Testing Neutrino Mass Models

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The existence of the neutrino masses and mixing would be an important window into the nature of physics beyond the Standard Model, which will be searched for in the forthcoming experiments such as LHC. In this talk, we discuss some examples of neutrino mass models which are testable through the observation of lepton flavor violating processes and/or electric dipole moments correlated with the neutrino mass structure.

INTRODUCTION AND SUMMARY

While the Standard Model is firmly established by various accelerator experiments in the past, there are also a number of theoretical and experimental reasons that it is a low energy effective theory of a more fundamental theory. The discovery of neutrino masses and mixing in astrophysical and reactor neutrino experiments would be one of the most important experimental evidences for physics beyond the Standard Model. A large number of new physics models have been suggested to explain the origin of the neutrino mass. Among them, low-energy models at TeV scale are of the special interest as they can be directly tested in the forthcoming colliders; LHC or LC. Some high-energy models, motivated by superstring phenomena can be derived in the triplet seesaw model.

In this talk, we present three examples of such testable neutrino mass models in anticipation of discovering new physics in the future collider or low-energy experiments:

1. Higgs Triplet model at TeV scale where the neutrino mass structure can be probed by observing doubly charged Higgs bosons decaying to two charged leptons.

2. Supersymmetric standard model with R-parity violation where the lightest supersymmetric particle (LSP) decay encodes the information of the neutrino masses and mixing.

3. Supersymmetric triplet seesaw model at high-energy scale which may provide testable correlations among lepton flavor violating (LFV) processes and electric dipole moments (EDMs) reflecting the neutrino flavor structure.

Let us now discuss the main features of each model which can be searched for in the future experiments to discover the new physics beyond the Standard Model.

TRIPLET SEESAW MODEL AT TEV SCALE

The first example is the Higgs triplet model in which a triplet scalar field $T = (T^+, T^0, T^-)$ with the mass $M$ is introduced to have the following renormalizable couplings:

$$
\mathcal{L}_T = \frac{1}{\sqrt{2}} [f_{ij} L_i L_j T + \mu \Phi \Phi T + h.c.] - M^2 |T|^2, \quad (1)
$$

where $L_i = (\nu_i, l_i)_L$ is the left-handed lepton doublet and $\Phi = (\phi^0, \phi^-)$ is the standard model Higgs doublet. Due to the “$\mu$” term in the above equation, the neutral component $T^0$ of the triplet gets the vacuum expectation value (VEV), $v_T = \mu v_\phi^2 / 2M^2$ where $v_\phi = (\phi^0) = 246$ GeV. This leads to the neutrino mass matrix,

$$
M'_{ij} = f_{ij} v_T. \quad (2)
$$

The above relation shows that the nine independent parameters contained in $f$ are in one-to-one correspondence with the low-energy neutrino parameters. As a consequence unambiguous predictions on the low-energy LFV phenomena can be derived in the triplet seesaw model.

We look for the possibility of the light triplet Higgs bosons, namely $M \sim$ TeV, so that observations of various lepton flavor violating processes can provide a probe for the neutrino masses and mixing through the relation (2), and thus a direct test of the model. In this “low-energy triplet Higgs model”, the small parameters $f$ and $\xi \equiv v_T / v_\phi$ are required;

$$
f_{ij} \xi \sim 10^{-12} \quad (3)
$$

for $M'_{ij} \sim 0.3$ eV. For sizable couplings $f$, the exchange of the triplet Higgs can induce the lepton flavor violating processes like $\mu \rightarrow e\gamma$. In Table I, the bounds on the couplings $f$ are summarized.

Some of striking collider signals in the triplet Higgs model come from the decays of a doubly charged Higgs boson, such as $T^{--} \rightarrow l_i l_j W^- W^-$, which have been studied extensively in the literature. We are interested in the situation that the decays $T^{--} \rightarrow l_i l_j$ are sizable so that the neutrino mass structure can be tested in colliders. Depending on the masses of the triplet components, the fast decay process like $T^{--} \rightarrow T^- W^(*)^-$ through gauge interactions can happen to over-dominate any other processes of our interest. The mass splitting among the triplet components arises upon the electroweak symmetry breaking and thus is of the order $M_W$. The most general scalar potential for a doublet and a triplet Higgs boson is

$$
V = m_2^2 (\Phi^\dagger \Phi) + \lambda_3 (\Phi^\dagger \Phi)^2 + M^2 \text{Tr}(\Delta^\dagger \Delta) + \lambda_2 [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_1 \text{Det}(\Delta^\dagger \Delta) + \lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 (\Phi^\dagger \tau_i \Phi) \text{Tr}(\Delta^\dagger \tau_i \Delta) + \frac{1}{\sqrt{2}} \mu (\Phi^T i \tau_3 \Delta \Phi) + h.c., \quad (4)
$$

