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Baryon number and strangeness: signals of a deconfined antecedent

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Abstract. The correlation between baryon number and strangeness is used to discern the nature of the deconfined matter produced at vanishing chemical potential in high-energy nuclear collisions at the BNL RHIC. Comparisons of results of various phenomenological models with correlations extracted from lattice QCD calculations suggest that a quasi-particle picture applies. At finite baryon densities, such as those encountered at the CERN SPS, it is demonstrated that the presence of a first-order phase transition and the accompanying development of spinodal decomposition would significantly enhance the number of strangeness carriers and the associated fluctuations.

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1. Introduction

The goal of high-energy heavy-ion collisions is the creation of a new state of strongly interacting matter at very high energy density where the degrees of freedom carry color charges. These may be quark and gluon quasi-particles \[ 1 \], colored excitations of hadrons, or more complicated composite structures. The creation of such a state of matter is ushered in by the high energy densities created in such collisions. The reconversion of such an excited state of matter back into ordinary hadrons necessarily requires the presence of a phase transformation. The presence of such an excited state of matter and a phase change between it and ordinary matter has been observed in lattice simulations of QCD (LQCD) at a temperature of 170 MeV and vanishing baryon density \[ 2 \]. The suppression of the number of high-momentum particles originating in the fragmentation of hard partonic jets requires the presence of a near opaque medium with densities at least 30 times that of conventional nuclear matter \[ 3 \]. Such densities, when compared with the results from LQCD require the system to lie firmly in the deconfined state.

The phase transformation at a vanishing baryon density has been ascertained to be continuous. As a result, one would expect that the various conserved quantities such as the baryon number, charge and strangeness remain more or less unchanged within various sub-volumes of the phase space occupied by the system. As a result event-by-event fluctuations of these quantities may be used as probes of a state prior to hadronization. This will be the topic of the first part of these proceedings. It will be demonstrated that different assumptions regarding the nature of the Baryon number and strangeness carrying degrees of freedom lead to different measurable observables.
At zero temperature, most models predict the occurrence of a first-order phase transition when the density is raised. To date, there exist no lattice calculations in this regime. Thus, if there exists a first-order phase transition at zero temperature and finite chemical potential, this transition line should extend into the region of finite temperature and would persist until the chemical potential has dropped below a certain critical value \( \mu_c \). There exist lattice QCD estimations \[5\] that suggest the presence of a first-order phase transition line at finite chemical potential as well as the existence of a critical end-point, though its precise location is not well determined. The second part of these proceedings will deal with how certain observables such as the ratio of strange to non-strange particles and the fluctuations of such ratios are influenced by the non-monotonic behaviour of the thermodynamic potential as the system traverses a first-order phase transition line in the course of its dynamical expansion.

2. Baryonic strangeness at RHIC: \( \mu << T \)

Prior to the appearance of data from RHIC, it was believed that results from LQCD could eventually be explained within a picture of the deconfined phase as a weakly interacting plasma of quasiparticles \[6\]. The large radial and elliptic flow of the bulk matter however is somewhat inconsistent with the picture of a weakly interacting plasma \[7\]. Such observables seem to require the presence of a strongly interacting liquid. Indeed, recent results from lattice QCD calculation on spectral functions suggest the presence of bound, color-neutral states above \( T_c \). This has led to the suggestion that at moderate temperatures, \( T \approx 1 - 2 T_c \), the system is composed of medium-modified (massive) quarks and gluons together with their (many) bound states \[8\]. In yet another picture, the system may be composed of strings between colored particles and their topological excitations \[9\]. At the present time, there exists no clear consensus on the nature of the degrees of freedom of the excited state of matter being created in RHIC collisions.

Imagine that highly excited matter has indeed been created in the mid rapidity region at RHIC. The matter is strongly interacting and as a result has thermalized in the course of its expansion. The thermodynamics of this system may, at this point, be described in terms of a temperature \( T \) and chemical potentials \( \mu_B, \mu_S, \mu_Q \) for each of the conserved quantities of baryon number, strangeness and electric charge \[10\]. Experimentally determined chemical freezeout temperatures place the chemical freeze out of the expanding gas of hadrons very close to the expected phase transition temperature. These features lead to the picture that the lifetime of the system as a gas of hadrons where the interactions are strong enough to change the chemistry of the state is rather short. It is this possibility which forms the basis of the assertion that the conserved quantities of baryon number \( B \), strangeness \( S \) and electric charge \( Q \), within a wide rapidity bin will receive minor contamination from neighbouring bins.

