MEASUREMENT OF THE DIFFERENTIAL CROSS-SECTION
AND THE SPIN-CORRELATION PARAMETERS P, A, AND R
IN THE BACKWARD PEAK OF $\pi^-p \rightarrow K^0\Lambda$ AT 5 GeV/c^{†)}

P. Astbury, J. Gallivan^{×)}, J. Jafar^{××)}, M. Letheren^{×××)},
V. Steiner and J.A. Wilson^{*)}

Imperial College, London, U.K.

W. Beusch, M. Borghini and D. Websdale^{**)}

CERN, Geneva, Switzerland

L. Fluri, K. Freudenreich, F.X. Gentit and W. Wetzel

ETH, Zürich, Switzerland

and

P. Le Du and O. Guisan^{***)}

CEN, Saclay, France

ABSTRACT

We present measurements for the baryon-exchange reaction $\pi^-p \rightarrow \Lambda K^0$ at 5 GeV/c over the range $-u < 2$ (GeV/c)². The differential cross-section is obtained from events produced on a liquid-hydrogen target. These data are combined with events produced on a butanol polarized target and the parameters P, A, and R are then determined from an analysis of the decay angular distribution of the forward-going Λ . From our data it is possible to determine the magnitudes and the relative phase of the two invariant amplitudes A' and B. The consequences for the ratio B/A' are discussed in detail.

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†) Work supported in part by the Swiss National Science Foundation.

×) Now at the Max-Planck Institute, Munich.

××) Now at the Nuclear Research Institute, Baghdad.

×××) Now at CERN, Geneva.

*) Now at the University of Birmingham.

***) Now at Imperial College, London.

***) Now at the University of Geneva.

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1. INTRODUCTION

We present a "complete" measurement in the backward peak of the reaction $\pi^- p \rightarrow K^0 \Lambda$ at 5 GeV/c using a butanol polarized target and a liquid-hydrogen target. The observed four-momentum transfer squared from the pion to the lambda was in the range $-u \leq 2 \text{ (GeV/c)}^2$. With this work we continue the earlier investigations^{1,2)} made with an unpolarized target. The main features observed previously were the rapid rise of the polarization P to $\approx +1$ at $u \approx -0.25$, followed by a significant decrease at the larger values of $-u$, and a break in the slope of the differential cross-section at $u \approx -0.7$.

The backward peak of this reaction is expected to be dominated by the exchange of Σ -trajectories. Barger, Cline and Matos³⁾ fitted P and $d\sigma/du$ by a weakly exchange-degenerate pair $(\Sigma_\alpha, \Sigma_Y)$ with simple but somewhat arbitrary residue functions. In contrast to this, Schmid and Storow⁴⁾ considered the exchange of the two strongly exchange-degenerate pairs $(\Sigma_\alpha, \Sigma_Y)$ and $(\Sigma_\beta, \Sigma_\delta)$. They showed that the observed sign and magnitude of the polarization are consistent with residue functions which are characterized by the elimination of a certain number of parity doublets.

2. PRINCIPLE OF THE EXPERIMENT

At any given energy the reaction $\pi^- p \rightarrow \Lambda K^0$ can be described by two independent helicity amplitudes $H_{++}(u)$ and $H_{+-}(u)$, where the subscripts refer to the final and initial baryon helicities. The axes shown in fig. 1 correspond to s-channel helicities. This specifies the z and z' axes for the target proton and the lambda to be opposite to the direction of the incident pion and the outgoing kaon, respectively. The normal to the production plane is defined by

$$\hat{n} = \frac{\vec{k}_{\pi^-} \times \vec{k}_{K^0}}{|\vec{k}_{\pi^-} \times \vec{k}_{K^0}|}, \quad (1)$$

where $\vec{k}_{\pi^-}$ and $\vec{k}_{K^0}$ are the momenta of the incoming pion and the outgoing kaon. The angle ϕ is measured between the normal $\hat{n}$ and the direction of positive target polarization.

When the target proton is polarized, the polarization vector of the Λ is not restricted to be in the direction of the normal but can acquire a component in the production plane. The parity-violating decay $\Lambda \rightarrow p\pi^-$ serves as the polarization analyser. The angles δ and η of the decay proton in the helicity frame of the Λ are indicated in fig. 1. In terms of these angles the Λ -decay distribution has the form

$$W(\phi, \delta, \eta) = \frac{1}{8\pi^2} \left[1 + \alpha P \cos \delta + P_T P \cos \phi + \alpha P_T \cos \phi \cos \delta + \right. \\ \left. + \alpha P_T \sin \phi \sin \delta (A_S \sin \eta + R_S \cos \eta) \right], \quad (2)$$

where the parameters P , A_s , and R_s are defined at fixed u by

$$\begin{aligned} P &= \frac{2 \operatorname{Im} (H_{++}^s H_{+-}^{s*})}{|H_{++}^s|^2 + |H_{+-}^s|^2} \\ A_s &= \frac{|H_{++}^s|^2 - |H_{+-}^s|^2}{|H_{++}^s|^2 + |H_{+-}^s|^2} \\ R_s &= \frac{2 \operatorname{Re} (H_{++}^s H_{+-}^{s*})}{|H_{++}^s|^2 + |H_{+-}^s|^2} . \end{aligned} \quad (3)$$

From this it follows that $P^2 + A_s^2 + R_s^2 = 1$. Here P_T is the target polarization, α is the asymmetry parameter of the Λ -decay, and the index s refers to the chosen helicity frame.

