On the $A$-dependence of nuclear generalized parton distributions

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Abstract

We perform a microscopic evaluation of nuclear generalized parton distributions (GPDs) for spin-0 nuclei in the framework of the Walecka model. We demonstrate that the meson (non-nucleon) degrees of freedom dramatically influence nuclear GPDs, which is revealed in the non-trivial and unexpected $A$-dependence of Deeply Virtual Compton Scattering (DVCS) observables. In particular, we find that the first moment of the nuclear D-term, $d_A(0) \propto A^{2.26}$, which confirms the earlier prediction of M. Polyakov. We find that in the HERMES kinematics, contrary to the free proton case, the nuclear meson degrees of freedom in large nuclei enhance the nuclear DVCS amplitude which becomes comparable to the Bethe-Heitler amplitude, and, thus, give the non-trivial $A$-dependence to the DVCS asymmetries: as a function of the atomic number the beam-charge asymmetry increases whereas the beam-spin asymmetry decreases slowly.
I. INTRODUCTION

During the last decade generalized parton distributions (GPDs) became a standard tool for the parameterization of the nonperturbative hadronic structure in hard processes. Although inaccessible directly, GPDs enable us to probe the 3-dimensional structure of the target and to study parton correlations in the target. GPDs are intensively studied both theoretically and experimentally. One of the key processes used for measurements of GPDs is Deeply Virtual Compton Scattering (DVCS).

While most of theoretical and experimental analyses of DVCS involve the nucleon target, there also exist ambitious projects to study DVCS on nuclei. Such experiments may provide us with valuable information on the nuclear forces as well as on the change of nucleon properties in the nuclear medium. A first DVCS experiment on neon nuclei was performed at HERMES (DESY). Another measurement of nuclear DVCS is planned at the future Electron Ion Collider (EIC).

By analogy with inclusive deep inelastic scattering (DIS) on nuclei one can expect that DVCS on nuclei will be sensitive to many nuclear phenomena such as shadowing, antishadowing, EMC-effect and Fermi motion. In addition, in the DVCS amplitude there maybe present other new nuclear effects which are absent in the imaginary part of the forward virtual photon-nucleon scattering amplitude probed in inclusive DIS on nuclei.

DVCS on different nuclei was a subject of investigation in. In the last two works nuclear GPDs are expressed in terms of convolution (sum) with GPDs of separate nucleons. This assumption is rather natural and is based on the well-known fact that nucleons in the nuclei are weakly bound and, thus, in hard processes one can consider nucleus as a collective of quasifree nucleons. However, as we will show, this assumption is not universal, i.e. it does not work for some of the observables.

A rather interesting property of nuclear GPDs was predicted in. Contrary to the expectations based on the quasifree nucleon model, the first moment of the D-term, which is intrinsically related to GPDs, has nonlinear A-dependence, \( d_A(0) \propto A^{26} \). Inspired by the result of, we performed a microscopic study of the nuclear GPDs in the framework of the well-established nuclear structure model developed by Walecka and collaborators. We confirmed the result of Ref. and found rapid A-dependence, \( d_A(0) \propto A^{26} \), largely due to mesonic degrees of freedom. We also found that mesons significantly enhance the DVCS amplitude compared to the nucleonic contribution. For large nuclei in the current HERMES kinematics, the DVCS amplitude squared increases as \( |A_{DVCS}|^2 \propto A^{429} \). We found that the nuclear asymmetries are very sensitive to the meson internal structure and thus in the study of the nuclear DVCS, one should pay particular attention to the meson distributions in the nuclei as well as to the quark distributions inside the mesons. We predict that, as a function of the atomic number, the beam-charge asymmetry grows as \( \propto A^{0.5} \), whereas the beam-spin asymmetry is a slowly decreasing function of the atomic number \( A: A_{LU} \propto A^{-0.03} \). In the absence of mesonic (non-nucleonic) degrees of freedom, the asymmetries are virtually independent of \( A \). Also, both asymmetries should have a maximum when the DVCS and BH amplitudes have comparable magnitudes. The position of the maximum is very sensitive to the nuclear constituents’ model. In the forward limit, our GPDs reproduce the earlier results for nuclear light-cone distributions.

The paper is organized as follows. In the Sect. we demonstrate that the first moment of the D-term evaluated in the Walecka model depends on the atomic number \( A \) as \( d_A(0) \propto A^{26} \). In Sect. we evaluate the nucleonic and mesonic off-forward light-cone distributions in nuclei. We discuss the influence of mesonic degrees of freedom on physical observables such as the ratio of nuclear-to-nucleon GPDs, beam-charge and beam-spin DVCS asymmetries in Sect. In Sect. we summarize our results and draw conclusions.
II. $d_A(0)$ AND ENERGY-MOMENTUM TENSOR

In this section we perform a microscopic calculation of the $A$-dependence of the first moment of the $D$-term, $d_A(0)$, which is intrinsically related to nuclear GPDs. In our analysis we use the connection of $d_A(0)$ to the form factors of the energy-momentum tensor introduced in [20]. As a framework we use the field-theoretical Walecka model. In this section nuclear constituents (nucleons and mesons) are treated as elementary pointlike objects. The influence of their internal structure is considered in the next sections.