As a result, the \( B, S \) and \( Q \) established in a given rapidity bin in the plasma phase will be approximately maintained by the hadron gas phase in each event. It may be argued that the correlation between the strangeness \( S \) and the baryon number \( B \) provides a useful diagnostic for the presence of strong correlations between quarks and anti-quarks. Considering a situation in which the basic degrees of freedom are non-interacting quarks and gluons, the strangeness is carried exclusively by the \( s \) and \( \bar{s} \) quarks which in turn carry baryon number in proportion to their strangeness, \( B_s = -\frac{1}{3}S_s \), thus rendering strangeness and baryon number strongly correlated. This feature is in stark contrast to a hadron gas in which the relation between strangeness
Baryon number and strangeness

and baryon number is less intimate. For example, at small chemical potential and
temperature, the strangeness is carried primarily by kaons which have no baryon
number.

These considerations lead us to introduce the correlation coefficient [14],

$$C_{BS} \equiv -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle}. \quad (1)$$

The last expression uses the fact that \(\langle S \rangle\) vanishes. We have chosen the normalization
such that \(C_{BS}\) is unity when the active degrees of freedom are individual quarks.

When the system consists of independent species \(k\) having baryon number \(B_k\) and
strangeness \(S_k\), its total baryon number is \(B = \sum_k n_k B_k\) and its total strangeness
is \(S = \sum_k n_k S_k\). The correlation coefficient may then be expressed in terms of the
multiplicity variances \(\sigma_k^2 \equiv \langle n_k^2 \rangle - \langle n_k \rangle^2 \approx \langle n_k \rangle\),

$$C_{BS} = -3 \sum_k \frac{\sigma_k^2 B_k S_k}{\sum_k \sigma_k^2} \approx -3 \sum_k \frac{\langle n_k \rangle B_k S_k}{\sum_k \langle n_k \rangle S_k^2} \quad (2)$$

Thus, in the hadronic gas, the numerator receives contributions from only (strange)
baryons (and anti-baryons), while the denominator receives contributions also from
(strange) mesons,

$$C_{BS} \approx 3 \frac{\langle \Lambda \rangle + \langle \bar{\Lambda} \rangle + \ldots + 3 \langle \Omega^- \rangle + 3 \langle \Omega^+ \rangle}{K^0 + \bar{K}^0 + \ldots + 9 \langle \Omega^- \rangle + 9 \langle \Omega^+ \rangle}. \quad (3)$$

The hadronic freeze-out value value of \(C_{BS}\) is shown in Fig. 1. At the relatively
high temperatures relevant at RHIC, the strange mesons significantly outnumber the
strange baryons, so \(C_{BS}\) is smaller than unity. Indeed, including hadrons up to \(\Omega^-\),
we find \(C_{BS} = 0.66\) for \(T = 170\) MeV and zero chemical potential, \(\mu_B = 0\).

This is no longer the case when the baryon chemical potential is raised however.

As an illustration, we trace the freeze-out line of Ref. [11] which reports freeze-out
temperatures and chemical potentials for a wide range of collision energies \(\sqrt{s}\). As we
go to lower energies the temperature decreases slightly however the chemical potential
rises sharply; as a result the population of baryons rises in comparison to the mesons.
This leads to an increase in the coefficient \(C_{BS}\) as shown in the left panel of of Fig. 1
as a function of the chemical potential at the corresponding \(\sqrt{s}\).

The value of unity for \(C_{BS}\) in a quark-gluon plasma was obtained within the
picture of a weakly coupled system. A more realistic value may be obtained from
lattice simulations of QCD on the basis of off-diagonal susceptibilities [12]. In these
simulations, the partition function \(Z\) for QCD with three quark flavors is estimated.
The input parameters are the temperature \(T\), and the three chemical potentials for the
the up, down and strange flavors (\(\mu_u, \mu_d, \mu_s\)). Calculations are performed at vanishing
chemical potentials.