where $\mu$ is the mass parameter of the triplet scalar triplet $\Delta$ and $\lambda$ are the coupling constants of the Lagrangian.
where $\Delta$ is the $2 \times 2$ matrix representation of the triplet $T$. In this model, the mass eigenstates consist of $T^{++}$, $H^+$, $H^0$, $A^0$ and $h^0$. Under the condition that $|\xi| \ll 1$, the first five states are mainly from the triplet sector and the last from the doublet sector.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Current limit</th>
<th>Future sensitivity</th>
<th>Bound on the couplings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu \to \gamma \gamma$</td>
<td>$1.2 \times 10^{-11}$</td>
<td>$\sim 10^{-14}$</td>
<td>$(f_{i}^{T})_{12} &lt; 1.2 \times 10^{-4} x_T$</td>
</tr>
<tr>
<td>$\tau \to \gamma \gamma$</td>
<td>$2.7 \times 10^{-6}$</td>
<td>$\sim 10^{-8}$</td>
<td>$(f_{i}^{T})_{13} &lt; 1.3 \times 10^{-1} x_T$</td>
</tr>
<tr>
<td>$\mu \to \mu \gamma$</td>
<td>$0.6 \times 10^{-6}$</td>
<td>$\sim 10^{-8}$</td>
<td>$(f_{i}^{T})_{23} &lt; 6.1 \times 10^{-2} x_T$</td>
</tr>
<tr>
<td>$\mu \to \tau \gamma$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>$\sim 10^{-15}$</td>
<td>$f_{11} f_{12} &lt; 9.3 \times 10^{-7} x_T$</td>
</tr>
<tr>
<td>$\tau \to \tau \gamma$</td>
<td>$2.7 \times 10^{-7}$</td>
<td>$\sim 10^{-8}$</td>
<td>$f_{11} f_{13} &lt; 1.1 \times 10^{-3} x_T$</td>
</tr>
<tr>
<td>$\tau \to \mu \mu$</td>
<td>$2.4 \times 10^{-7}$</td>
<td>$\sim 10^{-8}$</td>
<td>$f_{12} f_{13} &lt; 1.5 \times 10^{-3} x_T$</td>
</tr>
<tr>
<td>$\tau \to \tau \mu$</td>
<td>$3.2 \times 10^{-7}$</td>
<td>$\sim 10^{-8}$</td>
<td>$f_{22} f_{13} &lt; 1.2 \times 10^{-3} x_T$</td>
</tr>
<tr>
<td>$\tau \to \mu \tau$</td>
<td>$2.8 \times 10^{-7}$</td>
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</tr>
<tr>
<td>$\tau \to \bar{\mu} \mu$</td>
<td>$3.8 \times 10^{-7}$</td>
<td>$\sim 10^{-8}$</td>
<td>$f_{22} f_{23} &lt; 1.4 \times 10^{-3} x_T$</td>
</tr>
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TABLE I: The experimental limits on the branching ratios of various modes and the corresponding upper bounds on the product of couplings taking $x_T = (M_T/200 \text{GeV})^2$.

When $\lambda_5 > 0$, we have $M_{T^{\pm \pm}} < M_{H^{\pm}} < M_{H^0, A^0}$, so that the doubly charged Higgs boson $T^{--}$ can only decay to $l_i l_j$ or $W^- W^-$ through the following interactions:

$$
\mathcal{L} = \frac{1}{\sqrt{2}} [ f_{ij} \bar{f}_i P_l l_j + g \xi M_W W^- W^- ] T^{++} + \text{h.c.} \tag{5}
$$

The corresponding decay rates are

$$
\Gamma(T^{--} \rightarrow l_i l_j) = S \frac{f_{ij}^2}{16 \pi} M_{T^{\pm \pm}}
$$

$$
\Gamma(T^{--} \rightarrow WW) = \frac{\alpha_2 \xi^2 M_{T^{\pm \pm}}^2}{32 M_W^2} (1 - 4r_W + 12r_W^2)
\frac{1}{(1 - 4r_W)^{1/2}} \tag{6}
$$

where $S = 2(1)$ for $i \neq j$ ($i = j$) and $r_W = M_W^2/M_{T^{\pm \pm}}^2$. In this case, the heavier states $H^+, H^0$ and $A^0$ can have the decay modes: $H^0, A^0 \to H^+W^-(i)$ and $H^+ \to T^{++}W^-(i)$ leading to the production of $T^{\pm \pm}$.