In the case of an unpolarized proton target ($P_T = 0$) the Λ -decay distribution reduces to the usual form:

$$W(\delta) = \frac{1}{2} (1 + \alpha P \cos \delta) . \quad (4)$$

The parameter P , which characterizes the Λ -decay angular distribution in the case $P_T = 0$, is independent of the specific choice of helicity axes.

3. EXPERIMENTAL PROCEDURE

Two experiments were performed; the first used a butanol polarized target and the second a liquid-hydrogen target. Apart from the system of counters surrounding the target, the remainder of the apparatus was common to both experiments.

A schematic layout of the polarized target experiment is shown in fig. 2. The negative 5 GeV/c incident beam was defined by a system of scintillation counters S_i in coincidence, and by halo counters A_i in veto. A threshold gas Čerenkov counter C_1 , containing hydrogen at 8 atmospheres, identified incident pions. The two final beam counters S_3 and S_4 (1 cm diameter) were operated inside the polarized target cryostat at a temperature of ≈ 4 K. The cryostat⁵⁾ maintained the butanol target (15 cm long $\times$ 2 cm diameter) at 0.5 K, and special pole pieces were installed at the magnet entrance to provide the 2.5 T magnetic field, of adequate uniformity, for the target polarization. A polarization $P_T = (63 \pm 5)\%$ was achieved by dynamic nuclear orientation. The polarization direction was normal to the beam and was frequently reversed, and approximately the same number of incident pions was recorded for each of the two directions. The butanol target was almost completely surrounded by a scintillator/tungsten sandwich of veto counters R_i and F_i , as in fig. 2. These counters were used to suppress triggers from the non-interacting beam particles and from interactions that produced charged particles

or π^0 's. The charged decay of a forward-going V^0 , with a vertex inside the magnetic spectrometer, was required by demanding at least one hit in the counter E.

The two hodoscope planes L and B (eight counters each) were positioned downstream of the magnet. The bending power of the spectrometer together with the $[L_1 B_1]$ coincidence requirement favoured the selection of positively charged particles with momenta $\gtrsim 1.6$ GeV/c. The trigger system thus favoured the selection of fast protons from the decay of forward-going Λ 's produced in the baryon-exchange reaction $\pi^- p \rightarrow \Lambda K^0$. The atmospheric pressure Čerenkov counter C_2 was filled with neopentane $[C(CH_3)_4]$ and used in veto. At this pressure the pion threshold is ≈ 2.6 GeV/c. This provided a useful suppression factor against those asymmetric K^0 decays which could fake Λ 's in the kinematic analysis. The additional electronic condition $\bar{C}_2 [L_1 B_1]$ suppressed the trigger rate by a factor of ≈ 3.5 without significant loss of lambdas from the backward peak.

The detector consisted of six modules of optical spark chambers of 12 gaps each, 0.8 cm gap width, and 2 cm spacing, operating in a magnetic field of ≈ 1 T. The electronic logic used for triggering the spark chambers is indicated in fig. 2. A camera with 50 msec dead-time recorded two stereo-views on 70 mm film. The spectrometer system is more fully described in previous papers⁶⁻⁸). The beam intensity passing through the spectrometer was chosen to be $\approx 7 \times 10^5$ particles per 400 msec burst. Only 50% of this beam appeared as telescope counts owing to the relatively large beam spot at the counters S_3, S_4 , and to some inefficiency in these counters whose 3 m light-guides stretched from inside the cryostat to a region of low field. The track memory time in the spark chambers was ≈ 2 μ sec. In order to suppress the number of beam tracks appearing on the picture, a small area of each spark chamber gap was desensitized in the beam region by attaching 50 μ m mylar patches to the HV plates. The mean number of beam tracks per picture was reduced by a factor of 2. The trigger rate was $\approx 7.8 \times 10^{-6}$ per incident pion, and approximately 200,000 pictures were taken with this arrangement.

In fig. 3 a schematic layout of the experiment using the liquid-hydrogen target is shown. The veto counters R_1 and F_1 surrounded the 53.5 cm long $\times$ 6 cm diameter target. The additional pole-pieces were removed, and the centres of the hodoscope planes L and B were positioned along the new mean axis of the decay proton trajectory. Almost 82% of the incident beam was counted by the beam telescope, and the electronic logic for triggering the spark chambers is indicated on the figure. The counter R_5 was not included in this trigger condition. Whenever R_5 gave a signal in coincidence with a trigger, a flag was set and recorded. This information is used in estimating the event losses due to K_L^0 interactions in the veto counters. The trigger rate was $\approx 4.5 \times 10^{-6}$ per incident pion, and $\approx 33,000$ pictures were taken using this layout.