In this system the density of a flavor \(f\) is obtained as the derivatives of \(F = \log Z/V\) with respect to the chemical potential \(\mu_f\). The quark number susceptibilities
are the second derivatives

$$\chi_{ff'} = T \frac{\partial^2 F}{\partial \mu_f \partial \mu_{f'}}. \quad (4)$$
It may be easily argued that the mean flavour densities are zero at vanishing chemical potential for all flavours: $\langle u \rangle = \langle d \rangle = \langle s \rangle = 0$. As a result, the coefficient $C_{BS}$ may be expressed in terms of the susceptibilities as

$$-3 \frac{\langle B \delta S \rangle}{\langle S^2 \rangle} = - \left. \frac{\partial^2 Z(\mu_u, \mu_d, \mu_s)}{\partial \mu_u \partial \mu_d \partial \mu_s} \right|_{\mu_u = \mu_d = \mu_s = 0} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_{ss}}. \quad (5)$$

From Ref. [12] we obtain $\chi_{ss}/T^2 = 0.53(1)$ and $\chi_{us} + \chi_{ds} = 0.00(3)$ at $T = 1.5 T_c$. As a result we obtain $C \simeq 1$ from lattice QCD, consistent with the estimate from a naive picture of a weakly interacting plasma. These results were obtained in a quenched approximation, but the effect of sea quarks is expected to be marginal above $T_c$. We may thus surmise that the lattice system has $C_{BS} \approx 1$, which is consistent with a plasma of quasi-particle quarks.

Recently there has appeared a model that purports to explain both the equation of state as obtained on the lattice as well as the large flow observed in heavy-ion collisions [5]. The model describes the chromodynamic system as a gas of massive quarks, antiquarks, and gluons together with a myriad of their bound states generated by a screened Coulomb potential. In order to assess the consistency of this model with lattice calculations, we estimate the coefficient $C_{BS}$ in such a scenario.

Our estimates are based on Ref. [5] and the assumption that the fluctuation coefficient $C_{BS}$ is set at a temperature $T = 1.5 T_c$. In this model, the plasma at a temperature $T_c < T < 3 T_c$ possess massive quasi-particle excitations. The attraction between such colored states is modelled via a Coulomb potential with a Debye screening mass which is derived from lattice simulations. The temperature dependence of the quasiparticle and Debye masses is obtained by parameterizing data from the lattice. The attraction at such temperatures is still sufficient to produce a variety of bound states. For three flavors of quarks there exist 749 such bound states. However, only the color-triplet $sg$ and the color-singlet $q \bar{s}$ states (and their conjugates) are of relevance here. (The color-hexaplet $sg$ states as well as the diquark states are very weakly bound and dissolve entirely at $1.5 T_c$.\) There are 2 $\pi$-like (spin-singlet) and 6 $\rho$-like (spin-triplet) $q \bar{s}$ states, and 36 $sg$ states. The abundances of these states are estimated in a grand canonical ensemble with vanishing chemical potentials. The $q \bar{s}$ multiplets carry no baryon number and thus contribute only to $\sigma_3^2$, hence drive $C_{BS}$ towards zero, while more degenerate $sg$ multiplet contributes also to $\sigma_{BS}$ and thus drives $C_{BS}$ towards unity. The resulting value obtained at $T = 1.5 T_c$, $C_{BS} = 0.62$, differs significantly from the value extracted directly from lattice QCD (see above), thus suggesting that the bound-state model, does not encode the same degrees of freedom that are pervasive in lattice simulations of QCD.