When $\lambda_5 < 0$, one has $M_{T^{\pm \pm}} > M_{H^0} > M_{H^0, A^0}$. In this case, the decay processes of $T^{--} \rightarrow H^- W^-$ and $H^- \rightarrow H^0 (A^0) W^-$ can be allowed through the usual gauge interactions:

$$
\mathcal{L} = ig W^+[H^+ \overline{\Phi} T^{--} + \frac{1}{\sqrt{2}} H^0 \overline{\Phi} H^- + \frac{i}{\sqrt{2}} A^0 \overline{\Phi} H^-] + \text{h.c.}, \tag{7}
$$

giving rise to the decay rate

$$
\Gamma(T^{--} \rightarrow H^- W^-) = \frac{g^2}{8\pi} M_W \left[ 1 + \frac{2y^2 - y - 1}{2} r_W \right]
\left[ \frac{(y + 1)^2}{4} r_W - 1 \right]^{1/2} \tag{8}
$$

where $y = 2|\lambda_5|/g^2$. To suppress this decay mode, we will require $M_{T^{\pm \pm}} < M_{H^0} + M_W$, that is, $M_{T^{\pm \pm}} > \frac{(y + 1)}{2} M_W$. For $M_{T^{\pm \pm}} = 200$ GeV, it implies $|\lambda_5| < 0.89$. Thus, the decay $T^{--} \rightarrow H^- W^-$ is forbidden unless the coupling $\lambda_5$ is extremely large. Now, the off-shell production of $W$, $T^{--} \rightarrow H^- W^-$, is allowed to have the rate:

$$
\Gamma(T^{--} \rightarrow H^- W^-) \approx \frac{3G_\text{F}^2 g^5 M_W^6}{160 \pi^3} M_{T^{\pm \pm}}^{11/2} \tag{9}
$$

in the leading term of $y M_W^2$. With the further requirement of $\Gamma(T^{--} \rightarrow H^- W^-, i) < \Gamma(T^{--} \rightarrow l_i l_j)$, we limit ourselves in the parameter space satisfying

$$
|\lambda_5| < 0.16 \left( \frac{M_{T^{\pm \pm}}}{200 \text{ GeV}} \right)^{6/5} \left( \frac{f_{ij}}{10^{-3}} \right)^{2/5}. \tag{10}
$$

Let us now note that the triplet Higgs decay is short enough to occur inside colliders. Assuming Eq. (6) as the main decay rates and recalling $\sum_{ij} f_{ij}^2 \propto \text{Tr}(M^2_{\tilde{T}})$ where $M_{\tilde{T}} = f_{ij} \xi \varphi$, one obtains the following form of the total decay rate:

$$
\Gamma_{T^{\pm \pm}} = M_{T^{\pm \pm}} \left( \frac{1}{16 \pi} \frac{\tilde{m}^2}{\xi^2 \varphi^2} \right)
+ \frac{\alpha_2}{32} \frac{\xi^2}{r_W} (1 - 4r_W + 12r_W^2)(1 - 4r_W)^{1/2} \tag{11}
$$

where $\tilde{m}^2 \equiv \sum_i m_{T_i}^2$. When $M_{T^{\pm \pm}} > 2 M_W$, one finds the minimum value of the total decay rate given by

$$
\Gamma_{T^{\pm \pm}}|_{\text{min}} = \frac{1}{8 \pi} \frac{M_{T^{\pm \pm}} \tilde{m}^2}{\xi^2 \varphi^2}
$$
where $\hat{\xi}^2 = (2\sqrt{2}/g) r_W^{1/2}(\tilde{m}/v_h)(1-4 r_W + 12 r_W^2)^{-1/2}(1-4 r_W)^{-1/4}$. Taking $\tilde{m} = 0.05$ eV and $M_{T^{\pm}} = 200$ GeV, we obtain $\Gamma_{\pm|\text{min}} \approx 6 \times 10^{-13}$ GeV and $\hat{\xi} \approx 6 \times 10^{-7}$, leading to $\tau_{\text{max}} \approx 0.03$ cm. When $M_{T^{\pm}} < 2 M_W$, only the first term in Eq. (12) contributes and the total decay rate is then $\Gamma > 8 \times 10^{-14}$ GeV for $M_{T^{\pm}} = 100$ GeV and $\hat{\xi} < 10^{-6}$. Thus, as far as $T^{-} \to l_{1}l_{2}$ are the main decay modes of the doubly charged Higgs boson, its decay signal should be observed in colliders.

In the linear collider with $\sqrt{s} = 1$ TeV, the pair production cross section is $\sigma \approx (100 - 10) \text{ fb}$ for $M_{T^{\pm}} = (100 - 450)$ GeV. Taking $L = 1000/$fb, the number of the produced $T^{\pm\pm}$ will be $N = (10^6 - 10^5)$. In LHC with $L = 1000/$fb, the number of the reconstructed pair production events is expected to be $N = (10^5 - 10^4)$ for $M_{T^{\pm\pm}} = (100 - 450)$ GeV and it becomes down to $N = 10$ for $M_{T^{\pm\pm}} = 1000$ GeV. Thus, both LC and LHC can produce enough numbers of $T^{\pm\pm}$ to probe the neutrino mass pattern if $M_{T^{\pm\pm}} \lesssim 450$ GeV. In this case, the precise measurement of the branching ratios can also reconstruct the neutrino mass matrix $f$. LHC has also a good potential to confirm the triplet Higgs model as the source of neutrino mass matrix up to the triplet mass around 1 TeV. Let us finally note that the observation of the leading decay modes will be enough to discriminate the neutrino mass patterns: hierarchy with $m_1 < m_2 < m_3$ (HI); Inverse Hierarchy with $m_1 \simeq m_2 \gg m_3$ (IH1) and $m_1 = -m_2 \gg m_3$ (IH2); Degeneracy with $m_1 \simeq m_2 \simeq m_3$ (DG1), $m_1 \simeq m_2 \simeq -m_3$ (DG2), $m_1 \simeq -m_2 \simeq m_3$ (DG3), $m_1 \simeq -m_2 \simeq -m_3$ (DG4), each of which predicts

