Scalar Sector in the Minimal Supersymmetric 3-3-1 model.

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We consider the minimal supersymmetric extension of the 3-3-1 model and we study the mass spectra in the scalar sector of this model without the anti-sextet. We show that all our lightest scalars are in agreement with the experimental limits.

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1. Introduction

The Higgs mechanism plays a central role in gauge theories, and still remains one of the most indefinite part of the standard model (SM) \(^1\). However, we know that the SM is not considered as the ultimate theory since neither the fundamental parameters, masses and couplings, nor the symmetry pattern are predicted. These elements are merely built into the model.

On another hand, the possibility of a gauge symmetry based on the following symmetry \(SU(3)_C \otimes SU(3)_L \otimes U(1)_N\) (3-3-1)\(^2\) is particularly interesting. The main motivations to study this kind of model are:

1. The family number must be three. This result comes from the fact that the model is anomaly-free only if we have equal number of triplets and antitriplets, counting the \(SU(3)_C\) colors, and further more requiring the sum of all fermion charges to vanish. However each generation is anomalous, the anomaly cancellation occurs for the three, or multiply of three, together and not generation by generation like in the SM. Therefore triangle anomalies together with asymptotic freedom imply that the number of generations must be three and only three. This may provides a first step towards answering the flavor question;

2. It explains why \(\sin^2 \theta_W < \frac{1}{4}\) is observed at the Z-pole. This point come from the fact that in the model of Ref.\(^2\) we have that the \(U(1)_N\) and \(SU(3)_L\) coupling constants, \(g'\) and \(g\), respectively, are related by

\[
t^2 = \left(\frac{g'}{g}\right)^2 = \frac{\sin^2 \theta_W}{1 - 4\sin^2 \theta_W}.
\]

Hence, this 331 model predicts that there exists an energy scale, say \(\mu\), at which the model loses its perturbative character. The value of \(\mu\) can be found through the condition \(\sin^2 \theta_W (\mu) = 1/4\), and according to recent calculation \(\mu \approx 4 \text{ TeV}\)\(^4\).
(3) It is the simplest model that includes bileptons of both types: scalar and vectors
ones. In fact, although there are several models which include doubly charged
scalar fields, not many of them incorporate doubly charged vector bosons: this
is a particularity of the 331 model of Ref. 2;

(4) The model has several sources of CP violation. In the 331 model 2 we can
implement the violation of the CP symmetry, spontaneously 6, 7 or explicitly 8.
In models with exotic leptons it is possible to implement soft CP violation 9;

(5) The extra neutral vector boson $Z'$ conserves flavor in the leptonic but not in the
quark sector. The couplings to the leptons are leptophobic because of the sup-
pression factor $(1 - 4 \sin^2 \theta_W)^{1/2}$ but with some quarks there are enhancements
because of the factor $(1 - 4 \sin^2 \theta_W)^{-1/2}$ 10.

Recently, we have proposed the supersymmetric extensions of the 331 models
11, 12, 13. Since it is possible to define the $R$-parity symmetry in both models,
the phenomenology with $R$-parity conserved has similar features to that of the
$R$-conserving MSSM: the supersymmetric particles are pair-produced and the light-
est neutralino is the lightest supersymmetric particle (LSP). However, there are
differences between this kind of models and the MSSM with or without $R$-parity
breaking.

In the case in the model of Ref. 12, there are doubly charged scalar and vec-
tor fields. Hence, we have doubly charged charginos which are mixtures of the
superpartners of the $U$-vector boson with the doubly charged scalars. 11, 13. This im-
plies new interactions that are not present in the MSSM, for instance: $\tilde{\chi}^- \tilde{\chi}^0 U^{++}$,
$\tilde{\chi}^- \tilde{\chi}^- U^{++}$, $\tilde{l}^- l^- \tilde{\chi}^{++}$ where $\tilde{\chi}^{++}$ denotes any doubly charged chargino. Moreover,
in the chargino production, besides the usual mechanism, we have additional con-
tributions coming from the $U$-bilepton in the s-channel. Due to this fact we have an
enhancement in the cross section of production of these particles in $e^- e^-$ collisors,
such as the NLC 13.