4. DATA ANALYSIS

All photographs were measured on the CERN flying-spot digitizer HPD1. The V^0 pattern recognition was performed without human guidance, and events with two or more tracks found were taken to be V^0 candidates. These were approximately 60% of the pictures taken. The V^0 candidates were passed through modified versions of the track reconstruction program THRESH and the kinematic hypothesis testing program GRIND. For each V^0 candidate, two tracks of opposite sign were required to converge to an apex downstream of the veto counters. Then the following hypotheses were tested for the V^0 at the decay vertex:

$$K_S^0 \rightarrow \pi^+ + \pi^-$$

$$\Lambda \rightarrow p + \pi^-$$

$$\bar{\Lambda} \rightarrow \pi^+ + \bar{p}$$

$$\text{and } \gamma + (Z) \rightarrow e^+ + e^- + (Z) .$$

The unshaded histograms in fig. 4 correspond to the square of the missing mass to all lambdas with a decay fit probability $P(> \chi^2) > 0.03$. The shoulder in fig. 4a at $\approx 1 \text{ GeV}^2$ is due to genuine K^0 from the forward peak of $\pi^- p \rightarrow K^0 \Lambda$ where the recoil Λ escapes the veto counters and the K^0 also fits the Λ -decay hypothesis. This shoulder is not present in fig. 4b since the escape of the recoil Λ is much less probable in the hydrogen target set-up with the larger diameter of the veto counters. The shaded histograms in fig. 4 correspond to unambiguously identified lambdas. We estimate from the shaded histogram, fig. 4b, that the background under the K^0 peak is $\approx 5\%$. This background is produced by $\pi^- p \rightarrow \Lambda K^0 \pi^0$ or $\pi^- p \rightarrow \Sigma^0 K^0$, where the γ -rays have not been detected by the veto counters.

If a fit was obtained to any of the first three decay hypotheses, then an attempt was made to fit the event to its corresponding hypothesis at the production vertex, namely

$$\pi^- p \rightarrow (K^0) \Lambda$$

$$\pi^- p \rightarrow (\Lambda) K^0 ,$$

and the forbidden process $\pi^- p \rightarrow (\bar{\Lambda}) \bar{K}^0$, where the decay of the V^0 in parentheses was previously fitted. Only those events which gave a probability $P(> \chi^2) > 0.03$ for the fits to both the decay and the production hypotheses were considered. In the butanol target experiment, events could also be produced from the bound (unpolarized) protons in the oxygen and carbon nuclei. The missing-mass resolution in our spectrometer was not sufficient for an effective separation of bound from free proton events. However, our estimated ratio of bound/free proton events of ≈ 1.2 (see later) is less than the 3.2 calculated for butanol ($C_4H_{10}O$) owing to shadow effects and nuclear break-up, where in the latter case a secondary charged particle or γ -ray is detected by the veto counters.

5. THE DIFFERENTIAL CROSS-SECTION

We have obtained 1042 events satisfying the production fit to $\pi^-p \rightarrow (\Lambda)K^0$ from the liquid-hydrogen target, where the K^0 was not seen and the Λ -decay vertex was inside the chosen fiducial volume of $50 \times 50 \times 120 \text{ cm}^3$. Of these, only seven events also gave satisfactory fits to $\pi^-p \rightarrow (K^0)\Lambda$. The empty target contribution was measured to be $(5.8 \pm 1.5)\%$. In order to deduce the cross-section, two types of correction factors were estimated, those which are independent of u and those which are u -dependent.

Estimates for the correction factors independent of u are listed in table 1. These include a correction for the accidental loss of beam particles due to the finite resolving time of the anticoincidence, the branching ratios $\Lambda \rightarrow p\pi^-$ and $K^0 \rightarrow K_L^0$, losses due to the inadequacy of the pattern recognition and the track reconstruction programs in the u -range considered, losses of events due to the $P(> \chi^2)$ cuts imposed on the data sample. Corrections are included for absorption of the incident beam and the outgoing Λ in the hydrogen target and the veto counters. The over-all u -independent correction factor is 4.35 ± 0.24 .

The u -dependent correction factors are given in table 2. These were evaluated with the aid of a Monte Carlo program simulating the experimental geometry. The dominant correction at small $u_0 - u$ *) is due to the Λ decay upstream of the F counters. At the larger values of $u_0 - u$ the increased correction is due to limiting counter apertures at E, L, and B. Events that registered a hit in the monitor counter R_5 were used for estimating the K_L^0 interactions in the veto counters R_1 to R_4 . For $u_0 - u > 0.10 \text{ (GeV/c)}^2$ this loss amounted to $\approx 30\%$. Smaller corrections were included for the escape of K_S^0 and the decay of K_L^0 within the system of anticoincidence counters. The resulting differential cross-section is given in table 2 and fig. 5. Near the forward direction the u -resolution was $\approx \pm 0.001 \text{ (GeV/c)}^2$ and is everywhere smaller than the bin width. The errors quoted on $d\sigma/du$ do not include the error assigned to the over-all u -independent corrections factor, or other possible normalization errors. The cross-section for $\pi^-p \rightarrow \Lambda K^0$ at 5 GeV/c integrated to $u_0 - u = 1.6 \text{ (GeV/c)}^2$ is $0.97 \pm 0.10 \text{ } \mu\text{b}$. The error includes our estimate of the uncertainties on all corrections. This value is in good agreement with the fit given for a previous series of experiments²⁾.

6. THE LIKELIHOOD METHOD

We have used eq. (2) as the basis for the maximum likelihood estimate of the parameters P , A_s , and R_s in a given u -bin. The number of events N_T that satisfy the production hypothesis $\pi^-p \rightarrow \Lambda K^0$ include N_B events produced from the bound (unpolarized) protons of the butanol target. Since we cannot separate these

*) At this beam momentum $u_0 = 0.073 \text{ (GeV/c)}^2$.

kinematically, their presence is equivalent to diminishing the target polarization P_T by the factor $N_T/(N_T - N_B)$. Therefore, P_T in eq. (2) is replaced by the effective target polarization $P'_T = P_T[1 - (N_B/N_T)]$, which is determined directly from the data.