\begin{align*}
\text{(HI)} & \quad B(\mu\mu) : B(\tau\tau) : B(\mu\tau) = \frac{1}{2} : \frac{1}{2} : 1 \\
\text{(IH1)} & \quad B(\mu\mu) : B(\tau\tau) : B(\mu\tau) = 1 : 1 : \frac{1}{4} \\
\text{(IH2)} & \quad B(\mu\mu) : B(\tau\tau) = 1 : 1 \\
\text{(DG1)} & \quad B(\mu\mu) : B(\tau\tau) = 1 : 1 : 1 \\
\text{(DG2)} & \quad B(\mu\mu) : B(\tau\tau) = 1 : 1 \\
\text{(DG3)} & \quad B(\mu\mu) : B(\tau\tau) = 1 : 1 : \frac{1}{2} \cot^2 \theta_3 \\
\text{(DG4)} & \quad B(\mu\mu) : B(\tau\tau) = 1 : 1 : \frac{1}{4} \cot^2 \theta_3 : 1 : 1
\end{align*}

SUPERSYMMETRIC STANDARD MODEL WITH R-PARITY VIOLATION

The general superpotential of the supersymmetric standard model allowing lepton number violation is

\[ W_0 = \mu H_1 H_2 + Y_e L H_1 E^c + Y_d Q H_1 D^c + Y_u Q H_2 U^c, \]

\[ W_1 = \lambda_i L_i L_3 E^c_3 + \lambda_i' L_i Q_3 D^c_3, \quad (12) \]

where $W_0$ is R-parity conserving part and $W_1$ is R-parity violating part written in the basis where the bilinear term $L_i H_2$ is rotated away. Here, we have taken only 5 trilinear couplings, $\lambda_i$ and $\lambda_i'$, assuming the usual hierarchy of Yukawa couplings. Among soft SUSY breaking terms, R-parity violating bilinear terms are given by

\[ V_0 = m_{L,H_1}^2 L_i H_1^* + B_i L_i H_2 + h.c., \quad (13) \]

where $B_i$ is the dimension-two soft parameter. We will denote the Higgs bilinear term as $B H_1 H_2$. In the mSUGRA model, the bilinear parameters, $m_{L,H_1}^2$ and $B_i$ vanishes at the supersymmetry breaking mediation scale, and their non-zero values at the weak scale are generated through renormalization group (RG) evolution which will be included in our numerical calculations. For the consistent calculation of the Higgs and slepton potential, we need to include the 1-loop contributions to the scalar potential as follows:

\[ V_1 = \frac{1}{64\pi^2} \text{Str} M^4 \left( \ln \frac{M^2}{Q'^2} - \frac{3}{2} \right). \quad (14) \]

As is well-known, the electroweak symmetry breaking gives rise to a nontrivial vacuum expectation values of sneutrino $\tilde{\nu}_i$ as follows:

\[ \xi_i \equiv \langle \tilde{\nu}_i \rangle / \langle H^0_1 \rangle = \frac{m_{L,H_1}^2 + B_i t_{3} + \Sigma^{(1)}_{L_i}}{m_{\tilde{\nu}_i}^2 + \Sigma^{(2)}_{L_i}}, \quad (15) \]

where the 1-loop contributions $\Sigma^{(1,2)}_{L_i}$ are given by $\Sigma^{(1)}_{L_i} = \partial V_1 / \partial H^*_1 \partial L_i$, $\Sigma^{(2)}_{L_i} = \partial V_1 / \partial L_i \partial L_i$. The bilinear R-parity violating parameters induce the mixing between the ordinary particles and superparticles, namely, neutrinos/neutralinos, charged leptons/charginos, neutral Higgs bosons/sneutrinos, as well as charged Higgs bosons/charged sleptons. The mixing between neutrinos and neutralinos particularly serves as the origin of the tree-level neutrino masses. We note that the parameters $\xi_i$ should be very small to account for tiny neutrino masses. While the effect of such small parameters on the particle and sparticle mass spectra (apart from the neutrino sector) are negligible, they induce small but important R-parity violating vertices between the particles and sparticles, which in particular destabilizes the LSP together with the original trilinear couplings, $\lambda_i$ and $\lambda_i'$. From the seesaw formulae associated with the heavy four neutrinos, we obtain the light tree-level neutrino mass matrix of the form:

\[ M^\text{tree}_{ij} = -\frac{M^2}{F_N} \xi_i \xi_j \cos^2 \beta, \quad (16) \]

where $F_N = M_1 M_2 / M_{\tilde{G}} + M_2^2 \cos 2\beta / \mu$ with $M_{\tilde{G}} = e_W^2 M_1 + s_W^2 M_2$. The R-parity violating vertices between particles and sparticles can give rise to 1-loop neutrino masses. Including all the 1-loop corrections, the loop mass matrix can be written as

\[ M^\text{loop}_{ij} = -\frac{M^2}{F_N} (\xi_i \delta_j + \delta_i \xi_j) \cos \beta + \Pi_{ij}, \quad (17) \]
where $\Pi_{ij}$ denotes the 1-loop contribution of the neutrino self energy and

$$\delta_i = \Pi_{i\nu} \tilde{\nu}_i \left( -\frac{M_2 \sin^2 \theta_W}{M_\gamma M_W \tan \theta_W} + \Pi_{i\nu} \tilde{\nu}_3 \left( \frac{M_1 \cos^2 \theta_W}{M_\gamma M_W} \right) \right) + \Pi_{i\nu} \tilde{\nu}_1 \left( \frac{\sin \beta}{\mu} \right) + \Pi_{i\nu} \tilde{\nu}_2 \left( -\frac{\cos \beta}{\mu} \right).$$

(18)

Based on the neutrino mass matrix presented in the above, we will discuss whether the above mass matrix $M''$ can account for both atmospheric and solar neutrino experimental data.

For the generic parameter space of the R-parity violating mSUGRA model, the tree mass is dominating well over the loop contribution so that the atmospheric neutrino mixing angle $\theta_{23}$ has a much smaller correlation with the parameter $\xi_i$ than $\lambda'_i$, confirming the relation, $\sin^2 2\theta_{23} = \frac{4|\xi_2|^2}{\left| \sum_i |\xi_i|^2 \right|^2}$. The condition, $|\lambda'_i| < |\lambda'_2| \approx |\lambda'_3|$, is required to arrange the large atmospheric angle $\theta_{23}$ and the small CHOOZ angle $\theta_{13}$. However, the ratio of the loop to tree masses is smaller than 0.1 so that it cannot account for the mild hierarchy between the solar and atmospheric neutrino mass scales. This implies that there must be some cancellation to reduce the tree mass. As a result, the clean correlation between $\xi_i$ and the angle $\theta_{23}$ is lost, which makes it difficult to probe neutrino oscillation through the LSP decays. In fact, $\lambda'_i$ have the better correlation than $\xi_i$ for the solution points. Another consequence of the tree mass suppression is that the loop correction to the sneutrino VEV should be taken properly into account for the determination of the neutrino oscillation parameters in mSUGRA models.

Let us now discuss how the trilinear R-parity violating couplings are constrained by the neutrino data. Even though $\lambda'_2/\lambda'_3$ has no analytic relation with $\theta_{23}$ for the solution points, we can obtain the following favorable ranges through the parameter scan: $|\lambda'_2/\lambda'_3|$ from the neutrino data

$$0.4 \lesssim \frac{\lambda'_2}{\lambda'_3} \lesssim 2.5 \quad \text{for} \quad \tan \beta = 3 - 15$$
$$0.3 \lesssim \frac{\lambda'_2}{\lambda'_3} \lesssim 3.3 \quad \text{for} \quad \tan \beta = 30 - 40. \quad (19)$$

It is also amusing to find the correlation between the ratio $\lambda_2/\lambda_3$ and the solar neutrino mixing angle $\theta_{12}$. Similar to the case of the atmospheric neutrino oscillation, we can get the constraints:

$$0.3 \lesssim |\lambda_1/\lambda_2| \lesssim 1.6 \quad \text{for} \quad \tan \beta = 3 - 15,$$
$$0.2 \lesssim |\lambda_1/\lambda_2| \lesssim 5.0 \quad \text{for} \quad \tan \beta = 30 - 40. \quad (20)$$

In addition, fitting the measured mass-squared values, we find the allowed regions as follows:

$$|\lambda_{1,2}|, \quad |\lambda_{2,3}'| = (0.1 - 2) \times 10^{-4}, \quad (21)$$
$$|\lambda'_1| < 2.5 \times 10^{-5}. \quad (22)$$

The above four equations are the key predictions of the mSUGRA model, some of which can be tested in the future colliders.