Turning to more experimental considerations, we outline how such correlations may be actually measured in RHIC collisions. Such estimations are made with the aid of the Monte-Carlo event generator HIJING [9]. The collisions occur in the center-of-mass frame at $\sqrt{s} = 200 A GeV$ and we estimate the coefficient $C_{BS}$ using the left hand side of Eq. (1). One measures all hadrons in a given event that lie within a range of rapidity from $-|y_{max}| < y_{accept} < |y_{max}|$, and computes the total baryon number $B$, the total strangeness $S$ and their products $BS$ and $S$. One then calculates the event averages of all these quantities and resurrects the coefficient $C_{BS}$. These results are represented by the closed circles in the right panel of Fig. 1.
As the rapidity range of the acceptance is increased the coefficient remains more or less constant at a rather low value of 0.45. This is primarily due to the well know problem of baryon production in string fragmentation models. However as the rapidity range is increased we note a large rise in the coefficient. This is due to the fact that the net baryon number in the large rapidity regions is much larger than at mid-rapidity and as a result the number of baryons in the computation of the coefficient increases. This rise should be compared to the rise of the coefficient with baryon chemical potential in the left hand side of Fig. Eventually, at very large acceptances, one begins to capture the entire system. Due to the strict conservation of baryon number and charge in such models, the fluctuations of $BS$ drop rapidly as the acceptance encompasses the full system. There remains a residual fluctuation in the strangeness content due to weak decays. As a result $C_{BS} \rightarrow 0$ at large acceptance.

In order to study the effect of the exact conservation of baryon number without the influence of the rather large net baryon number, we repeat the above study on JETSET ($e^+e^-$ annihilation to two jets) events where the net baryon number is zero. As would be expected, at the lower rapidity range of the acceptance, it is consistent with the results from HIJING, which also displays a very small net baryon number at mid-rapidity. The coefficient $C_{BS}$ extracted from JETSET events shows a monotonous drop to zero as the acceptance is increased to encompass the full system. The value of $C_{BS}$ for asymptotically small ranges of the acceptance may estimated as simply the ratio of the probability to observe a strange baryon to that of strange meson.

Figure 1. Calculations for the coefficient $C_{BS} = \langle \delta B \delta S \rangle / \langle \delta S^2 \rangle$ in various scenarios. The dot-dashed line represents $C_{BS}$ on the lattice and in a quasiparticle plasma; the dashed line is the estimate for a hadron gas and for a Shuryak-Zahed (SZ) plasma. The filled circles represent the estimate from JETSET at center-of-mass energy = 20 GeV for various ranges of the maximum absolute rapidity of acceptance. The green triangles represent the results from HIJING simulations for Au-Au at 200AGeV for various ranges of acceptance.

In these proceedings, we have proposed an experimentally observed ratio $C_{BS} = -3\langle \delta B \delta S \rangle / \langle \delta S^2 \rangle$, which may be used not merely to detect the prior existence of a chromodynamic phase of QCD, but indeed to decipher the strangeness and baryon
number carrying degrees of freedom prevalent in such an environment. Estimates based on a quasi-particle picture of such degrees of freedom tend to favor a value of unity for the coefficient. This is also consistent with the estimates from lattice simulations. In a more strongly interacting scenario, which possesses light bound states of quarks and antiquarks the coefficient is reduced to approximately $2/3$ which is consistent with the coefficient from a hadronic resonance gas with no prior existence of a deconfined phase. Estimates from Monte Carlo models of string fragmentation place the coefficient at an even lower value of 0.45 nearly independent of the rapidity range of the acceptance up to $\pm 2$ units of rapidity. Thus measurements of the coefficient $C_{BS}$ allow for a resolution of the baryonic and strange degrees of freedom in a quark gluon plasma. For further details see Ref. [14].

3. Strangeness at the SPS: $\mu > T$

In this section, the focus is turned to systems produced at the lower beam energies of the SPS. Deriving a freeze-out temperature $T$ and a chemical potential $\mu$ from fits to particle ratios leads to a $T \simeq 165 - 160$ MeV and $\mu_B \simeq 240 - 300$ MeV for a collision energy $\sqrt{s} = 8 - 17$ GeV. Most models [4] indicate that at such large baryon chemical potentials the system may indeed undergo a first order phase transition from a partonic phase to a hadronic phase. As the collision energy is increased, the chemical potential tends to drop. As a result, the line of first-order phase transitions should eventually terminate in a second-order end point followed by the regime of continuous cross over as expected from lattice simulations at vanishing chemical potential. Current lattice estimates do indicate the presence of such a critical end point [5]. The location of this point is however not well established.

