$Q_T$ RESUMMATION IN TRANSVERSELY POLARIZED DRELL-YAN PROCESS *

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We calculate QCD corrections to transversely polarized Drell-Yan process at a measured $Q_T$ of the produced lepton pair in the dimensional regularization scheme. The $Q_T$ distribution is discussed resumming soft gluon effects relevant for small $Q_T$.

Keywords: $Q_T$ resummation, Transversity, Drell-Yan process

Hard processes with polarized nucleon beams enable us to study spin-dependent dynamics of QCD and the spin structure of nucleon. The helicity distribution $\Delta q(x)$ of quarks within nucleon has been measured in polarized DIS experiments, and $\Delta G(x)$ of gluons has also been estimated from the scaling violations of them. On the other hand, the transversity distribution $\delta q(x)$, i.e. the distribution of transversely polarized quarks inside transversely polarized nucleon, can not be measured in inclusive DIS due to its chiral-odd nature and remains as the last unknown distribution at the leading twist. Transversely polarized Drell-Yan (tDY) process is one of the processes where the transversity distribution can be measured, and has been undertaken at RHIC-Spin experiment.

We compute the 1-loop QCD corrections to tDY at a measured $Q_T$ and azimuthal angle $\phi$ of the produced lepton in the dimensional regularization scheme. For this purpose, the phase space integration in $D$-dimension, separating out the relevant transverse degrees of freedom, is required to extract the $\propto \cos(2\phi)$ part of the cross section characteristic of the spin asymmetry of tDY. The calculation is rather cumbersome compared with the corresponding calculation in unpolarized and longitudinally polarized cases, and has not been performed so far. We obtain the NLO

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\( \mathcal{O}(\alpha_s) \) corrections to the tDY cross section in the \( \overline{\text{MS}} \) scheme. We also include soft gluon effects by all-order resummation of logarithmically enhanced contributions at small \( Q_T \) ("edge regions of the phase space") up to next-to-leading logarithmic (NLL) accuracy, and obtain the first complete result of the \( Q_T \) distribution for all regions of \( Q_T \) at NLL level.

We first consider the NLO \( \mathcal{O}(\alpha_s) \) corrections to tDY: \( h_1(P_1, s_1) + h_2(P_2, s_2) \rightarrow l(k_1) + \bar{l}(k_2) + X \), where \( h_1, h_2 \) denote nucleons with momentum \( P_1, P_2 \) and transverse spin \( s_1, s_2 \), and \( Q = k_1 + k_2 \) is the 4-momentum of DY pair. The spin dependent cross section \( \Delta_T d\sigma \equiv (d\sigma(s_1, s_2) - d\sigma(s_1, -s_2))/2 \) is given as a convolution

\[
\Delta_T d\sigma = \int dx_1 dx_2 \delta H(x_1, x_2; \mu_F) \Delta_T d\hat{\sigma}(s_1, s_2; \mu_F),
\]

where \( \mu_F \) is the factorization scale, and

\[
\delta H(x_1, x_2; \mu_F) = \sum_i e_i^2 \delta q_i(x_1; \mu_F) \delta \hat{q}_i(x_2; \mu_F) + \delta \hat{q}_i(x_1; \mu_F) \delta q_i(x_2; \mu_F)
\]
is the product of transversity distributions of the two nucleons, and \( \Delta_T d\hat{\sigma} \) is the corresponding partonic cross section. Note that, at the leading twist level, the gluon does not contribute to the transversely polarized process due to its chiral odd nature.

We compute the one-loop corrections to \( \Delta_T d\hat{\sigma} \), which involve the virtual gluon corrections and the real gluon emission contributions, e.g., \( q(p_1, s_1) + \bar{q}(p_2, s_2) \rightarrow l(k_1) + \bar{l}(k_2) + g \), with \( p_1 = x_1 P_1 \). We regularize the infrared divergence in \( D = 4 - 2\epsilon \) dimension, and employ naive anticommuting \( \gamma_5 \) which is a usual prescription in the transverse spin channel. In the MS scheme, we eventually get \( \delta \overline{\text{MS}} \) to NLO accuracy,

\[
\frac{\Delta_T d\sigma}{dQ^2 dQ_T dy d\phi} = N \cos(2\phi) \left[ X(Q_T^2, Q^2, y) + \hat{Y}(Q_T^2, Q^2, y) \right],
\]

where \( N = \alpha^2/(3 N_c S Q^2) \) with \( S = (P_1 + P_2)^2 \), \( y \) is the rapidity of virtual photon, and \( \phi \) is the azimuthal angle of one of the leptons with respect to the initial spin axis. For later convenience, we have decomposed the cross section into the two parts: the function \( X \) contains all terms that are singular as \( Q_T \rightarrow 0 \), while \( Y \) is of \( \mathcal{O}(\alpha_s) \) and finite at \( Q_T = 0 \). Writing \( X = X^{(0)} + X^{(1)} \) as the sum of the LO and NLO contributions, we have \( \frac{\Delta_T d\sigma}{dQ^2 dQ_T dy d\phi} = X^{(0)} + X^{(1)} \)

\[
X^{(0)} = \delta H(x_1, x_2; \mu_F) \delta(Q_T^2),
\]