Since the LSP is destabilized by the R-parity violating interactions, the structure of the R-parity violating couplings shown above may be probed by observing the lepton number violating signals of the LSP decay. Based on the parameter sets constrained by the neutrino data, one can calculate the cross section for the pair production of the LSP, which can be either a neutralino or a stau, and then its decay length and branching ratios. Taking the luminosity of $1000/\text{fb/yr}$ in the future colliders, the branching ratios of the order $10^{-4} - 10^{-3}$ will be measurable as the LSP production cross sections are of the order 10-100 fb.

**Stau LSP**: When the LSP is the stau, $\tilde{\tau}_1$, it mainly decays into two leptons through the coupling $\lambda_i$. For small $\tan \beta$, $\tilde{\tau}_1$ is almost the right-handed stau $\tilde{\tau}_R$ due to the small left-right mixing mass. Then the light stau almost decays into leptons via $\lambda_i L_i L_j E^c_i$ terms in the superpotential. Thus, one can expect that the branching ratios of those decay channels depend on the parameter $\lambda_i$. In this case, the following relation holds,

$$Br(e\nu) : Br(\mu\nu) : Br(\tau\tau) \approx |\lambda_1|^2 : |\lambda_2|^2 : |\lambda_3|^2 + |\lambda_3|^2. \quad (23)$$

The corresponding decay length is much smaller than micro-meter($\mu$m) so that the stau LSP production and decay occur instantaneously. The R-parity violating signals in the linear collider will be

$$e^+ e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1 \rightarrow l^+_i l^-_j \nu \nu,$$

which are identical to the Standard Model background,

$$e^+ e^- \rightarrow W^+ W^- \rightarrow l^+_i l^-_j \nu \nu.$$

But this is a flavor independent process and can be deduced from the total number of events to establish the flavor dependent quantities $Br(l_i \nu)$. Therefore, the observation of the following relations:

$$Br(\tau\tau) = Br(e\nu) + Br(\mu\nu),$$

$$\frac{Br(e\nu)}{Br(\mu\nu)} \approx 0.1 - 2.6. \quad (24)$$

will be a strong indication of the R-parity violation.

The situation is more complicated for large $\tan \beta$. A characteristic feature of this case is that there is a sizable fraction of the stau LSP decay into top and bottom quarks, if available kinematically, which is a consequence of a large left-right stau mixing. However, the above relation becomes obscured by the large $\tan \beta$ Yukawa coupling effect. The deviation from comes from the large mixing between the stau and the charged Higgs.
Another effect would be the charged Higgs contribution to the event:

\[ e^+ e^- \to H^+ H^- \to \tau^+ \tau^- \nu \bar{\nu}. \]

Even though a clean prediction for the \( \tau \) sector is lost, we are still able to establish the lepton number violating signals in the first two generations and measure the quantity:

\[ \frac{Br(e\nu)}{Br(\mu\nu)} = \left| \frac{\lambda_1}{\lambda_2} \right|^2 \approx 0.04 - 25. \]  

**Neutralino LSP**: The lepton number violating signatures from the neutralino decay have been studied extensively in the literature as the LSP is a neutralino in the most parameter space. A characteristic feature of the neutralino LSP is that the vertex for the process \( N_1^0 \to l_i W \) is proportional to \( \xi \) which determines the tree-level neutrino mass (17). As a result, measuring the branching ratios \( Br(l_i jj) \) through either on-shell or off-shell \( W \) bosons will determine the ratio of \( |\xi|^2 \), that is,

\[ Br(ejj) : Br(\mu jj) : Br(\tau jj) = |\xi_1|^2 : |\xi_2|^2 : |\xi_3|^2. \]  

If the tree mass dominates over the loop mass, which is usually the case in the gauge-mediated supersymmetry breaking models, the neutrino mixing angles \( \theta_{23} \) and \( \theta_{13} \) can be cleanly measured in colliders. Unfortunately, this is not the case in the mSUGRA model under consideration.

As we discussed in the previous section (see Fig. 3), the tree mass has to be suppressed and thus the variables \( \xi_i \) are not correlated with the mixing angles \( \theta_{23} \) and \( \theta_{13} \) in general and \( \lambda_1' \) maintain better correlations (see Fig. 6 and 7). Although \( Br(l_i jj) \) or \( Br(l_i W) \) are proportional to \( \xi_i^2 \), the correlation with the mixing angles is lost. On the other hand, similarly to the stau LSP case, we can extract the information on \( \lambda_i \) from the measurement of \( v \ell^0 \tau^\mp \) branching ratios for small \( \tan \beta \) because the following relation,

\[ Br(ve^\mp \tau^\mp) : Br(\nu \ell^\pm \tau^\mp) : Br(\nu \tau^\mp \tau^\mp) \approx |\lambda_1|^2 : |\lambda_2|^2 : |\lambda_1|^2 + |\lambda_2|^2, \]  

holds. Likewise, if we measure the above branching ratios, the models can be tested by comparing them with the values allowed by the neutrino experimental data. Note that this is the case for small \( \tan \beta < 15 \). For large \( \tan \beta \), the relation (21) is invalidated again because of the large tau Yukawa coupling effect.