The Lagrangian of the model in the simplest form is [30, 31]

$$\mathcal{L} = \bar{\psi}(i\hat{\partial} - M - g_v\hat{V} + g_s\phi)\psi + \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 \right) - \frac{1}{4} V_{\mu\nu}V^{\mu\nu} + \frac{m_V^2}{2} V^2, \tag{1}$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$; $\psi$ corresponds to the field of nucleons; the massive vector field $V$ and scalar field $\phi$ are effective fields, which represent the empirically observed dominant vector and scalar components of the $NN$ interaction. The relativistic MFT models based on Lagrangians of type (1) succeeded in description of such important characteristics as nuclear densities, the level structure of the nuclear shell model, the spin dependence of nucleon-nucleus scattering etc. The simplest version of the model used in the work consists of baryons and isoscalar scalar and vector mesons. A virtue of such a simple model is that it has only two independent parameters (coupling constants) which can be fixed from the infinite nuclear matter properties. After this, all the model predictions for finite nuclei become unambiguous. Extensions of the model with more independent degrees of freedom (and consequently more free parameters) can give better quantitative description of nucleus [53]. Note that we neglect the pseudoscalar pion degrees of freedom since the effects of pions in the ground-state of spin-zero nuclei essentially average to zero due to the spin-dependence of the pion-nucleon coupling constant. In addition to the pions, in order to achieve a truly quantitative description of nuclear properties, one should ultimately include the rho-meson and electromagnetic fields. However, those will lead to small corrections, which we neglect in our exploratory study. The values of the coupling constants are fixed from phenomenological parameters of the nuclear matter [34]. Numerically the constants are large. Therefore, the perturbative methods cannot be used. Instead, observing that for sufficiently large nuclei, the nuclear density becomes large, one can use the mean-field approximation, when the quantum meson fields are replaced by their classical ground-state expectation values. The model with Lagrangian (1) is not renormalizable and should be understood as an effective one.

Solving classical equations of motion in the nuclear rest frame [55] and making Lorentz boosts to the infinite momentum frame, we can obtain the light-cone wave functions necessary for evaluation of light-cone matrix elements.

As it has been shown in [2], one can parameterize the matrix element of the energy-momentum tensor in terms of two form factors $M_A(t)$ and $d_A(t)$:

$$\langle P' | \hat{T}_{\mu\nu}(0) | P \rangle = M_A(t) \tilde{P}_\mu \tilde{P}_\nu + \frac{1}{5} d_A(t) (\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2), \tag{2}$$

where $\tilde{P} = (P + P')/2$, $\Delta = P' - P$.

Acting with the traceless operator

$$\left( \frac{\partial}{\partial \Delta^i} \frac{\partial}{\partial \Delta^j} - \frac{\delta_{ij}}{3} \frac{\partial}{\partial \Delta^k} \frac{\partial}{\partial \Delta^k} \right) |_{\Delta=0, P=\text{const}}$$

on Eq. (2), one can express $d_A(0)$ in terms of the matrix element of the energy-momentum tensor $\tilde{T}_{\mu\nu}$ as [20]

$$d_A(0) = -\frac{m_A}{2} \int d^3r \left( r_i r_j - \frac{\delta_{ij}}{3} r^2 \right) T_{ij}(r; 0), \tag{3}$$
where \( m_A \) is the mass of the nucleus; \( T_{ij}(\vec{r}; \Delta) \) is a shorthand notation for the Fourier of the matrix element

\[
\frac{1}{2E} \int \frac{d^3\Delta}{(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \langle P + \Delta/2 | \hat{T}_{ij}(0) | P - \Delta/2 \rangle
\]

between the nuclear ground-states. One can split the energy-momentum tensor, which can be derived from the Lagrangian \( \mathcal{L} \), into three parts

\[
\hat{T}_{\mu\nu} = \left( -V^\lambda V_{\lambda\nu} + \frac{g_{\mu\nu}}{2} V_{\lambda\nu} V^\lambda - m_N^2 V_{\mu\nu} - g_{\mu\nu} m_N^2 V^\rho V_\rho \right) + \left( \partial_\mu \phi \partial_\nu \phi - \frac{g_{\mu\nu}}{2} (\partial_\rho \phi \partial_\rho \phi - m_N^2 \phi^2) \right)
\]

\[
+ \left( \frac{i}{2} (\bar{\psi}(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi) - g_V V_\mu \bar{\psi} \gamma_\nu \psi \right),
\]

which will be referred to as \( \hat{T}^{V}_{\mu\nu}, \hat{T}^{\phi}_{\mu\nu} \) and \( \hat{T}^{N}_{\mu\nu} \), respectively. In complete analogy with \( \mathcal{L} \) one can define the form factors \( M_i/A(t), d_i/A(t) \) for each of the three contributions \( \hat{T}^i_{\mu\nu} \) to the total \( \hat{T}_{\mu\nu} \). The form factors \( M_i/A(t), d_i/A(t) \) are connected with \( M_A(t), d_A(t) \) as

\[
M_A(t) = \sum_i M_{i/A}(t) = M_{N/A}(t) + M_{V/A}(t) + M_{\phi/A}(t),
\]

\[
d_A(t) = \sum_i d_{i/A}(t) = d_{N/A}(t) + d_{V/A}(t) + d_{\phi/A}(t).
\]

In Sect. \( \text{III} \) we shall show that the form factors \( M_i/A(t), d_i/A(t) \) are closely related to the moments of GPDs of nucleons and mesons. Assuming spherical symmetry of the considered nuclei, the final answer for \( d_{i/A}(0) \) is

\[
d_{N/A}(0) = -\frac{m_A}{2} \int d^3r \left( r_i r_j - \frac{\delta_{ij}}{3} \right) \sum_n \Phi_n(\vec{r}) \gamma_i \gamma_j \Phi_n(\vec{r}) - \frac{g_V}{2m_A} \int d^3r \rho_B(r) V_0(r),
\]

\[
d_{\phi/A}(0) = -\frac{m_A}{3} \int d^3r r^2 \phi'(r)^2,
\]

\[
d_{V/A}(0) = \frac{m_A}{2} \int d^3r r^2 V_0'(r)^2,
\]

(7)

where the sum over \( n \) in the first equation goes over the occupied levels; \( \Phi_n(\vec{r}), \phi(\vec{r}), V_0(\vec{r}) \) are the (classical) self-consistent solutions of the Hartree-Fock equations; \( \rho_B(\vec{r}) = \sum_n \Phi_n^\dagger(\vec{r}) \Phi_n(\vec{r}) \) is the baryon density.