We will also have the singly charged charginos and neutralinos, as in the MSSM,
where there are processes like $\tilde{l}^- l^+ \tilde{\chi}^0$, $\nu_L l^- \tilde{\chi}^+$, with $\tilde{l}$ denoting any slepton; $\tilde{\chi}^-$
denotes singly charged chargino and $\nu_L$ denotes any sneutrino. The only difference
is that in the MSSM there are five neutralinos while in our model there are eight
neutralinos.

In the case of model in Ref. 13 there is no double charged chargino. The mech-
anism of production of charginos and neutralinos are the same as in the MSSM,
but in the case of $R$-parity conservation there are fifteen neutralinos and six singly
charged chargino in this model.

The scalar sector of the minimal 331 model 2 was studied on. 14 While the
scalar sector of the 331 model with right handed neutrinos is given at. 15 Recently
it was presented the scalar sector of the supersymmetric 331 model with right-
handed neutrinos. 16 On this work we will analyse the scalar sector of the minimal
supersymmetric 331 model.

This paper is organized as follows. In Sec. 2 we review the minimal supersymmetry-
2. Minimal Supersymmetric 331 Model

In the nonsupersymmetric 331 model, the fermionic representation content is as follows: left-handed leptons \( L_aL \sim (1, 3, 0), a = e, \mu, \tau \); left-handed quarks \( Q_{aL} \sim (d_a, u_a, j_0) \sim (3, 3^*, -1/3) \), \( \alpha = 1, 2 \), \( Q_{3L} \sim (u_3, d_3, J)_L \sim (3, 3, 2/3) \); and in the right-handed components we have \( u^c_i \sim (3^*, 1, -2/3), d^c_i \sim (3^3, 1, 1/3), i = 1, 2, 3 \), and the exotic quarks \( j^c_i(L) \sim (3^*, 1, 4/3), j^c_1 \sim (3^*, 1, -5/3) \).

While in the supersymmetric version of this model we have to add their supersymmetric partners \( \tilde{L}_aL, \tilde{Q}_{aL}, \tilde{Q}_{3L}, \tilde{d}^c_i, \tilde{d}^c_j \), and \( \tilde{j}^c_i \).

The minimal scalar representation content is formed by three scalar triplets: \( \eta \sim (1, 3, 0) = (\eta^0, \eta^+_1, \eta^+_2)^T \); \( \rho \sim (1, 3, +1) = (\rho^+, \rho^0, \rho^{++})^T \) and \( \chi \sim (1, 3, 1) = (\chi^-, \chi^0, \chi^0)^T \), and their supersymmetric partner \( \tilde{\eta}, \tilde{\rho} \), and \( \tilde{\chi} \). However, we have to introduce the followings extras scalars \( \eta', \rho', \chi' \) and their higgsinos \( \tilde{\eta}', \tilde{\rho}' \) and \( \tilde{\chi}' \).

In the nonsupersymmetric 331 model to give arbitrary mass to the leptons we have to introduce one scalar antisextet \( S \sim (1, 6^*, 0) \). We can avoid the introduction of the antisextet by adding a charged lepton transforming as a singlet. Notwithstanding, here we will omit both the antisextet and the exotic lepton, because we have showed in \cite{17} that in this model the \( R \)-violating interactions give the correct mass spectra to \( e, \mu \), and \( \tau \).

The complete set of fields in the minimal supersymmetric 331 model (MSUSY331) has been given in \cite{12, 13}, see these articles to the complete Lagrangian of the model. On this article we will write only the lagrangian necessary to construct the mass spectra in the scalar sector.

The gauge sector’s lagrangian is writting as

\[
\mathcal{L}^{gauge} = \frac{1}{4} \int d^2 \theta \text{Tr}[W_C W_C] + \frac{1}{4} \int d^2 \theta \text{Tr}[W_L W_L] + \frac{1}{4} \int d^2 \theta W' W' \\
+ \frac{1}{4} \int d^2 \theta \text{Tr}[\tilde{W}_C \tilde{W}_C] + \frac{1}{4} \int d^2 \theta \text{Tr}[\tilde{W}_L \tilde{W}_L] + \frac{1}{4} \int d^2 \theta \tilde{W}' \tilde{W}'
\]