Another interesting aspect is that the parameters \( \lambda_i' \) can be probed if the neutralino LSP is heavy enough to allow the decay modes, \( l_i t b \). The main contributions to these final states come from the couplings \( \lambda_i' \) and thus one obtains the following approximate relation:

\[ Br(e \bar{t} b) : Br(\mu t \bar{b}) : Br(\tau t \bar{b}) \approx |\lambda_1'^2| : |\lambda_2'^2| : |\lambda_3'^2| \]  

which should be consistent with the predictions (20), (22) and (23). One also finds the branching ratios for \( l_i t \bar{b} \) large enough to be measured in the colliders with the integrated luminosity of 1000/fb. The branching ratios get too small if the LSP mass is more than twice top quark mass. This is because the \( \nu t \bar{b} \) mode occurring through the light Higgs exchange becomes dominating.

**SUPERSYMMETRIC TRIPLET SEESAW MODEL AT HIGH-ENERGY SCALE**

Supersymmetric extensions of the Standard Model exhibit plenty of new CP violating (CPV) phases in addition to the unique CKM phase. Although new sources of CP violation are welcome to dynamically achieve an adequate matter - antimatter asymmetry, it is known that they constitute a threat for very sensitive CP tests like those of the EDMs, at least for supersymmetric masses which are within the TeV region. In view of such constraints as well as of those coming from flavor changing neutral current processes, one can safely assume that all the terms which softly break supersymmetry are real and flavor universal at the scale where supersymmetry breaking is communicated to the visible sector. In spite of this, their renormalization group running down to the electroweak scale feels the presence of the Yukawa couplings in the superpotential which can induce flavor and CP violation in the soft breaking sector at low energy.

In addition to the logarithmically divergent RG corrections, there are additional finite contributions to the soft terms in the seesaw mechanism. In type-I seesaw, these contributions are induced by the bilinear soft-term \( \mathbf{B}_N N \tilde{N} \), associated with the Majorana mass matrix \( \mathbf{M}_N \) for the heavy singlet states \( N \). The superpotential of the type-I seesaw mechanism reads: \( W = W_0 + W_N \) with

\[ W_N = Y_N H_2 NL + \frac{1}{2} \mathbf{M}_N NN. \]  

Assuming flavor universality and CP conservation of the supersymmetric sector, finite and infinite LFV and CPV radiative corrections are induced by the new flavor structures \( (Y_N, M_N) \). Such contributions are proportional to the quantity \( Y_N^\dagger Y_N \). In spite of this dependence on the “leptonic” quantity \( Y_N \), it is worthwhile emphasizing that these contributions affect also the hadronic EDMs, a point which was missed in the literature. In particular, this applies to the finite contributions to the trilinear \( \mathbf{A}_u \) term which is corrected as

\[ \delta A_u = -\frac{1}{16\pi^2} Y_u \text{tr}(Y_N^\dagger \mathbf{B}_N Y_N), \]  

leading to quark EDMs, and thus to a nonzero neutron
EDM. The scheme yields the following ratios of EDMs:

\[ \frac{d_{\mu}}{d_e} \approx \frac{m_{\mu}}{m_e} \langle Y_N^t B_N Y_N \rangle_{22}, \quad \frac{d_d}{d_e} \approx C \frac{m_{\mu} \text{tr}(Y_N^t B_N Y_N)}{m_e \langle Y_N^t B_N Y_N \rangle_{11}}, \]

where \( C \) is a factor depending on the soft mass parameters. The above ratios are strongly model-dependent given their dependence on the combination \( Y_N^t B_N Y_N \).

A more predictive picture for LFV and CPV can emerge in the triplet seesaw case. Here the MSSM superpotential \( W_0 \) is augmented by

\[ W_T = \frac{1}{\sqrt{2}} (Y_T LTL + \lambda_1 H_1 T H_1 + \lambda_2 H_2 T H_2) + M_T T \bar{T} \]

(32)

where the supermultiplets \( T = (T^0, T^+, T^{++}), \bar{T} = (T^0, \bar{T}^-, \bar{T}^{--}) \) are in a vector-like \( SU(2)_W \times U(1)_Y \) representation, \( T \sim (3, 1) \) and \( \bar{T} \sim (3, -1) \). \( Y_T \), a complex symmetric matrix, is characterized by 6 independent moduli and 3 physical phases, while the parameters \( \lambda_2 \) and \( M_T \) can be taken to be real, and \( \lambda_1 \) is in general complex. After integrating out the triplet states at the scale \( M_T \), the resulting neutrino mass matrix becomes

\[ M_{\nu} = U^* m_D^0 U^\dagger = \frac{\sqrt{2} \lambda_2}{M_T} Y_T, \]

(33)

where \( m_D^0 \) is the diagonal neutrino mass matrix and \( U \) is the neutrino mixing matrix.