The numerical evaluation of \( d_A(0) \) for different nuclei according to Eq. \( \text{III} \) gives the results presented in Table \( \text{III} \) and Fig. \( \text{III} \). The most interesting feature of the obtained result is that \( d_A(0) \) receives the dominant contribution from the mesons. Notice that \( d_A^{\text{mes}}(0) = d_{\phi/A}(0) + d_{V/A}(0) \) and \( d_A(0) \) for all nuclei lie on the straight lines when plotted in the logarithmic coordinates. The only exception is \( d_A(0) \) for \( ^{12}\text{C} \), where we observe compensation of the nucleon and meson contributions. The least-square fit to the values presented in Table \( \text{III} \) gives

\[
d_A(0) \approx -0.308 \, A^{2.26}.
\]

(8)

Note that the obtained parameters contain uncertainties due to the numerical nature of our analysis and the fitting method. One can see that this result agrees with the estimate based on the liquid drop model \( 20 \)

\[
d_A(0) \propto -0.2 \, A^{7/3} \left( 1 + \frac{3.8}{A^{2/3}} \right),
\]

(9)
TABLE I: Values of $d_A(0)$ for different nuclei. Contribution of mesons $d_A^{\text{mes}}(0) = d_{\phi/A}(0) + d_{V/A}(0)$ and total result $d_A(0)$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$d_A^{\text{mes}}(0)$</th>
<th>$d_A(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>-100.9</td>
<td>-7.77</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>-189.9</td>
<td>-143.8</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>-1622.5</td>
<td>-1525</td>
</tr>
<tr>
<td>$^{90}$Zr</td>
<td>-11270.2</td>
<td>-8258</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>-64187.3</td>
<td>-49195</td>
</tr>
</tbody>
</table>

where the last $O(1/A^{2/3})$-term takes into account finite width of the nuclear border. Thus the simple power dependence $d_A(0) \sim A^n$ exists only for relatively large-$A$ nuclei. In our analysis we cannot separate contributions of different parts of the nucleus. This explains why the point $^{12}$C on the Fig. 1 does not lie on the straight line.

The $A$-dependence of the $D$-term was also examined in [36], where it was found that

$$d_A(0) \propto A \left(1 + O(\ln A)\right).$$

(10)

As it is discussed in Sect. III, the result of [36], which is inconsistent with [20] and the result of our work, is due to the nonrelativistic approximation used by the authors.

It was emphasized in [20] that the nuclear surface responsible for stability of the liquid drop gives the dominant negative contribution to $d_A(0)$ with rapid $A$-dependence. In the Walecka model mesons compensate the positive contribution of the nucleons to the total pressure and, thus, provide stability of the nucleus. A similar phenomenon happens in the calculation of the nucleon $D$-term in the chiral quark soliton model, where the dominant negative contribution does also come not from the valent quarks but from the Dirac sea [37]. Thus in all three models we obtain a negative value of $d_A(0)$ which comes from "complementary" degrees of freedom.

To measure $D$-term, one should study hard processes sensitive to the real part of the DVCS amplitude, such as the beam-charge asymmetry [38] or the DVCS total cross section in properly chosen kinematics, where the BH amplitude is suppressed [39]. We address this issue in Sect. IV and show that the $A$-dependence of the GPDs is observable in DVCS asymmetries.

III. EVALUATION OF NUCLEAR GPDS

In dealing with hard processes on nuclear targets one often assumes that the hard scattering occurs on quasifree nuclear constituents. For the weakly-bound system it also seems quite natural to assume that one can ignore possible "distortion" of the constituents in the nuclear medium and neglect offshellness of the nucleons and mesons. While this assumption is commonly believed to be justified for the nucleons, we cannot estimate its accuracy for the meson degrees of freedom. However, since the meson GPDs are unknown even for the onshell case, it should be a sufficient first approximation to use the parametrization which only satisfies general GPDs properties. A straightforward application of these ideas to the DVCS
process results in the convolution formula (we consider only the quark nuclear GPDs)

\[ H_q/A(x, \xi, t) = \sum_i \int_1^x \frac{dy}{y} H_{i/A}(y, \xi, t) H_{q/i} \left( \frac{x}{y}, \frac{\xi}{y}, t \right), \tag{11} \]

where \( i \) labels nuclear constituents (\( \psi, V \) and \( \phi \) in case of the Walecka model); \( H_{i/A} \) describe the distribution of the constituents in the nucleus; \( H_{q/i} \) are the GPDs of the free constituents. (For a more detailed discussion, see e.g. [40, 41] and references therein. A generalization to the off-forward case and non-nucleonic constituents is straightforward.) Equation (11) complies with the intuitive picture of the nuclear hard scattering as a two-step process depicted in Fig. 2: the hard scattering amplitude on the nuclear target equals the hard amplitude on the free constituent convoluted with the off-forward distribution of the constituent in the nuclear target. Note that the convolution approximation of Eq. (11) neglects the simultaneous coherent interaction of the virtual photon with several nuclear constituents. Therefore, strictly speaking, Eq. (11) is applicable only for \( x > 0.1 \).

Assuming that the polynomiality conditions

\[ \int_{-1}^1 x^n dx H_{i/A}(x, \xi, t) = \sum_{k=0,2,...}^{[n]} B_n^k(t) \xi^k, \]

FIG. 1: \( d_A(0) \) as a function of \( A \). We can see that \( d_A(0) \) is mostly due to the meson contribution.
fulfil separately for the constituents’ GPDs in the nuclear medium and off-forward nuclear distributions, from Eq. (11) one can obtain that the polynomiality condition for the total GPD reads

$$\int_{-1}^{1} x^n \, dx \, H_q(x, \xi, t) = \sum_{k=0,2,\ldots}^{[n]} C_n^k(t) \xi^k,$$

(12)

where

$$D_n^k = \sum_{l=0}^{[n]} C_n^k B_n^{k-l}.$$  

(14)

For the case of the zero-order moment, summation with the quark charges $e_q$ of Eq. (13) gives the nuclear electromagnetic form factor

$$F_{em}(t) = \sum_q e_q F_{N/A}(t) F_{q/N}(t) = F_{N/A}(t) F_N(t),$$

(15)

where $F_N(t)$ is the nucleon electromagnetic form factor. Since mesons are not charged, they do not contribute to the nuclear electromagnetic form factor. Mathematically it follows from the antisymmetry of the mesonic off-forward distributions with respect to the variable $x$, as it will be shown below.