while the lagrangian of the scalar sector is written as

\[
\mathcal{L}^{scalar} = \int d^4 \theta \left[ \hat{\eta} e^{2g \hat{V}} \tilde{\eta} + \hat{\rho} e^{(2g \hat{V} + g' \hat{V}')} \tilde{\rho} + \hat{\chi} e^{(2g \hat{V} - g' \hat{V}')} \tilde{\chi} + \hat{\eta}' e^{2g \hat{V}} \tilde{\eta}' + \hat{\rho}' e^{(2g \hat{V} - g' \hat{V}')} \tilde{\rho}' + \hat{\chi}' e^{(2g \hat{V} + g' \hat{V}')} \tilde{\chi}' \right] + \int d^2 \theta W + \int d^2 \theta \tilde{W}
\]

here \( g \) and \( g' \) are the gauge coupling constants of \( SU(3) \) and \( U(1) \) respectively and \( W \) is the superpotential of the model given by \( W = W_2 + W_3 \).
From the equation described above we can construct

\[ W_2 = \mu_0 L_L^3 + \mu_\ell L_L^3 + \mu \phi \phi' + \mu \chi \chi', \]

\[ W_3 = \lambda_{abc} L_L^3 b_L^3 + \lambda_{2abc} L_L^3 b_L^3 + \lambda_{3a} L_L^3 \phi + f_1 \phi \phi' + f_2 \phi' \phi'' \]

\[ + \kappa_{1a} \bar{Q}_a L_L^3 \phi + \kappa_{2a} \bar{Q}_a L_L^3 \phi + \kappa_{3a} \bar{Q}_a L_L^3 \phi' \]

\[ + \epsilon \xi_{ijk} \bar{d}_i L_L^3 c_j + \epsilon \xi_{ijs} \bar{u}_i L_L^3 c_j + \epsilon \xi_{ijs} \bar{d}_i L_L^3 c_j. \]

(4)

The superpotential is written explicitly as

\[ W = \mathcal{W} F + \mathcal{W} W \]

The coefficients \( \mu_0, \mu_\ell, \mu_\phi \) and \( \mu \) have mass dimension, while all the coefficients in \( W_3 \) are dimensionless.

To get the scalar potential of our model we have to eliminate the auxiliary fields \( F \) and \( D \) that appear in our model. We are going to pick up the \( F \) and \( D \)-terms, from Eqs. (4.2, 4.3), we get

\[ \mathcal{L}_D^{\text{gauge}} = \frac{1}{2} D^2 + \frac{1}{2} D D, \]

\[ \mathcal{L}_F^{\text{scalar}} = |F_0|^2 + |F_1|^2 + |F_2|^2 + |F_3|^2, \]

\[ \mathcal{L}_W^{\text{scalar}} = \frac{g}{2} [\bar{\eta} \lambda^a \phi + \bar{\phi} \lambda^a \lambda' - \bar{\phi} \lambda^a \lambda' - \bar{\chi} \lambda^a \chi'] D^a \]

\[ + \frac{g'}{2} [\bar{\phi} \phi - \bar{\chi} \chi - \bar{\chi} \chi] D, \]

\[ \mathcal{L}_W^{W} = \frac{\mu_0}{2} (\eta F_1 + \eta F_2 + \eta F_3 + \eta F_4) + \frac{\mu_\ell}{2} (\phi F_1 + \phi F_2 + \phi F_3 + \phi F_4) \]

\[ + \frac{\mu_\phi}{2} (\lambda^a \phi + \lambda^a \phi + \lambda^a \phi + \lambda^a \phi), \]

\[ \mathcal{L}_W^{W_3} = \frac{1}{3} [f_1 \epsilon (F_0 \chi \eta + \rho F_0 \chi \eta + \rho F_0 \chi \eta + \bar{F}_0 \chi \eta + \bar{F}_0 \chi \eta + \bar{F}_0 \chi \eta)] . \]

(5)