\[
X^{(1)} = \frac{\alpha_s}{2\pi} C_F \left\{ \delta H(x_1^0, x_2^0; \mu_F) \left[ 2 \left( \frac{\ln Q^2/Q_T^2}{Q_T^2} \right) + \frac{3}{(Q_T^2)^+} + (-8 + \pi^2) \delta(Q_T^2) \right] \right.
\]

\[
+ \left\{ \frac{1}{(Q_T^2)^+} + \delta(Q_T^2) \ln \frac{Q_T^2}{\mu_F^2} \right\} \int_{z_1^0}^{1} \frac{dz}{z} \delta P_{qq}^{(0)}(z) \delta H \left( \frac{x_1^0}{z}, \frac{x_2^0}{z}; \mu_F \right) \left[ (x_1^0 \leftrightarrow x_2^0) \right] \}
\]

where \( x_1^0 = \sqrt{T} e^y, x_2^0 = \sqrt{T} e^{-y} \) are the relevant scaling variables with \( \tau = Q^2/S \), and \( \delta P_{qq}^{(0)}(z) = 2z/(1-z) + (3/2) \delta(1-z) \) is the LO transverse splitting function. In the analytic expression of \( Y \), see Ref. 9 Eq. 4 gives the first NLO result in the MS scheme.
scheme. We note that there has been a similar NLO calculation of tDY cross section in massive gluon scheme. We also note that, integrating over $Q_T$, our result coincides with the corresponding $Q_T$-integrated cross sections obtained in previous works employing massive gluon scheme and dimensional reduction scheme via the scheme transformation relation.

The cross section becomes very large when $Q_T \ll Q$, due to the terms behaving $\sim \alpha_s \ln(Q^2/Q_T^2)/Q^2$ and $\sim \alpha_s/Q_T^2$ in the singular part $X$. It is well-known that, in unpolarized and longitudinally polarized DY, large “recoil logs” of similar nature appear in each order of perturbation theory as $\alpha_s^n \ln^{n-1}(Q^2/Q_T^2)/Q^2$, $\alpha_s^n \ln^{2n-2}(Q^2/Q_T^2)/Q_T^2$, and so on, corresponding to LL, NLL, and higher contributions, respectively, and that the resummation of those “double logarithms” to all orders is necessary to obtain a well-defined, finite prediction of the cross section. Because the LL and NLL contributions are universal, we can work out the all-order resummation of the corresponding logarithmically enhanced contributions in up to the NLL accuracy, based on the general formulation of the $Q_T$ resummation. This can be conveniently carried out in the impact parameter space, following the method introduced in the joint resummation. Obviously prescription to define the $b$ integration is not unique reflecting IR renormalon ambiguity, e.g., “$b_*$ prescription” to “freeze” effectively the $b$ integration along the real axis is frequently used. The

$$X \rightarrow \sum_i e_i^2 \int_0^\infty db b J_0(bQ_T)e^{S(b,Q)}(C_{qq} \otimes \delta q_i) \left( x_1^0, \frac{b_0^2}{b^2} \right) (C_{\bar{q}q} \otimes \delta \bar{q}_i) \left( x_2^0, \frac{b_0^2}{b^2} \right)$$

(5)

Here $b_0 = 2e^{-\gamma_E}$, and the large logarithmic corrections are resummed into the Sudakov factor $e^{S(b,Q)}$ with $S(b,Q) = -\int_{b_0^2/b^2}^{Q^2} (d\kappa^2/\kappa^2) \{ \ln(\frac{Q^2}{\kappa^2})A_q(\alpha_s(\kappa)) + B_q(\alpha_s(\kappa)) \}$. The functions $A_q, B_q$ as well as the coefficient functions $C_{qq}, C_{\bar{q}q}$ are calculable in perturbation theory, and at the present accuracy of NLL, we get

$$A_q(\alpha_s) = (\alpha_s/\pi)C_F + (\alpha_s/2\pi)^22C_F \{ (67/18 - \pi^2/6)C_G - 5N_f/9 \}, B_q(\alpha_s) = -3C_F(\alpha_s/2\pi), C_{qq}(z, \alpha_s) = C_{\bar{q}q}(z, \alpha_s) = \delta(1-z) \{ 1 + (\alpha_s/4\pi)C_F(\pi^2 - 8) \}.$$
renormalon ambiguity should be eventually compensated in the physical quantity by the power corrections \(\sim (\Lambda_{\text{QCD}} b)^n\) \((n = 2, 3, \ldots)\) due to non-perturbative effects. Correspondingly, we make the replacement \(e^{S_b(Q)} \rightarrow e^{S_b(Q)} F^{NP}(b)\) in \((5)\) and the “minimal” ansatz for non-perturbative effects \(F^{NP}(b) = \exp(-gb^2)\) with a non-perturbative parameter \(g\). Fig.1 shows the \(Q_T\) distribution of \(t\)DY at \(\sqrt{S} = 100\) GeV, \(Q = 10\) GeV, \(y = \phi = 0\), and with a model for the transversity \(\delta q(x)\) that saturates the Soffer’s inequality at a low scale. \([13]\) Solid line shows the NLO result using \([3]\), and the dashed and dot-dashed lines show the NLL result using \([3]\), \([12]\), \(F^{NP}(b) = \exp(-gb^2)\), with \(g = 0.5\) GeV\(^2\) and \(g = 0\), respectively.

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