From Eq. (14), one obtains the first moment

$$\int_{-1}^{1} x \, dx \, H_q(x, \xi, t) = M_q/A(t) + \frac{4}{9} \xi^2 d_q/A(t),$$

(16)

where

$$M_q/A(t) = \sum_i M_{qj}(t) M_{ji}/A(t),$$
\[ d_{q/A}(t) = \sum_i \left( M_{q/i}(t)d_{i/A}(t) + d_{q/i}(t) \int_{-1}^{1} \frac{dy}{y} H_{i/A}(y, \xi, t) \right), \]  

(17)

and we introduced the conventional notation \( M_i(t), d_i(t) \) for the form factors \( B_1(t), C_1(t) \). These form factors already appeared in the Lorentz-covariant expansion of the energy-momentum tensor in the previous section. Note that in our numerical analysis, we use a simple parameterization of the meson GPDs, see Eqs. (24) and (26), which corresponds to the vanishing \( d_{q/\phi} \) and \( d_{q/V} \).

Contrary to naive expectations, as it follows from Eq. (17), the total \( d_A(t) \) is not reducible to a mere sum of the free constituents’ \( d_{i/A}(t) \)-terms but consists of two parts. The first term in (17) is a "collective" effect: in the previous section it was shown that \( d_{i/A}(0) \propto A^{2.26} \). To evaluate the last term in (17) one can use the approximation \( H_{N/A}(y, 0, 0) \approx A \delta \left( y - \frac{1}{A} \right) \) which corresponds to an ensemble of quasifree nucleons and is justified for a weakly bound nucleus. Then the last term in (17) is \( \propto A^2 \).

A direct evaluation with the nucleon off-forward distribution obtained in the Walecka model gives the same \( A \)-dependence. Thus the predicted \( A \)-dependence for the total \( d_A(0) \) is due to the first term in Eq. (17), which dominates in the large-\( A \) limit and defines asymptotics, whereas the second term is a \( O(A^{-0.26}) \)-correction. Notice also that the essential contribution to the coefficient in front of \( A^{2.26} \) comes from the mesonic degrees of freedom.

Now we evaluate the nuclear part of the DVCS amplitude \( H_{i/A}(x, \xi, t) \). In complete analogy with Ref. [7] one can define off-forward distributions of nucleons and mesons in the nucleus:

\[ H_{N/A}(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P' | \bar{\psi} \left( \frac{z}{2} \right) e^{ix/2} V(\lambda) - d \lambda \gamma_+ \bar{\psi} \left( -\frac{z}{2} \right) | P \rangle, \]

\[ H_{\phi/A}(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P' | \phi \left( \frac{z}{2} \right) | P \rangle, \]

\[ H_{V/A}(x, \xi, t) = \frac{1}{4xP^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P' | V_+ \left( \frac{z}{2} \right) V_+ \left( -\frac{z}{2} \right) | P \rangle, \]

(18)

The definitions of the nucleon and meson GPDs in Eq. (20) are chosen such that there is one-to-one correspondence to the ++ component of the energy momentum tensor of the model, see Eq. 5. This automatically guarantees conservation of the momentum sum rule

\[ \sum_i \int_{-1}^{1} dx \ x \ H_{i/A}(x, 0, 0) = 1. \]  

(19)

Notice that the large-\( A \) nucleus in the Mean Field Theory approach in several respects resembles the nucleon in the large-\( N_c \) chiral quark soliton model. A large number of particles in both models enables us to represent the interaction of a single particle with the ensemble of other particles as the interaction with the central potential. Forward distributions of nucleons (quarks) in both cases are strongly peaked at \( 1/A (1/N_c) \). So we expect that the off-forward distributions should have similar qualitative features.

Using standard steps [11, 22], see also the Appendix for more details, we can obtain from Eq. (18)

\[ H_{N/A}(x, \xi, t) = \frac{m_A}{2\pi} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \int d^3X \sum_n \Phi_n \left( \frac{z}{2} - \vec{X} \right) P e^{i \int_{-1/2}^{1/2} V(\lambda) - d \lambda \gamma_+ \Phi_n \left( -\frac{z}{2} - \vec{X} \right)}. \]  

(20)

To simplify evaluations, numerically it seems reasonable to use the approximation \( V_\mu(x) \approx \delta_{\mu0} \nu \) in the path exponent since actually we have "cutoff" due to the wave functions \( \Phi_n(r) \). We define the average
value $\bar{V}$ as

$$\bar{V} \simeq \frac{1}{A} \int d^3 X V_+(X) \rho_B(X),$$  \hspace{1cm} (21)

which ensures that the approximate and exact formulas give the same results for the first moment of $H_{N/A}$ (conservation of the nuclear momentum sum rule). This approximation implies that the Wilson link just results in a shift of the whole parton distribution by the value $\delta x \propto \bar{V}$. The evaluation with the exact expression shows that Wilson links results not only in the shift but also in the broadening of the nucleon distribution. However such ”broadening” is negligible, especially for large nuclei. The expressions for off-forward distributions look more compact in the momentum representation:

$$H_{N/A}(x, \xi, t) =$$

$$\int \frac{d\bar{p}^+ d^2\bar{p}_\perp}{(2\pi)^3} \delta \left( x + 8x - \frac{\bar{p}^+}{P^+} \right) \sum_n \Phi_n \left( (x - \xi), \bar{p}_\perp + \frac{\Delta_\perp}{2} \right) \gamma_+ \Phi_n \left( (x + \xi), \bar{p}_\perp - \frac{\Delta_\perp}{2} \right),$$

$$H_{\phi/A}(x, \xi, t) =$$

$$\int \frac{d\bar{p}^+ d^2\bar{p}_\perp}{(2\pi)^3} \delta \left( x - \bar{p}^+/P^+ \right) \bar{p}^+ \phi \left( (x - \xi), \bar{p}_\perp + \frac{\Delta_\perp}{2} \right) \phi \left( (x + \xi), \bar{p}_\perp - \frac{\Delta_\perp}{2} \right),$$

$$H_{V/A}(x, \xi, t) =$$

$$\int \frac{d\bar{p}^+ d^2\bar{p}_\perp}{(2\pi)^3} \delta \left( x - \bar{p}^+/P^+ \right) \bar{p}^2 \phi \left( (x - \xi), \bar{p}_\perp + \frac{\Delta_\perp}{2} \right) \phi \left( (x + \xi), \bar{p}_\perp - \frac{\Delta_\perp}{2} \right).$$  \hspace{1cm} (22)