The proximity between the expected first-order phase transition line and the chemical freeze-out line at these collision energies suggests that a quark-gluon plasma could possibly be formed at these energies. This system will then traverse the phase transition line during its subsequent evolution into a hadronic gas. In this section, we outline the observational consequences of a first order phase transition on the multiplicity and fluctuations of strange particles produced in such collisions.

A universal feature of first-order phase transitions is the occurrence of spinodal decomposition, which occurs as a result of the presence of a convex anomaly in the associated thermodynamic potential. This phenomenon occurs when bulk matter, by a sudden expansion or cooling, is brought into the region of phase coexistence. Since such a configuration is thermodynamically unfavorable the uniform system prefers to reorganize itself into spatially separate single-phase domains. These blobs of plasma immersed in an environment of hadrons tend to have a characteristic scale [15]. The essential feature of such a scenario is that if the breakup is sufficiently rapid, then whatever strangeness happens to reside within the region of the plasma that forms a given blob will effectively become trapped and, consequently, the resulting statistical hadronization of the blob will be subject to a corresponding canonical constraint on the strangeness. This will lead to an enhancement of the multiplicity of strangeness-carrying hadrons, as compared to the conventional scenario where global chemical equilibrium is assumed to be maintained through freeze-out. (This occurrence of an enhancement is qualitatively easy to understand, since the presence of a finite strangeness in the hadronizing blob enforces the production of a corresponding minimum number of strange hadrons.) The fluctuations in the multiplicity of strange hadrons, such as kaons, are enhanced even more, thus offering a possible means for
the experimental exploration of the phenomenon. Indications of an enhancement in the $K$-to-$\pi$ ratio has already been seen experimentally around $20 - 30$ AGeV beam energy \[16\]. Furthermore, at the same energy, the fluctuation of this ratio are strongly enhanced if expressed relative to mixed events.

Strangeness, as opposed to other conserved quantities such as charge and baryon number, is not brought in by the initial states in a heavy-ion collision. It is produced in the dense systems created in such collisions and thus the net strangeness in the full participant region is necessarily vanishing. There is however no constraint on the number of strangeness carriers e.g., the strange quarks and antiquarks or on the resulting strange mesons and baryons. For such an enhancement to occur the phase transition in question must necessarily be from a state with a greater number of strangeness carrying degrees of freedom (compared to the dominant carriers of entropy e.g., in a quark-gluon plasma) to a state with a fewer number of strangeness carriers as compared to the number of carriers of entropy (e.g., a hadron gas). The rapid expansion curtails the global chemical re-equilibration of the hadronic strangeness carriers emanating from different plasma blobs and results in a persistence of the overall enhancement of the mean number of strangeness carriers in the rarer hadronic phase.

As in most statistical estimations of the multiplicities of produced particles, we consider a volume $V$ in which the produced strange quarks and antiquarks are assumed to reach full global chemical equilibrium (at plasma temperature $T_q$, and baryon and charge chemical potentials $\mu_B$, $\mu_Q$) under the canonical constraint of net total strangeness $S_{tot} = 0$, i.e., we assume that the expansion is not rapid enough to offset the chemical or kinetic equilibrium of the produced quarks, antiquarks and gluons. The multiplicity distribution of the total number of strange quarks (identical to the distribution of anti-quarks) is given by the expression

$$P_q(N) = P_{\bar{q}}(N) = \frac{\xi_q^N \xi_{\bar{q}}^N}{N! N!} \sum_{N=0}^{\infty} \frac{\xi_q^N \xi_{\bar{q}}^N}{N! N!}.$$  \hspace{1cm} (6)

The single quark (anti-quark) partition function $\xi$ is given as

$$\xi_s = \frac{g_s V T_q^3}{2 \pi^2 \hbar^3 c^3} \tilde{K}_2\left(\frac{m_s c^2}{T_q}\right) e^{(\mu_B + \mu_Q)/T_q}.$$  \hspace{1cm} (7)

Where $g_s$ is the spin-color degeneracy of a strange quark and $\tilde{K}_2(x) = x^2 K_2(x)$ [$K_2(x)$ is the modified Bessel function of the second kind]. We denote the denominator of Eq. 6 which represents the canonical partition function of a system of strange quarks and antiquarks with net strangeness zero as $Z_{0}^{S_q=\pm 1} = Z_{0}^{S_q}$. 