From the equation described above we can construct

\[ \mathcal{L}_F = \mathcal{L}_W^{\text{scalar}} + \mathcal{L}_W^{W} + \mathcal{L}_W^{W_3} \]

\[ = |F_0|^2 + |F_1|^2 + |F_2|^2 + |F_3|^2 + |F_4|^2 \]

\[ + \frac{1}{2} [\mu_\ell (\eta F_1 + \eta F_2 + \eta F_3 + \eta F_4) + \mu_\phi (\phi F_1 + \phi F_2 + \phi F_3 + \phi F_4) + \mu_\chi (\lambda^a \phi + \lambda^a \phi + \lambda^a \phi + \lambda^a \phi)] + \frac{1}{3} [f_1 \epsilon (F_0 \chi \eta + \rho F_0 \chi \eta + \rho F_0 \chi \eta + \bar{F}_0 \chi \eta + \bar{F}_0 \chi \eta + \bar{F}_0 \chi \eta)] . \]

\[ \mathcal{L}_D = \mathcal{L}_D^{\text{gauge}} + \mathcal{L}_D^{\text{scalar}} \]

\[ = \frac{1}{2} D^2 + \frac{1}{2} D D + \frac{g}{2} [\bar{\eta} \lambda^a \eta + \bar{\phi} \lambda^a \lambda' - \bar{\phi} \lambda^a \lambda'] \]

\[ - \bar{\chi} \lambda^a \chi' D^a + \frac{g'}{2} [\bar{\phi} \phi - \bar{\chi} \chi - \bar{\chi} \chi] D. \]
We will now show that these fields can be eliminated through the Euler-Lagrange equations
\[
\frac{\partial \mathcal{L}}{\partial \phi} - m^2 \frac{\partial \mathcal{L}}{\partial (\partial_m \phi)} = 0 ,
\]
where \( \phi = \eta, \rho, \chi, \eta', \rho', \chi' \). Formally auxiliary fields are defined as fields having no kinetic terms. Thus, this definition immediately yields that the Euler-Lagrange equations for auxiliary fields simplify to \( \frac{\partial \mathcal{L}}{\partial \phi} = 0 \).

Applying these simplified equations to various auxiliary \( F \)-fields yields the following relations
\[
\begin{align*}
\tilde{F}_\eta &= - \left( \frac{\mu_\eta}{2} \eta' + \frac{f_1}{3} \epsilon \rho \chi \right) ; \\
\tilde{F}_\rho &= - \left( \frac{\mu_\rho}{2} \rho' + \frac{f_1}{3} \epsilon \chi \eta \right) ; \\
\tilde{F}_\chi &= - \left( \frac{\mu_\chi}{2} \chi' + \frac{f_1}{3} \epsilon \rho \eta \right) ; \\
\tilde{F}_{\eta'} &= - \left( \frac{\mu_\eta'}{2} \eta' + \frac{f_1}{3} \epsilon \rho' \chi' \right) ; \\
\tilde{F}_{\rho'} &= - \left( \frac{\mu_\rho'}{2} \rho' + \frac{f_1}{3} \epsilon \chi \eta' \right) ; \\
\tilde{F}_{\chi'} &= - \left( \frac{\mu_\chi}{2} \chi' + \frac{f_1}{3} \epsilon \rho' \eta' \right) ,
\end{align*}
\]
using these equations, we can rewrite Eq. (8) as
\[
\mathcal{L}_F = - \left| \tilde{F}_\eta \right|^2 + \left| \tilde{F}_\rho \right|^2 + \left| \tilde{F}_\chi \right|^2 + \left| \tilde{F}_{\eta'} \right|^2 + \left| \tilde{F}_{\rho'} \right|^2 + \left| \tilde{F}_{\chi'} \right|^2 .
\]
If we perform the same program to \( D \)-fields we get
\[
\begin{align*}
D^a &= - g \left[ \bar{\eta} \lambda^a \eta + \bar{\rho} \lambda^a \rho + \bar{\chi} \lambda^a \chi - \bar{\eta}' \lambda^a \eta' - \bar{\rho}' \lambda^a \rho' - \bar{\chi}' \lambda^a \chi' \right] , \\
D &= - g' \left[ \bar{\rho} \rho - \bar{\chi} \chi - \bar{\rho}' \rho' + \bar{\chi}' \chi' \right] ,
\end{align*}
\]
According with Eq. (10)
\[
\mathcal{L}_D = - \frac{1}{2} \left( D^a D^a + D D \right) .
\]

3. The scalar potential

The pattern of the symmetry breaking of the model is given by the following scheme
\[
\begin{array}{c}
\text{MSUSY331} \xrightarrow{\text{electro}} \text{SU(3)}_C \otimes \text{SU(3)}_L \otimes \text{U(1)}_N (\lambda \lambda') \text{SU(3)}_C \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y \xrightarrow{\text{phys}} \text{SU(3)}_C \otimes \text{U(1)}_Q \\
\langle \rho, \eta, \rho', \eta' \rangle
\end{array}
\]
From this pattern of the symmetry breaking comes the constraint
\[
V_\eta^2 + V_\rho^2 = (246 \text{ GeV})^2
\]
coming from $M_W$, where, we have defined $V_\eta^2 = v_\eta^2 + \bar{v}_\eta^2$ and $V_\rho^2 = v_\rho^2 + \bar{v}_\rho^2$.