We can see that similarly to the gluonic GPD, the vector off-forward distribution is singular at $x = 0$. Using explicit expressions for the off-forward distributions, one can check that they fulfil required sum rules. For instance, the momentum sum rule reads:

$$\int x dx H_{N/A}(x, 0, 0) + \int x dx H_{\phi/A}(x, 0, 0) + \int x dx H_{V/A}(x, 0, 0) =$$

$$= \frac{1}{P^+} \int d^3 X \left( -V_+(X)V_A(X) + m_v^2 V_+(X)V_+(X) + \partial_+ \phi(X) \partial_+ \phi(X) \right)$$

$$+ \bar{\psi}(X) \gamma_+ i \partial_+ \psi(X) - g_V \bar{\psi}(X) \gamma_+ V_+(X) \psi(X) \right) = \frac{\int d^3 X \delta^{++}}{P^+} = 1, \hspace{1cm} (23)$$

where $\delta^{++}$ is the $++$-component of the energy-momentum tensor and the integral is performed over the hypersurface $X_+ = \text{const}$. One can easily check that the coefficients in front of $\xi^2$ in the first moments $\int x dx H_{i/A}(x, \xi, t)$ of the off-forward distributions coincide with the results obtained in the previous section from the energy-momentum tensor form factors, see Eq. 7.

Plots of the off-forward distributions $H_{i/A}$ as functions of the variable $Ax$ at fixed values of the other parameters are given in Fig. 8. We can see that all the off-forward distributions have a very strong $t$-dependence, which is similar to the $t$-dependence of the nuclear form factor $F_{N/A}(t)$. The nucleonic off-forward distribution (the leftmost panel of Fig. 8) has a pronounced maximum at the point $Ax \approx 1$.

IV. PREDICTIONS FOR PHYSICAL OBSERVABLES

To make predictions for physical observables, we should use a model describing the internal structure of the constituents, which is parametrized by $H_{q/A}(x, \xi, t)$ in Eq. 10. While the resulting GPDs
FIG. 3: Off-forward distributions of nuclear constituents for $^{40}$Ca at fixed $\xi = \xi_{HERMES} \approx 0.045$ and different $t$; $t_{\text{min}} = -4\xi^2m_A^2/(1 - \xi^2)$.

$H^{q/A}(x, \xi, t)$ as well as all the quantities including them (cross sections, asymmetries etc.) are very sensitive to the details of the model used to parameterize $H_{q/i}(x, \xi, t)$, we expect that the ratios of the nuclear-to-nucleon quantities should be less model dependent. Since modelling of the nucleon GPDs (especially in external meson fields) is beyond the scope of the current work, we use the simplest known parameterization - the double distributions [5] supplemented with the $D$-term [38].

$$H_N(x, \xi, t) \equiv \sum_q e_q^2 H_{q/N}(x) = F_N(t) \int_{-1}^{1-|\beta|} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) h(\beta, \alpha) q_N(\beta) + \theta \left(1 - \frac{x^2}{\xi^2}\right) D_N \left(\frac{x}{\xi}, t\right),$$

where $q_N(x) \equiv \sum_q e_q^2 q_{q/N}(x)$ is a standard flavour combination measured in DIS; $h(\beta, \alpha) = 3\sqrt{2} \frac{(1 - |\beta|^2)^{1/2}}{(1 - |\beta|^2)^{3/2}}$; $D_N(z, t) = -\sum_q e_q^2/N_f 4 (1 - z^2) C_i^{3/2}(z)$ [38] (we used $N_f = 2$ and neglected the $D^u - D^d$ difference, which is formally suppressed by the $1/N_c$ factor). For the parton distributions $q_{q/N}(x)$ we used the CTEQ5L parameterization [43].

Since to the best of our knowledge, the fit for quark distributions in mesons was done only for pions [14], but not for mesons we need, we used a simple model for the mesonic GPDs

$$H_V(x, \xi, t) = H_\Phi(x, \xi, t) = H_{\text{mes}}(x, \xi, t) \equiv \sum_q e_q^2 H_{q/\text{mes}}(x, \xi, t) = \frac{5}{18} \left(\delta \left(x - \frac{1}{2}\right) - \delta \left(x + \frac{1}{2}\right)\right).$$

This model corresponds to a GPD of the weakly bound quark-antiquark pair. We used $N_f = 2$ and the same parameterization for both flavours since the considered mesons have isospin zero. The parameterization [24] has correct symmetry properties, which can be derived from the $C$-parity, and satisfies the momentum sum rule

$$\int_{-1}^{1} \sum_q x dx H_{q/\text{mes}}(x, 0, 0) = 1.$$
We have neglected possible $\xi$-dependence for the meson GPDs $H_{\text{mes}}(x, \xi, t)$ since in the kinematics of nuclear DVCS $\xi \ll 1$, $t/\langle r^2_{\text{mes}} \rangle \ll 1$, and give only small corrections compared to the forward case. Notice that this is not true for the nuclear form factors since the radii of the considered nuclei are $\langle r^2 \rangle^{1/2} \sim 3 \div 5$ fm and, thus, values $t \sim 0.1 \text{GeV}^2$ are not small and the $t$-dependence cannot be neglected. For the sake of comparison we have also investigated how our predictions change if for $H_V$ and $H_\phi$, we use a different model,

$$H_{q/mes}(x, \xi, t) = q_\pi(x) - q_\pi(-x),$$

where $q_\pi$ are the pion PDFs parameterized by Owens [44].

