite} to appear in the optional argument of \cite{bibitem};
authordate adds the syntax of the \cite{citename} command.

This package file is intended to act as a module for other class files written for specific journals, in which case the flexible \cite{bibstyle}@bst commands are not wanted. Punctuation and other style features are to be rigidly fixed. These journal options are

agu for journals of the American Geophysical Union;
egs for journals of the European Geophysical Society, in particular Nonlinear Processes in Geophysics.

The remaining options are:

package to produce a .sty package file with most comments removed;

209 (together with package) for a style option file that will run under the older \LaTeX{} 2.09;

subpack (together with package) for coding that is to be included inside a larger package; even more comments are removed, as well as \LaTeX{} 2e option handling and identification; produces a basic natbib package for author–year only, fixed citation style (punctuation);

notes extracts a summary of usage to be used as a reference sheet; the resulting file is to be \LaTeX{}ed;
nopreonly allows \citereuse and \bibpunct to be called anywhere in the text; this is considered possibly useful with the chapterbib package where different chapters might have different bibliography and citation styles; is only provided in case I change my mind about this feature, but for now I refuse to implement it;

driver to produce a driver .drv file that will print out the documentation under \LaTeX\textsuperscript{2ε}. The documentation cannot be printed under \LaTeX\textsuperscript{2.09}.

The source file natbib.dtx is itself a driver file and can be processed directly by \LaTeX\textsuperscript{2ε}. 