Following the dynamical scenario outlined above, we imagine that the system decomposes into $p$ boxes of equal size $V_q = V/p$ (here and in the rest of this section we will use blobs and boxes interchangeably). The number of ways of distributing $N$ non-identical quarks into $p$ boxes with $n_1$ quarks in box 1, $n_2$ in box 2 etc. , is given by the combinatorial factor,

$$C_{\{n\}} = \frac{N!}{n_1! n_2! \ldots n_p!} \left(\frac{1}{p}\right)^N \sum_{n_1 + \ldots + n_p = N}.$$  \hspace{1cm} (8)
In the above equation, \( \{ \mathbf{n} \} \) refers to the set of occupation numbers of the various boxes or blobs, i.e., \( n_1, n_2, \ldots, n_p \).

With the aid of the above expression and Eq. (6) we obtain the probability distribution of events with quarks and anti-quarks distributed in the \( p \) compartments according to the occupation vectors of quarks and antiquarks i.e., \( \mathbf{n}^q \) and \( \mathbf{n}^\bar{q} \), as

\[
P(\mathbf{n}^q, \mathbf{n}^\bar{q}) = \frac{1}{Z_0^q} \prod_i \left( \frac{\xi}{n_i!} \frac{\bar{\xi}}{\bar{n}_i!} \right)^{n_i} \sum_{n_i=0}^{\infty} \frac{1}{Z_0^q}.
\] (9)

These blobs of plasma now undergo expansion and hadronization. The final hadronic populations achieve chemical equilibration within a volume \( V_h \sim \chi V_q \) (at a temperature \( T_h \)) in the vicinity of the now well separated plasma blobs. The expansion factor is estimated to be \( \chi \sim 1.5 - 4 \). As a result, the net strangeness of a blob \( i \) \( (s_i = \bar{n}_i - n_i) \) is maintained post hadronization by the hadronic resonance gas. Thus a computation of the properties of the final hadronic gas requires the probability distribution of the net strangeness in the various blobs,

\[
Q^0_p(s_1, \ldots, s_p) = \sum_{n_1, \ldots, n_p, \bar{n}_1, \ldots, \bar{n}_p} P(\mathbf{n}^q, \mathbf{n}^\bar{q}) \prod_i \delta(s_i + n_i - \bar{n}_i).
\] (10)

In an experimental setup one never measures all the particles produced in the full phase space, but instead views a sub-sample of a given event. As a result, the focus will lie on the set of partially summed quantities,

\[
Q^S_p(s_1, \ldots, s_k) = \prod_{i=1}^k Z^\pm_{s_i} \sum_{s_{k+1}, \ldots, s_p} \prod_i Z^\pm_i \delta \left( \sum_{j=1}^p s_j - S \right).
\] (11)

Where \( Z^\pm_{s_i} \) represents the canonical partition function of only strange quarks and antiquarks in a single blob \( i \) with net strangeness \( s_i \). The above equation represents the probability distribution, that a system of strange quarks and antiquarks is distributed into \( p \) separate compartments and the strangeness in the first \( k \) compartments is specified. The strangeness contents of the remaining compartments is summed over all allowed values.

Given the strangeness \( S_0 \) in a compartment, the distribution of strange hadrons in that compartment may be computed using the canonical partition function for strange hadrons including the six classes of such hadrons with strangeness \( \pm 1, \pm 2, \pm 3 \) and with total net strangeness \( S_0 \),

\[
Z^\pm_{S_0} = \prod_{S=\pm 1, \pm 2, \pm 3} \sum_{N_s} \zeta_{NS}^{N_S} \frac{\delta \left( \sum_{S} N_S S - S_0 \right)}{N_S!}.
\] (12)