The first quantity that we consider is the ratio

$$R(x, \xi, t) = \frac{\sum_q e_q^2 H_q/A(x_A, \xi, t)}{F_{N/A}(t) H_N(x, \xi, t)},$$

which in the forward limit reduces to the ratio of the structure functions (to the leading order in $\alpha_S$)

$$R(x) = \frac{F_2 A(x_A)}{A F_2 N(x)};$$

where $x = A x_A$. The dependence of the function $R(x, \xi, t)$ on $x$ at fixed $x_{Bj} = 0.09$ (HERMES) and different values of $t$ is given in Fig. 4. For comparison we also present the SLAC [45] and NMC [46] data for the ratio $R(x)$. Notice that the meson contribution enhances $R(x, \xi, t)$ at small $x$ above the pure nucleonic contribution previously obtained in [32, 33]. It is important to emphasize that the convolution

FIG. 4: Typical behaviour of the function $R(x, \xi, t)$ (off-forward EMC effect) for $^{40}$Ca and different values of $t$. The left panel corresponds to the parameterization (24), the right panel corresponds to the parameterization (26). Experimental data are from SLAC [45] and NMC [46] for the forward case.
approximation, which implies the interaction with the nuclear constituents one at a time, ignores the physics of the simultaneous coherent interaction with several constituents, which becomes important for $x < 0.1$, see e.g. [47]. This explains the fact that our calculation gives $R(x) > 1$ for $x < 0.1$, in dramatic contrast to the data. While our calculations provide a fairly good description of $R(x)$ for $x > 0.1$, our model predicts a significant enhancement of the nuclear anti-quark distributions, which contradicts the experiment [48]. This problem is typical for any model of nuclear structure, which involves a significant fraction of non-nucleonic meson degrees of freedom [19].

The difference between $H^{q/A}(x,\xi,t)$ and $H^{q/N}(x,\xi,t)$ can be also probed through the DVCS beam-charge and beam-spin asymmetries

$$A_C(\phi) = \frac{\sigma^+(\phi) - \sigma^-(\phi)}{\sigma^+(\phi) + \sigma^-(\phi)}, \quad A_{LU}(\phi) = \frac{\overline{\sigma}(\phi) - \sigma(\phi)}{\overline{\sigma}(\phi) + \sigma(\phi)},$$

(29)

where $\phi$ is the angle between the lepton and hadron scattering planes; $\sigma^\pm$ and $\overline{\sigma}$ denote cross-sections of unpolarized electron/positron and longitudinally polarized leptons, respectively [10]. The asymmetries $A_C(\phi), A_{LU}(\phi)$ directly measure interference between Bethe-Heitler and DVCS amplitudes. We perform the evaluation of the beam-charge and beam-spin asymmetries in the kinematics similar to that used at HERMES [19]: $\langle x_B \rangle_{\text{per nucleon}} = 0.09, (Q^2) = 2.2 \text{ GeV}^2, (t) = -0.01 \text{ GeV}^2$. In this kinematics due to the small value of the prefactor $-t/4m_N^2$, one can safely ignore the contributions of the GPD $E(x,\xi,t)$ and magnetic form factor $F_2(t)$ compared to those of $H(x,\xi,t)$ and $F_1(t)$, see Ref. [11] for more details and a complete set of formulas describing DVCS process both for unpolarized and polarized cases.

In HERMES kinematics, $1/Q$ is a modest expansion parameter for the nuclear hard scattering and one should check if the expansion really works. For instance, due to mesons the nuclear DVCS amplitude squared is \textit{NOT} negligible compared to the Bethe-Heitler amplitude squared (in the denominator of the expression for the DVCS asymmetries), though numerically it is suppressed by the factor $t/Q^2$.

Using our results for nuclear off-forward distributions, we calculate the dominant (leading-twist) harmonics of the beam-charge and beam-spin harmonics,

$$A_C^{\cos} = \frac{1}{\pi} \int_0^{2\pi} d\phi \cos \phi A_C(\phi); \quad A_{LU}^{\sin} = \frac{1}{\pi} \int_0^{2\pi} d\phi \sin \phi A_{LU}(\phi).$$

(30)

In order to study the role of the mesons, we give the answer for the full calculation and for the calculation, where the contribution of the $\phi$ and $V$ mesons was neglected. The results are summarized in Table 11 in terms of the ratios of the nuclear to the free proton asymmetries. The second and third columns correspond to the calculation without the nuclear mesons; the fourth and fifth columns correspond to the full calculation with the meson GPDs parameterized by Eq. (24); the sixth and seventh columns correspond to the full calculation with the meson GPDs parameterized by Eq. 26.

One can see from Table 11 that, in the absence of the meson contributions, both asymmetries are practically independent of the atomic number. In this case the DVCS amplitude squared is small compared to the BH amplitude squared for all nuclei.

In the presence of mesons (the full calculation) we can see that beam-charge asymmetry is a \textit{growing} function of the atomic number whereas the beam-spin asymmetry is weakly-dependent of the atomic number; the DVCS amplitude \textit{increases}. A least-square fit gives the following approximate $A$-dependence: $A_C^{\cos} A_N^{\cos} \propto A^{0.5}$, $A_{LU}^{\sin} A_N^{\sin} \propto A^{-0.03}$ for all $A$; the ratio of the DVCS amplitudes squared $|A_{DVCS} A_N A_{DVCS} N|^2 \propto A^{1.29}$. The natural explanation of such a behaviour is that the asymmetries are large when both DVCS and BH amplitudes have comparable magnitudes and decrease when one of the amplitudes essentially overcomes the other. Thus in the considered kinematics the asymmetries as functions of the atomic number $A$ should be small for small $A$ (when the Bethe-Heitler amplitude dominates); they should raise when DVCS amplitude becomes comparable with the Bethe-Heitler amplitude.
As a consequence, the asymmetries should have their maxima. However the position of the maxima is strongly model dependent: the model (24) predicts the maximum at moderate values of $A \propto 40 - 50$ for the beam-charge asymmetry, whereas the model (26) predicts it for $A > 208$ (i.e. that for all nuclei the $A$-dependence of the beam-charge asymmetry is homogeneous).