We now turn to the EDM predictions in this model. First of all, out of the three phases present in the neutrino sector, only the Dirac phase \( \delta \) may entail CP-violating effects in the LFV entries (this is due to the symmetric nature of \( Y_T \)). However, the contributions to physical observables such as the EDMs turn out to be quite suppressed in general. Indeed, due to the hermeticity of \( Y_T \), the phase of the electron EDM amplitude is always proportional to the small neutrino mixing angle \( \theta_{13} \) and to a high power of the Yukawa couplings. Only in very special circumstances with \( \theta_{13} \) close to the present experimental limit and very large \( \tan \beta \), these contributions could become sizeable.

On the other hand, a single CP phase residing in the soft term \( B_T M_T \bar{T} \bar{T} \) can play a significant role in generating non-zero EDMs, once we assume vanishing CP phases in \( \mu \) and tree-level \( A \)-terms. In such a case, the trilinear couplings \( A_e, A_d, A_u \) receive finite ‘complex’ radiative corrections at the decoupling of the heavy states \( T, \bar{T} \), exhibiting the common phase from the soft-term \( B_T \). In Fig. 1 we show the diagrammatic contribution to \( A_e \) proportional to \( Y_T^t Y_T \). Similar diagrams generate other contributions proportional to \( |\lambda_1|^2 \), relevant for \( A_e, A_d \) and \( A_u \). Thus we obtain:

\[ \delta A_e = - \frac{3}{16\pi^2} Y_e \left( Y_T^t Y_T + |\lambda_1|^2 \right) B_T, \]

\[ \delta A_d = - \frac{3}{16\pi^2} Y_d |\lambda_1|^2 B_T, \]

\[ \delta A_u = - \frac{3}{16\pi^2} Y_u |\lambda_1|^2 B_T. \]

(34)

The lepton (quark) EDMs arise from one-loop diagrams that involve the exchange of sleptons (squark) of both chiralities and Bino (gluino) (at leading order in the electroweak breaking effects). The parametric dependence of the EDMs at the leading order in the trilinear coupling goes as follows

\[ \frac{(d_e)}{e} \approx - \frac{\alpha}{4\pi c_W} m_{\nu_e} M_1 \text{Im} (\delta \hat{A}_e)_{ii} F(x_1), \]

(35)

where \( M_1 \) and \( M_3 \) are the Bino and gluino masses, respectively, the trilinear couplings have been parameterized as \( \delta A_f = Y_f \delta \hat{A}_f (f = e, u, d) \), and \( F(x) \approx x_1 = M_1^2/m_{\nu_e}^2, x_3 = M_3^2/m_{\nu_e}^2 \) is a loop function of order one. Finally, by using Eqs. (31), (33), we arrive at the peculiar result, namely the ratio of the leptonic EDMs can be predicted only in terms of the neutrino parameters:

\[ \frac{d_{\mu}}{d_e} \approx \frac{m_{\mu}}{m_e} \left[ U(m_D^0)^2 U^\dagger \right]_{22}, \quad \frac{d_{d}}{d_e} \approx \frac{m_{\mu}}{m_e} \left[ U(m_D^0)^2 U^\dagger \right]_{33}, \quad \frac{d_{\mu}}{d_e} \approx \frac{m_{\mu}}{m_e} \left[ U(m_D^0)^2 U^\dagger \right]_{22}, \]

(36)

where \( d_e \equiv (d_e)_1, d_{\mu} \equiv (d_e)_2 \) etc, and for simplicity we have assumed \( |\lambda_1|^2 \ll (Y_T^t Y_T)_{ii} \). Notice that the presence of extra CPV phases would alter the simple form of the above ratios and in general the result would be more model dependent. Regarding some numerical insight, we can consider three different neutrino mass patterns as before. For each case, the relative size of the entries in Eq. (36) is given as follows:

\[ \left[ V(m_D^0)^2 V^\dagger \right]_{ii} = \begin{cases} c_{13}^2 s_{12}^2 + s_{13}^2 c_{12}^2 & : (H) \\ c_{13}^2 s_{23}^2 & : (I) \\ c_{13}^2 s_{23}^2 & : (D) \end{cases} \]

(37)

where \( \rho = \frac{m_{\nu_e}^2}{m_{\nu_e}^2} \approx 25 \) and \( s_{ij} (c_{ij}) = \sin \theta_{ij} (\cos \theta_{ij}) \). Therefore, according to Eq. (36) and using the present
that also the branching ratios $B(\mu \rightarrow e\gamma)$ can be related in terms of only the low-energy neutrino parameters and we find

$$B(\mu \rightarrow e\gamma) : B(\tau \rightarrow e\gamma) : B(\tau \rightarrow \mu\gamma) \sim 1 : 10^{-1} : 300.$$  

(41)