The probability that an incident pion on the butanol target, polarized with spin-up (+) or spin-down (-), results in a $\pi^- p \rightarrow \Lambda K^0$ event with the angles ϕ , δ , and η , is

$$P_{\pm}(\phi, \delta, \eta) d\phi d(\cos \delta) d\eta = KC(\phi, \delta, \eta) [1 + \alpha P \cos \delta \pm P'_T P \cos \phi \pm \alpha P'_T \cos \phi \cos \delta \pm \alpha P'_T A_S \sin \phi \sin \delta \sin \eta \pm \alpha P'_T R_S \sin \phi \sin \delta \cos \eta] d\phi d(\cos \delta) d\eta, \quad (5)$$

where $C(\phi, \delta, \eta)$ is the experimental acceptance function and K is proportional to the expected event rate. The probability $\bar{P}_{\pm}$ that no $\pi^- p \rightarrow \Lambda K^0$ event is observed can be expressed as

$$\bar{P}_{\pm} = 1 - K(c_1 + c_2 \alpha P \pm c_3 P'_T P \pm c_4 \alpha P'_T \pm c_5 \alpha P'_T A_S \pm c_6 \alpha P'_T R_S). \quad (6)$$

The coefficients c_i are the moments of C with respect to the different angular functions in $P_{\pm}$, e.g.

$$c_2 = \int C(\phi, \delta, \eta) \cos \delta d\phi d(\cos \delta) d\eta. \quad (7)$$

In terms of $p_{\pm}$ and $\bar{p}_{\pm}$ the likelihood function of the experiment reads:

$$\begin{aligned} \ln L = & \sum_{i=1}^{n_+} \ln p_+(\phi_i^+, \delta_i^+, \eta_i^+) + (N_+ - n_+) \ln \bar{p}_+ + \\ & + \sum_{j=1}^{n_-} \ln p_-(\phi_j^-, \delta_j^-, \eta_j^-) + (N_- - n_-) \ln \bar{p}_-, \end{aligned} \quad (8)$$

where $N_{\pm}$ are the beam counts and $n_{\pm}$ the event counts for the respective directions of target polarization.

Differentiation of eq. (8) with respect to the parameters α , P_T , P , A_S , and R_S shows that when $N_+ = N_-$ the only effect of the acceptance is via the moment c_2 . To first order a non-zero c_2 results in a correction for P . We estimate this effect by evaluating the asymmetry in the decay distribution with respect to the production normal for our by-product sample of forward-produced K^0 's. In fig. 6 we plot this asymmetry, with reference to the π^+ , against the four-momentum transfer squared from the incident π^- to the K^0 . The value obtained by an average over all t -bins is 0.009 ± 0.005 . For Λ 's this asymmetry corresponds to a polarization of 0.028 ± 0.015 .

7. DETERMINATION OF P , A_s , AND R_s

We consider $\approx 8500 \pi^- p \rightarrow \Lambda K^0$ events in the range $-u \leq 2 \text{ (GeV/c)}^2$. In the butanol target experiment the u -resolution was $\approx \pm 0.003 \text{ (GeV/c)}^2$ in the forward direction and was everywhere smaller than the bin width chosen. Before applying the maximum likelihood method, each of the five coefficients of eq. (2) is directly determined by a least-squares fit to its corresponding projected distribution. The two plots in fig. 7 show the decay proton asymmetry relative to the production normal for the butanol and hydrogen target events. The two best-fit values for αP , which are in agreement, are indicated on the figure. We have also checked that the u -dependence of the Λ polarization obtained from these two event samples is consistent. Therefore, it is reasonable to assume that bound and free proton events yield the same Λ polarization. The four remaining coefficients PP'_T , $\alpha P'_T$, $\alpha P'_T A_s$, and $\alpha P'_T R_s$ are obtained from the spin-up/spin-down asymmetry plotted against corresponding functions of the angles ϕ , δ , and η as shown in figs. 8 and 9, where the best-fit values are also indicated. These results when combined with the value for αP yield $P \approx 0.80$, $A_s \approx -0.93$, $R_s \approx 0.03$, $P'_T \approx 0.23$, and $\alpha \approx 0.55$, which are averages over the u -range considered. Using the same data, a maximum likelihood search gave $P \approx 0.76$, $A_s \approx -0.86$, $R_s \approx 0.02$, $P'_T \approx 0.24$, and $\alpha \approx 0.56$, in good agreement with the least-squares fit. This procedure serves as a check of the likelihood method, and the adequate straight line fits to the projected distributions show that the data are not biased significantly.

In order to impose the constraint $P^2 + A_s^2 + R_s^2 = 1$ in the maximum likelihood search, the parameters P , A_s , and R_s of eq. (3) were expressed in terms of two angles μ and ν :

$$\begin{aligned} P &= \cos \mu \\ A_s &= \sin \mu \sin \nu \\ R_s &= \sin \mu \cos \nu . \end{aligned} \tag{9}$$

The current best value for α reported by the Particle Data Group⁹⁾ is 0.647 ± 0.013 and can be included in a new likelihood function $\mathcal{L}$:

$$\ln \mathcal{L} = \ln L + \frac{1}{2} \left(\frac{\alpha - 0.647}{0.013} \right)^2 , \tag{10}$$

where L is defined in eq. (8). The value of α at the maximum of $\mathcal{L}$ represents a new best value. The events produced on liquid-hydrogen were included in the likelihood estimator of eq. (10) after setting $P'_T = 0$ for these events. To first order they should improve the determination of P .