In Fig. 5 we present the $t$-dependence of the ratio of the nuclear to the free proton beam-charge and beam-spin asymmetries at fixed $\xi \approx 0.045$ (per nucleon). From Fig. 5 one can see that the qualitative $t$-dependence of both asymmetries does also change in the presence of mesons.

It is interesting to extrapolate our results to the point $t = 0$ to eliminate the "pure kinematical" $A$-dependence coming from the difference of the $\langle r^2 \rangle$ in the nuclear formfactor $F_A(t) = Z e^{-\langle r^2 \rangle |t|/6}$. The fit to data gives $|A_{DVCS_A}/A_{DVCS_N}|^2 \propto A^{4.57 \pm 0.17}$, which is remarkably close to the value $d_A^2(0) \propto A^{4.52}$ obtained in Section III. In the asymmetries nuclear formfactors contract and we get for $t = 0$ approximately the same power as for finite $t$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$A_{C,\text{spin}}^{\text{cos}}/A_{C,\text{N}}^{\text{cos}}$</th>
<th>$A_{C,\text{spin}}^{\text{sin}}/A_{C,\text{N}}^{\text{sin}}$</th>
<th>$A_{U,\text{spin}}^{\text{cos}}/A_{C,\text{N}}^{\text{cos}}$</th>
<th>$A_{U,\text{spin}}^{\text{sin}}/A_{C,\text{N}}^{\text{sin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>2.45</td>
<td>1.85</td>
<td>4.61</td>
<td>2.49</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>2.43</td>
<td>1.83</td>
<td>5.41</td>
<td>2.33</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>2.38</td>
<td>1.79</td>
<td>7.34</td>
<td>1.60</td>
</tr>
<tr>
<td>$^{90}$Zr</td>
<td>2.59</td>
<td>1.93</td>
<td>6.80</td>
<td>0.81</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>2.42</td>
<td>1.07</td>
<td>6.12</td>
<td>0.31</td>
</tr>
</tbody>
</table>

TABLE II: The ratios of the nuclear to the free proton asymmetries for different nuclei. The second and third columns correspond to the calculation without the nuclear mesons; the fourth and fifth columns correspond to the full calculation including the meson contribution with the internal structure model given by (24); the sixth and the seventh columns correspond to the parameterization (26).

It is interesting to compare our predictions for the DVCS asymmetries to the HERMES preliminary result on Neon [19]. Taking the ratio of the nuclear to the proton single-spin asymmetries measured by HERMES, one readily finds

$$ \frac{A_{LU,\text{spin}}^{\text{sin}}}{A_{LU,\text{spin}}^{\text{cos}}} = 1.22 \pm 0.26, $$

where we have added the proton and neon experimental errors in quadrature. In the similar kinematics, the linear interpolation of our results from Table III gives

$$ \frac{A_{LU,\text{spin}}^{\text{sin}}}{A_{LU,\text{spin}}^{\text{cos}}} \approx 2.1 \pm 0.2. $$

A possible explanation of the discrepancy between the experimental result and the theoretical prediction (a simple combinatoric counting also suggests the enhancement) was suggested in [27]. The explanation consists in the observation that, depending on $t$ and the experimental resolution, the contribution of incoherent nuclear scattering might significantly reduce the nuclear DVCS asymmetries.

V. CONCLUSIONS

We performed a microscopic evaluation of nuclear GPDs for spin-0 nuclei in the framework of the Walecka model. We found that the meson (non-nucleonic) degrees of freedom of the Walecka model play
a dramatic role in nuclear GPDs.

We observed a non-trivial $A$-dependence of the first moment of the nuclear $D$-term, $d_A(0) \propto A^{2.26}$, which confirmed the prediction of M. Polyakov made in the framework of the nuclear liquid drop model [20]. This result demonstrated that contrary to the assumptions of the random phase approximation, mesonic degrees of freedom dominate $d_A(0)$ and cannot be neglected.

Using the resulting nuclear GPDs, we studied the $A$ and $t$-dependence of the beam-charge and beam-spin DVCS asymmetries in the HERMES kinematics. We found that due to the mesonic degrees of freedom, the nuclear DVCS amplitude squared has a rapid $A$-dependence, $|A_{DVCS}|^2 \propto A^{4.29}$, and becomes much larger than the Bethe-Heitler amplitude squared. This dramatically affects the $A$-dependence of the asymmetries. We found that $A_{C_{A}/C_{N}}^{cos} A_{C_{A}/C_{N}}^{sin} \propto A^{0.5}$, $A_{L_{A}/L_{N}}^{sin} A_{L_{A}/L_{N}}^{sin} \propto A^{-0.03}$ for all $A$. This behaviour should be compared to the case, when the meson degrees of freedom are neglected. In this case, the DVCS asymmetries are virtually $A$-independent.