The scalar potential is written as

$$V_{MSUSY331} = V_D + V_F + V_{\text{soft}}$$

where

$$V_D = -\mathcal{L}_D = \frac{1}{2} (D^a D^a + DD)$$

$$= \frac{g^2}{2} (\bar{\rho} \rho - \bar{\rho}' \rho' - \bar{\chi} \chi + \chi' \chi')^2 + \frac{g^2}{8} \sum_{i,j} (\bar{\eta}_i \lambda^a_{ij} \eta_j + \bar{\rho}_i \lambda^a_{ij} \rho_j + \bar{\chi}_i \lambda^a_{ij} \chi_j$$

$$- \bar{\eta}_i \lambda^a_{ij} \eta_j - \bar{\rho}_i \lambda^a_{ij} \rho_j - \bar{\chi}_i \lambda^a_{ij} \chi_j)^2$$

$$V_F = -\mathcal{L}_F = \sum_m \bar{F}_m F_m$$

$$= \sum_{i,j,k} \left[ \frac{\mu_\eta}{2} \bar{\eta}_i^2 + \frac{f_1}{3} \epsilon_{ijk} \rho_j \chi_k \right]^2 + \left[ \frac{\mu_\rho}{2} \rho_i^2 + \frac{f_1}{3} \epsilon_{ijk} \chi_j \eta_k \right]^2 + \left[ \frac{\mu_\chi}{2} \chi_i^2 + \frac{f_1}{3} \epsilon_{ijk} \rho_j \eta_k \right]^2$$

$$V_{\text{soft}} = -\mathcal{L}_{\text{soft}}$$

$$= m_\eta^2 \bar{\eta} \eta + m_\rho^2 \bar{\rho} \rho + m_\chi^2 \bar{\chi} \chi + m_\eta' \bar{\eta}' \eta' + m_\rho' \bar{\rho}' \rho' + m_\chi' \bar{\chi}' \chi'$$

$$+ [k_1 \epsilon \rho \chi \eta + k_1' \epsilon \rho' \chi' \eta' + H.c.]$$

where, $m_\eta^2, m_\rho^2, m_\chi^2, m_\eta', m_\rho', m_\chi'$, and $k_1$ and $k_1'$ have mass dimension.

All the six neutral scalar components $\eta^0, \rho^0, \chi^0, \eta'^0, \rho'^0, \chi'^0$ gain non-zero vacuum expectation values. Making a shift in the neutral scalars as

$$< \eta > = \begin{pmatrix} v_\eta + H_\eta + iF_\eta \\ 0 \\ 0 \end{pmatrix}, \quad < \eta' > = \begin{pmatrix} v_{\eta'} + H_{\eta'} + iF_{\eta'} \\ 0 \\ 0 \end{pmatrix},$$

$$< \rho > = \begin{pmatrix} v_\rho + H_\rho + iF_\rho \\ 0 \\ 0 \end{pmatrix}, \quad < \rho' > = \begin{pmatrix} v_{\rho'} + H_{\rho'} + iF_{\rho'} \\ 0 \\ 0 \end{pmatrix},$$

$$< \chi > = \begin{pmatrix} v_\chi + H_\chi + iF_\chi \\ 0 \\ 0 \end{pmatrix}, \quad < \chi' > = \begin{pmatrix} v_{\chi'} + H_{\chi'} + iF_{\chi'} \\ 0 \\ 0 \end{pmatrix}.$$  (16)