The likelihood function of eq. (10) was used in evaluating the parameter P'_T . For this purpose the events were divided into 15 bins in u , and for each bin the parameters μ , ν , α , and P'_T were allowed to vary in the search. The resulting values of P'_T at the maxima in $\mathcal{L}$ are plotted in fig. 10. These results indicate that there is no significant P'_T dependence on u . The weighted mean value of P'_T is 0.286 ± 0.022 . In all subsequent likelihood searches the parameter P'_T was held at this value. Similarly, a best value of α can be determined from this experiment, by ignoring the second term in eq. (10), and this was 0.584 ± 0.046 .

Using eq. (10) with μ , ν , and α as variable parameters, the polarization P was determined in 16 bins over the u -range. The results are given in table 3 and fig. 11. The polarization rises rapidly to near its maximum value at $u_0 - u \approx 0.3 \text{ (GeV/c)}^2$ and falls off to a small value near 0.9 (GeV/c)^2 and probably increases beyond. A similar likelihood search in each of seven u -bins yielded the parameters μ , ν , α and therefore P , A_s , and R_s in the range $-u < 1.2 \text{ (GeV/c)}^2$. These results are shown in table 4 and fig. 12. The error correlation between μ and ν was not significant, and the errors quoted for P , A_s , and R_s were determined by projecting the surface bounded by the spherical coordinates $(1, \mu \pm \sigma_\mu, \nu \pm \sigma_\nu)$ onto the three axes corresponding to P , A_s , and R_s . The boundaries of the projected surface on each of the three axes correspond to $\pm\sigma$ from the most likely value.

8. AMPLITUDE ANALYSIS

In this section we discuss the amplitude structure suggested by our data. For comparison with exchange models we consider the invariant amplitudes¹⁰⁾ A' and B . The kinematic factors and crossing functions which relate P , R_s , and A_s to B/A' are given in the Appendix. In order to make full use of their u -dependence, a separate fit in terms of B/A' was made. The results, plotted in the complex B/A' plane, are shown in fig. 13. The orientation of the error bars gives an indication of the error correlations. Figure 14 contains, besides a plot of $|B/A'|$, the corresponding values for the t -channel polarization parameters R_t and A_t , which are defined in the Appendix. The kinematic factor relating them to B/A' is evaluated at the centre of each bin.

For the subsequent discussion we distinguish two u -regions:

$$\text{i) } u_0 - u \leq 0.4 \text{ (GeV/c)}^2 \quad \text{and} \quad \text{ii) } u_0 - u \geq 0.5 \text{ (GeV/c)}^2 .$$

For region (i) we conclude from fig. 13 that the rapid variation of P , A_s , or A_t is due to the spin-flip factor $\sqrt{u_0 - u}$, since $|B/A'|$ and $\sin \phi$ [$\phi = \arg(B/A')$] are approximately constant [$|B/A'| = 2 \text{ (GeV/c)}^{-1}$, $\sin \phi \approx 1$]. Note that the values of the polarization parameters at $u = u_0$ are independent of the dynamics. For the

strict value $\phi = 90^\circ$, R_t vanishes and R_s is small. The non-zero values for R_t and R_s in the bin around 0.25 (GeV/c)^2 therefore demand a somewhat larger value for ϕ .

In the second region (ii) we observe another kind of rapid variation of all polarization parameters, part of which occurs at u -values where $d\sigma/du$ is still decreasing exponentially.

The situation is in fact quite complicated, since according to figs. 11, 12, and 14 all the parameters seem to cross zero at nearly the same u in spite of the constraint $P^2 + R^2 + A^2 = 1$. In order to see what may happen in this u -region, where the statistics do not allow us to use small u -bins, we have tried to interpolate the measured points in the B/A' plane. The continuous line in fig. 13 is a conjecture for this contour. B/A' goes clockwise with increasing $u_0 - u$. This interpolation takes into account the fact that B/A' has to return to the upper half plane since P is positive above $u_0 - u = 1.2 \text{ (GeV)}^2$ (fig. 11). It has furthermore the feature that P passes twice through zero in the interval $0.7 \leq u_0 - u \leq 1.2 \text{ (GeV/c)}^2$ with a zero of R_t lying in between. This zero structure is in fact characteristic of a monotonic decrease of ϕ with increasing $u_0 - u$. The detailed implications of the chosen contour for the t -channel polarization parameters can be seen from the continuous lines in fig. 11 and figs. 14b and 14c.

From a theoretical point of view⁴⁾, B/A' might be expected to be dominated by the two degenerate pairs $\Sigma_\alpha - \Sigma_\gamma$, $\Sigma_\beta - \Sigma_\delta$ of Σ Regge poles, i.e. $A' = A'_{\alpha\gamma} + A'_{\beta\delta}$, $B = B_{\alpha\gamma} + B_{\beta\delta}$.