The effective single-particle partition function for the class of strange hadrons with strangeness \( S \), \( \zeta_S \), is a sum of the single-particle partition functions off all the hadrons belonging to this class. The general criteria for the inclusion a given strange hadron in the above equation will be that its mass be below 1680 MeV and its decay width be below 200 MeV.
The net strangeness $S_0$ in Eq. (12) is given by the probability distribution $Q_p^0(S_0)/Z_q^0$ computed above. The chemical potentials involved in the single-particle partition functions of the hadrons ($\zeta_k$) are in principle also influenced by the value of $S_0$ in the given blob. One may also note that the non-strange sector will be influenced by the traversal of the system through the first order phase transition line. In these proceedings we will ignore such complications and refer the reader to Ref. [17] for details. In what follows, we will assume the chemical potentials to be independent of the net strangeness in the blob. It will also be assumed that the non-strange hadron multiplicities and their fluctuations may be obtained by grand canonical estimates based on the measured freeze-out temperature and chemical potentials.

As a result of the above simplifications, we may obtain the mean and variance of particles of strangeness $k$ from a compartment of net strangeness $S_0$ as [18]

$$\langle n_k \rangle_{S_0} = \zeta_k \frac{Z_{S_0-k}^{\pm1,\pm2,\pm3}}{Z_{S_0}^{1,1,1,2,3}}, \quad (\sigma_k^2)^{S_0} = \zeta_k^2 \frac{Z_{S_0-2k}^{\pm1,\pm2,\pm3}}{Z_{S_0}^{1,1,1,2,3}} + \langle n_k \rangle_{S_0}^2 - (\langle n_k \rangle_{S_0})^2. \quad (13)$$

The mean number of such particles from a single blob produced in a scenario as outlined above may be obtained by simply taking the convolution of the above equation with the probability distribution of strangeness,

$$\langle n_k \rangle = \zeta_k \frac{\sum_{S_0} Z_{S_0-k}^{\pm1,\pm2,\pm3} Q_p^0(S_0)}{Z_q^0}. \quad (14)$$

As all the different blobs or compartments are considered equivalent, we present results for the $K^\pm$ multiplicities and variances from a single such blob of volume $V_q = 50 \text{ fm}^3$ in the plasma phase in Fig. 2. For comparison, two other scenarios are presented. Besides the mean and variance that result from a spinodal decomposition during the phase transition, we also present the results from a simple grand canonical reference scenario. Using the baryon chemical potential $\mu_B$ as a control parameter, we obtain the freeze-out temperature $T_h$ from the fit to the data obtained in Ref. [10]. Subsequently, we perform a grand-canonical iteration to determine those values of $\mu_Q$ and $\mu_S$ that ensure $\bar{Q} = \alpha \bar{B}$ and $\bar{S} = 0$, where $\alpha = 0.4$ which is representative of $Z/A$ for gold. The last is referred to as the restricted canonical scenario where the blob strangeness is always fixed to zero.

The general trend in all the three scenarios is similar. As $\mu_B$ is increased, the $K^+$ yield initially increases to balance the strangeness of the increasing number of strange baryons. As the freeze-out temperature begins to decrease noticeably, the hadron production generally decreases, leading to a decreasing behavior of the $K^+$ curve. The multiplicities are insensitive to the value adopted for the plasma temperature $T_q$, which governs the fluctuations in the blob strangeness $S_0$. For the variances, the overall behavior is qualitatively similar to the behavior of the averages, however, with some differences. First, in the restricted scenario (where only $S_0 = 0$ is included) the suppression of the variance is significantly larger. This is the well known effect of canonical suppression. Furthermore, at the larger values of $\mu_B$, where the net baryon density becomes significant, the grand-canonical variance suffers more from the decreasing temperature than the canonical variance. This important divergence is a result of the fact that the larger average baryon number implies a correspondingly larger variance in the baryon number and therefore also a larger
Baryon number and strangeness

Figure 2. The average $K^\pm$ multiplicities and the $K^+$ variances as functions of the baryon chemical potential $\mu_B$ in three three scenarios: 1) the standard grand-canonical treatment, 2) the canonical treatment in which the blob strangeness $S_0$ is conserved through the hadronization (the mean multiplicities are not sensitive to the plasma temperature $T_q$), and 3) the restricted canonical treatment admitting only $S_0 = 0$.

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