FIG. 5: The ratio of the nuclear to the free proton beam-charge and beam-spin asymmetries as functions of the momentum transfer squared $t$ at fixed $\xi \approx 0.045$ (per nucleon) for the $^{12}$C nucleus. The results are presented for the full calculation with the parameterization [26] (solid curves) and for the calculation ignoring the meson contribution (dashed curves).
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APPENDIX: DERIVATION OF THE GPDS

In this section we establish the formulas used for evaluation of GPDs in the functional integrals approach [50]. The generalized parton distributions are defined as

\[ H_\psi(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP_+z^-} \langle P' \bar{\psi}(\frac{-z}{2}n_-) \gamma_+ \psi(\frac{z}{2}n_-) | P \rangle, \]

\[ H_\phi(x, \xi, t) = \frac{1}{2xP_+} \int \frac{dz^-}{2\pi} e^{ixP_+z^-} \langle P' | \partial_+ \phi(\frac{-z}{2}n_-) \partial_+ \phi(\frac{z}{2}n_-) | P \rangle, \]

\[ H_V(x, \xi, t) = \frac{1}{2xP_+} \int \frac{dz^-}{2\pi} e^{ixP_+z^-} \langle P' | V_+ \phi(\frac{-z}{2}n_-) V_+ \phi(\frac{z}{2}n_-) | P \rangle + \frac{m_V^2}{2xP_+} \int \frac{dz^-}{2\pi} e^{ixP_+z^-} \langle P' | V_+ \phi(\frac{-z}{2}n_-) V_+ \phi(\frac{z}{2}n_-) | P \rangle. \]

(A.1)

For the sake of brevity we denote \( \Phi(x) = \{ \phi(x), V(x), \bar{\Psi}(x), \Psi(x) \} \) and notice that in all three cases we have to evaluate the general matrix element

\[ \langle P' | \Phi \left( \frac{z}{2} \right) \Phi \left( -\frac{z}{2} \right) | P \rangle = \frac{\int D\phi(x) D\bar{\Psi}(x) D\Psi(x) D\bar{\Psi}(x) D\Psi(x) \Phi \left( \frac{z}{2} \right) \Phi \left( -\frac{z}{2} \right) e^{iS}}{\int D\phi(x) D\bar{\Psi}(x) D\Psi(x) D\bar{\Psi}(x) D\Psi(x) e^{iS}}, \]

(A.2)

which, up to inessential kinematical factors and integration over the light-cone separation \( z \), will give us the GPDs (A.1).

By definition we should integrate in (A.2) over all configurations which satisfy the asymptotic conditions [50]

\[ \lim_{t \to \pm \infty} \Phi(\vec{x}, t) \approx \Phi^{P', P}(\vec{x}, t), \]

(A.3)

or in more explicit form

\[ \lim_{t_1 \to +\infty} \Phi(\vec{x}, t) \approx \Phi_s(\vec{x} - \vec{V}t_1), \quad \lim_{t_2 \to -\infty} \Phi(\vec{x}, t) \approx \Phi_s(\vec{x} - \vec{V}t_2). \]

(A.4)

We reparameterize the field configurations as

\[ \Phi(x) \to \Phi_s(\vec{x} - \vec{X}(t)) + \delta \Phi(\vec{x} - \vec{X}(t)), \]

\[ \int D\Phi \to \int D\delta \Phi D\vec{X}(t), \]

(A.5)

where the integral over the zero mode - position of the soliton \( \vec{X}(t) \) - should be taken exactly, while the integral over the shifts \( \delta \Phi(x) \) may be evaluated in the saddle-point approximation. Notice that the
standard mean field approximation ignores all the loop corrections and thus implicitly uses the smallness of the coupling constant $g$. In this case the evaluation of a simple path integral for the center of mass motion gives

$$\int D\Phi(t) \Phi \left( \frac{z}{2} \right) \Phi \left( -\frac{z}{2} \right) e^{is} \approx e^{is_{cl}} \int d\vec{X}_1 \int d\vec{X}_2 \Phi \left( \frac{z}{2} - \vec{X}_1 \right) \Phi \left( -\frac{z}{2} - \vec{X}_2 \right) \times$$

$$\times \int \prod_{t > z_0/2} d\vec{X}(t) e^{iM_s \vec{X}(t)^2/2} \int \prod_{t < -z_0/2} d\vec{X}(t) e^{iM_s \vec{X}(t)^2/2} \int \prod_{-z_0/2 < t < z_0/2} d\vec{X}(t) e^{iM_s \vec{X}(t)^2/2}$$

$$\sim const \int d\vec{X}_1 d\vec{X}_2 \exp \left[ \frac{iM_s (\vec{X}_1 - \vec{X}_2)^2}{2z_0} \right] \exp(i(\vec{P}' \vec{X}_1 - \vec{P} \vec{X}_2)) \Phi_s \left( \frac{z}{2} - \vec{X}_1 \right) \Phi_s \left( -\frac{z}{2} - \vec{X}_2 \right)$$

(A.6)

where $\vec{X}_1 = \vec{X}(t = z_0/2); \vec{X}_2 = \vec{X}(t = -z_0/2)$ and continuity of the path as well as proper boundary conditions

$$\vec{X}(t_1) = \vec{q}_1, \quad \vec{X}(t_2) = \vec{q}_2, \quad \lim_{t_1 \to -\infty} \vec{q}_1 = \vec{X}', \quad \lim_{t_2 \to +\infty} \vec{q}_2 = \vec{X}$$

are implied; the const represents (divergent) kinematical factors which will contract with the same factors in the denominator of (A.2). From the definition (A.1) we can see that the most essential are small $z \sim 1/\vec{P}^+$. The term $\lim_{z \to 0} = \sqrt{z_0}$ is not analytical at the point $z_0 = 0$ and is strongly suppressed for $z_0 \neq 0$. Hence we may safely replace it with its limit

$$\lim_{z \to 0} \frac{1}{\sqrt{z_0}} \exp \left[ \frac{iM_s (\vec{X}_1 - \vec{X}_2)^2}{2z_0} \right] \approx \delta(\vec{X}_1 - \vec{X}_2)$$

Thus finally we arrive to the well-known result 51, 52

$$\langle P'| \Phi \left( \frac{z}{2} \right) \Phi \left( -\frac{z}{2} \right) | P \rangle \approx \frac{M_s}{2\pi} \int d^3X \Phi_s \left( \frac{z}{2} - \vec{X} \right) \Phi_s \left( -\frac{z}{2} - \vec{X} \right) e^{i\Delta \vec{X}}$$

(A.7)

which is valid in the leading order in the coupling constant: to evaluate the matrix element, one should replace all the fields with the solutions of the Hartree-Fock equations centered at the point $\vec{X}$ and integrate over the center of mass.

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