The amplitudes factorize into real residues (b, a') and a Regge phase as shown below:

$$\begin{pmatrix} B_{\alpha\gamma} \\ A'_{\alpha\gamma} \end{pmatrix} = \begin{pmatrix} b_{\alpha\gamma} \\ a'_{\alpha\gamma} \end{pmatrix} e^{-i\pi\alpha_{\alpha\gamma}} \begin{pmatrix} B_{\beta\delta} \\ A'_{\beta\delta} \end{pmatrix} = \begin{pmatrix} b_{\beta\delta} \\ a'_{\beta\delta} \end{pmatrix} e^{-i\pi\alpha_{\beta\delta}}.$$

In terms of these quantities we have

$$\frac{B}{A'} = S_+ + S_- \frac{1 - \eta_\Sigma \exp \{-i\pi(\alpha_{\beta\delta} - \alpha_{\alpha\gamma})\}}{1 + \eta_\Sigma \exp \{-i\pi(\alpha_{\beta\delta} - \alpha_{\alpha\gamma})\}} \quad (11)$$

with the definitions

$$S_\pm = \frac{1}{2} (b_{\alpha\gamma}/a'_{\alpha\gamma} \pm b_{\beta\delta}/a'_{\beta\delta}), \quad \eta_\Sigma = a'_{\beta\delta}/a'_{\alpha\gamma}.$$

The fact that the polarization reaches 100% requires the spacing of the trajectories $\alpha_{\beta\delta} - \alpha_{\alpha\gamma}$ to be near 0.5, which is consistent with their separation for positive u . As a consequence the complex factor multiplying S_- in eq. (11) has modulus 1. The real functions $S_\pm$ are assumed in ref. 4 to be characterized

by the property of the residues to eliminate certain parity doublets. In the approximation of a common mass series (M_i , $i = 1, \dots, k$) for the eliminated doublets on the two trajectories, one has $S_+ = 0$, $S_- = \gamma^{\text{odd}}(u)/\gamma^{\text{even}}(u)$, where γ^{even} and $\sqrt{u} \gamma^{\text{odd}}$ are the even and odd parts of $\gamma = \prod_{i=1}^k [1 + (\sqrt{u}/M_i)]$. In this case $|B/A'|$ is predictable from γ , and the phase ϕ only depends on η_Σ .

The three curves in fig. 14a were obtained with the masses $M_1 = 1.29$ GeV, $M_2 = 1.72$ GeV, and $M_3 = 1.97$ GeV. These masses, in ascending order, are the average of the masses of the Σ and the $Y^*(1385)$ and of the next two corresponding pairs in the two trajectories. Both $k = 2$ and $k = 3$ show reasonable agreement for $u_0 - u \leq 0.6$ (GeV/c)². The tendency of the data is, however, opposite above 0.6 (GeV/c)². For a phase value ϕ around 90° the ratio η_Σ , which is expected to be positive for small $u_0 - u$ (ref. 4), lies near 1. The subsequent decrease of ϕ in the region $u_0 - u \leq 0.8$ (GeV/c)² corresponds to a decrease of η_Σ by roughly a factor of 4.

This u -dependence of hyperon Regge residues is similar to that of nucleon residues. We can compare the decrease of η_Σ with the behaviour of η_N which is the analogue of η_Σ for the two pairs $N_\alpha - N_\gamma$ and $N_\beta - \Delta_\delta$. By an analysis of backward scattering data ($\pi^- p \rightarrow p \pi^-$, $\pi^+ p \rightarrow p \pi^+$, $K^- n \rightarrow \Lambda \pi^-$) Halzen¹¹⁾ has determined effective residues. Under the assumption that these residues are real (no residual dependence on $\sqrt{u}$) we can find an estimate for η_N ; η_N decreases by at least a factor of 4 over the u -range in question.

A final remark concerns the dip in $d\sigma/du$ in the backward direction (fig. 5). With the observed flip/non-flip ratio $|B/A'| \approx 2$ (GeV/c)⁻¹ the kinematical factor $\sqrt{u_0 - u}$ does not produce a sizeable flattening of the exponential behaviour of $d\sigma/du$ observed in the interval $0.1 \leq u_0 - u \leq 1$ (GeV/c)². An unexpectedly rapid variation near the backward direction in A' would therefore be needed. In a previous experiment at 4 and 6.2 GeV/c we have found a modest flattening only. We cannot exclude therefore that the small value of $d\sigma/du$ in the first bin is due to a statistical fluctuation.

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Table 1

Correction factors independent of u

Electronic counting losses	1.08	± 0.01
Losses in pattern recognition	1.15	± 0.06
Cuts on $P(> \chi^2)$	1.06	
Target empty correction	0.942	± 0.015
Branching ratios $\Lambda \rightarrow p\pi^-$ and $K^0 \rightarrow K_L^0$	3.115	± 0.024
π^- and Λ absorption in target	1.074	± 0.008
Λ absorption in F counters	1.120	± 0.020
Contamination from Σ^0 and $\Lambda\pi^0$	0.95	± 0.03
Muon contamination in the beam	1.05	± 0.03
Over-all correction factor = 4.62 ± 0.36		

Table 2

u-dependent correction factors and the differential cross-section for $\pi^-p \rightarrow \Lambda K^0$ at 5 GeV/c

$u_0 - u$ $(\text{GeV}/c)^2$	u-dependent correction factor	$d\sigma/du$ $\mu\text{b}/(\text{GeV}/c)^2$
0.00 - 0.025	3.11	2.91 ± 0.38
0.025 - 0.050	3.06	3.93 ± 0.48
0.050 - 0.075	3.64	4.21 ± 0.56
0.075 - 0.10	3.64	3.71 ± 0.52
0.10 - 0.15	3.10	2.92 ± 0.27
0.15 - 0.20	3.09	1.97 ± 0.20
0.20 - 0.25	3.05	1.48 ± 0.18
0.25 - 0.30	3.03	1.31 ± 0.16
0.30 - 0.40	3.16	0.90 ± 0.095
0.4 - 0.5	3.27	0.41 ± 0.065
0.5 - 0.6	3.23	0.29 ± 0.055
0.6 - 0.7	3.07	0.142 ± 0.032
0.7 - 0.8	3.32	0.096 ± 0.032
0.8 - 1.0	4.08	0.053 ± 0.017
1.0 - 1.2	5.29	0.038 ± 0.016
1.2 - 1.4	8.69	0.038 ± 0.022
1.4 - 1.6	16.24	0.071 ± 0.045

Table 3

The polarization in $\pi^-p \rightarrow \Lambda K^0$ at 5 GeV/c

$u_0 - u$ (GeV/c) ²	Polarization
0.0 - 0.025	0.211 $\pm$ 0.092
0.025 - 0.050	0.350 $\pm$ 0.096
0.050 - 0.075	0.581 $\pm$ 0.094
0.075 - 0.10	0.699 $\pm$ 0.091
0.10 - 0.15	0.753 $\pm$ 0.070
0.15 - 0.20	0.838 $\pm$ 0.080
0.20 - 0.25	0.836 $\pm$ 0.083
0.25 - 0.30	0.960 $\pm$ 0.066
0.3 - 0.4	0.927 $\pm$ 0.071
0.4 - 0.5	0.974 $\pm$ 0.109
0.5 - 0.6	0.454 $\pm$ 0.158
0.6 - 0.8	0.310 $\pm$ 0.156
0.8 - 1.0	-0.222 $\pm$ 0.235
1.0 - 1.2	0.098 $\pm$ 0.294
1.2 - 1.6	0.67 $\pm$ 0.27
1.6 - 2.0	-0.09 $\pm$ 0.51

Table 4
The parameters P , A_s , and R_s

$u_0 - u$ (GeV/c) ²	μ (degrees)	ν (degrees)	P	A_s	R_s
0.00 - 0.05	74.1 $\pm$ 4.0	278 $\pm$ 42	0.274 $\pm$ 0.066	-0.952 $\pm$ 0.329 - 0.017	0.136 $\pm$ 0.605 - 0.676
0.05 - 0.10	50.2 $\pm$ 4.9	269 $\pm$ 22	0.639 $\pm$ 0.066	-0.769 $\pm$ 0.062 - 0.052	-0.007 $\pm$ 0.296 - 0.295
0.10 - 0.20	37.8 $\pm$ 4.8	294 $\pm$ 22	0.790 $\pm$ 0.051	-0.557 $\pm$ 0.140 - 0.057	0.257 $\pm$ 0.192 - 0.232
0.20 - 0.30	26.2 $\pm$ 6.9 - 8.6	190 $\pm$ 37	0.897 $\pm$ 0.052 - 0.067	-0.080 $\pm$ 0.279 - 0.246	-0.434 $\pm$ 0.137 - 0.115
0.30 - 0.50	19.5 $\pm$ 9.2 - 13.8	44 $\pm$ 70	0.942 $\pm$ 0.045 - 0.075	0.233 $\pm$ 0.114 - 0.384	0.240 $\pm$ 0.118 - 0.383
0.50 - 0.80	67.6 $\pm$ 6.9	34 $\pm$ 48	0.382 $\pm$ 0.111	0.510 $\pm$ 0.406 - 0.752	0.771 $\pm$ 0.153 - 0.646
0.80 - 1.20	96.0 $\pm$ 10.9	235 $\pm$ 41	-0.105 $\pm$ 0.186	-0.817 $\pm$ 0.582 - 0.177	-0.567 $\pm$ 0.684 - 0.400

APPENDIX

The crossing angle between the s- and t-channel helicity amplitudes in

$$\begin{bmatrix} H_{++}^t \\ H_{+-}^t \end{bmatrix} = \begin{bmatrix} \cos \chi & \sin \chi \\ -\sin \chi & \cos \chi \end{bmatrix} \begin{bmatrix} H_{++}^s \\ H_{+-}^s \end{bmatrix}$$

is given by $\chi = (\chi_\Lambda - \chi_p)/2$, where χ_p and χ_Λ represent the following individual crossing angles of the two spin 1/2 particles p and Λ :

$$\cos \chi_\Lambda = \frac{(s + m_\Lambda^2 - m_k^2)(t + m_\Lambda^2 - m_p^2) - 2m_\Lambda^2 \cdot D}{\lambda_{1/2}(s, m_\Lambda^2, m_k^2) \cdot \lambda_{1/2}(t, m_p^2, m_\Lambda^2)}; \quad \sin \chi_\Lambda \geq 0$$

$$\cos \chi_p = \frac{(s + m_p^2 - m_\pi^2)(t + m_p^2 - m_\Lambda^2) + 2m_p^2 \cdot D}{\lambda_{1/2}(s, m_p^2, m_\pi^2) \cdot \lambda_{1/2}(t, m_p^2, m_\Lambda^2)}; \quad \sin \chi_p \geq 0$$

$$D = m_\Lambda^2 + m_\pi^2 - m_p^2 - m_K^2$$

$$\lambda_{1/2}(x_1, x_2, x_3) = (x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_1x_3)^{1/2}.$$

The relation between H_{+-}^t/H_{++}^t and B/A' is

$$\frac{H_{+-}^t}{H_{++}^t} = - \frac{\sqrt{s} \sqrt{t_0 - t} \sqrt{u_0 - u}}{4\bar{m}^2 - t} \cdot \frac{B}{A'},$$

where the constants $\bar{m}$ and $\sqrt{s}$ are the average external baryon mass and the total energy in the centre of mass, respectively.

The t-channel polarization parameters R_t and A_t are defined as

$$R_t = \frac{2 \operatorname{Re} H_{++}^t H_{+-}^{t*}}{|H_{++}^t|^2 + |H_{+-}^t|^2}$$

$$A_t = \frac{|H_{++}^t|^2 - |H_{+-}^t|^2}{|H_{++}^t|^2 + |H_{+-}^t|^2}.$$

Figure captions

- Fig. 1 : The coordinate system and angles used in the analysis; they correspond to s-channel helicities.
- Fig. 2 : Schematic layout of the beam and the magnet-spark-chamber spectrometer. We show the special pole-pieces for the butanol polarized target and detailed views of the veto counters surrounding the target. The electronic logic used for the trigger is indicated.
- Fig. 3 : Schematic layout of the beam and the magnet-spark-chamber spectrometer with a liquid-hydrogen target. Detailed views of the veto counters surrounding the target are shown and the electronic logic used for the trigger is indicated.
- Fig. 4 : Histograms of the missing-mass squared to all Λ 's with probability $P(> \chi^2) > 0.03$ for the fit to the decay hypothesis $\Lambda \rightarrow p\pi^-$. The shaded areas correspond to unambiguously identified Λ 's for a) the butanol target and b) the hydrogen target.
- Fig. 5 : The differential cross-section for $\pi^-p \rightarrow \Lambda K^0$ at 5 GeV/c. The u-resolution is everywhere smaller than the bin width.
- Fig. 6 : The asymmetry in the decay distribution with respect to the production normal for the by-product sample of forward-produced unambiguously identified K^0 's. This asymmetry is evaluated with reference to the π^+ and plotted against the four-momentum transfer squared from the incident π^- to the outgoing K^0 . The small average asymmetry observed shows that the effect of the acceptance is not significant. Note the scale change at 0.7 (GeV/c)^2 .
- Fig. 7 : The Λ -decay distribution relative to the production normal is plotted against $\cos \delta$ for a) the butanol target and b) the hydrogen target. The straight lines are from a least-squares fit to the data. The best fit coefficients indicated are in agreement.
- Fig. 8 : The spin-up/spin-down asymmetry is plotted against a) $\cos \phi$ and b) $\cos \phi \cos \delta$. The straight lines are from a least-squares fit to the data. The best fit coefficients together with the $\chi^2/\text{degree of freedom}$ are indicated.

- Fig. 9 : The spin-up/spin-down asymmetry is plotted against a) $\sin \phi \sin \delta \sin \eta$ and b) $\sin \phi \sin \delta \cos \eta$. The straight lines are from a least-squares fit to the data. The best fit coefficients together with the $\chi^2/\text{degree of freedom}$ are indicated.
- Fig. 10 : The effective target polarization P'_T determined from a maximum likelihood analysis is plotted against $u_0 - u$. The weighted average value and the $\chi^2/\text{degree of freedom}$ are indicated. Note the scale change at $0.7 (\text{GeV}/c)^2$.
- Fig. 11 : The polarization in $\pi^- p \rightarrow \Lambda K^0$ at 5 GeV/c. The continuous line is a model curve discussed in Section 8.
- Fig. 12 : The parameters P , A_s , and R_s for $\pi^- p \rightarrow \Lambda K^0$ at 5 GeV/c.
- Fig. 13 : The results of the likelihood fit in terms of B/A' are plotted in the complex plane. The orientation of the error bars reflect the error correlations of the fit. For each point the centre of the corresponding $u_0 - u$ bin in $(\text{GeV}/c)^2$ is indicated. The continuous line is a conjecture for the behaviour of B/A' , with the values of $u_0 - u$ in $(\text{GeV}/c)^2$ attached (see text).
- Fig. 14 : The data points shown in these figures represent the following projections of those of fig. 13:
- a) the magnitude $|B/A'|$
 - b) spin polarization parameter A_t
 - c) spin polarization parameter R_t .
- The dashed curves in (a) are the predictions of an exchange model. The continuous lines in (b) and (c) result from the interpolation shown in fig. 13 and discussed in the text.

Fig. 1

$$\text{TRIGGER} \equiv S_1 \cdot C_1 \cdot S_2 \cdot S_3 \cdot S_4 \cdot S_5 \cdot (\sum A_i + R_i + F_i) \cdot \overline{C_2} \cdot E \cdot (\sum L_i \cdot B_i)$$

Fig. 2

10 cm

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14