### 4. Constraint Equations

Here in this section we give the constraint equations, due to the requirement the potential to reach a minimum at the chosen VEV’s. We get this equation requiring
that in the shifted potential the linear terms in fields must be absent

\[ 0 = \frac{g^2}{12}(2v^2 - 2v^2 - v^2 + v^2 - v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{f^2}{18}(v^2 + v^2) \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} + \frac{1}{6v^2}(f_1 \frac{\mu v}{v} v y) + f_1 \frac{\mu v}{v} v y \]

\[ + \frac{f^2}{18}(v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{k_1}{\sqrt{2}} \frac{v y}{v} + \frac{f_1}{6v^2}(f_1 \frac{\mu y}{v} v y) + f_1 \frac{\mu y}{v} v y \]

\[ - \frac{f^2}{6v^2} v y \]

\[ 0 = \frac{g^2}{12}(2v^2 - 2v^2 - v^2 + v^2 - v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{f^2}{18}(v^2 + v^2) \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} + \frac{1}{6v^2}(f_1 \frac{\mu v}{v} v y) + f_1 \frac{\mu v}{v} v y \]

\[ + \frac{f^2}{18}(v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{f^2}{18}(v^2 + v^2) \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} + \frac{1}{6v^2}(f_1 \frac{\mu v}{v} v y) + f_1 \frac{\mu v}{v} v y \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} \]

\[ 0 = \frac{g^2}{12}(2v^2 - 2v^2 - v^2 + v^2 - v^2 + v^2) + \frac{g^2}{2}(v^2 - v^2 + v^2 + v^2) \]

\[ + \frac{f^2}{18}(v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{f^2}{18}(v^2 + v^2) \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} + \frac{1}{6v^2}(f_1 \frac{\mu v}{v} v y) + f_1 \frac{\mu v}{v} v y \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} \]

\[ 0 = \frac{g^2}{12}(2v^2 - 2v^2 - v^2 + v^2 - v^2 + v^2) + \frac{g^2}{2}(v^2 - v^2 - v^2 + v^2) \]

\[ + \frac{f^2}{18}(v^2 + v^2) + m^2 + \frac{1}{4} \mu^2 + \frac{f^2}{18}(v^2 + v^2) \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} + \frac{1}{6v^2}(f_1 \frac{\mu v}{v} v y) + f_1 \frac{\mu v}{v} v y \]

\[ + \frac{k_1}{\sqrt{2}} \frac{v v y}{v} \]

The mass matrices, thus, can be calculated, using

\[ M_{ij}^{2} = \frac{\partial^2 V_{\text{MSUSY-331}}}{\partial \phi_i \partial \phi_j} \]

evaluated at the chosen minimum, where \( \phi_i \) are the scalars of our model described above.

For the sake of simplicity, here we assume that vacuum expectation values (VEVs) are real. This means that the CP violation through the scalar exchange
is not considered in this work. In literature, a real part $H$ is called CP-even scalar or scalar, and an imaginary one $F$ - CP-odd scalar or pseudoscalar field. In this paper we call them scalar and pseudoscalar, respectively.

5. Mass Spectrum general case

We will use below the following set of parameters in the scalar potential:

$$f_1 = 1.2, \quad f_1' = 10^{-6}, \quad \text{(dimensionless)}$$

and

$$-k_1 = k_1' = 10, \quad -\mu_\eta = \mu_\rho = \mu_\chi = 1000, \quad \text{(in GeV)},$$

we also used the constraint in Eq.(13). Assuming that $v_\eta = 20, \quad v_\chi = 1000, \quad v_\eta' = v_\rho' = v_\chi' = 1$ in GeV, the value of $v_\rho$ is fixed by the constraint in Eq.(13).

Diagonalizing the matrices we got the mass eigenstates. On the case of the neutral real scalar, Eq.(A.1), we got in (GeV):

$$m_{H^0_1} = 110.5, \quad m_{H^0_2} = 249, \quad m_{H^0_3} = 701.3,$$

$$m_{H^0_4} = 1220, \quad m_{H^0_5} = 1683.9, \quad m_{H^0_6} = 4951.7,$$

in fig.(1) we show the behavior of the lightest scalar $H^0_1$ as a function of $v_\chi$, although in fig.(2) we have done the same analyses as function of $v_\chi$ and $v_\chi'$. The current 95% CL mass bound on the lightest scalar at MSSM is 89.8GeV [19].

To the neutral imaginary, Eq.(B.1), we got the following values in (GeV)

$$m_{A_1} = 248.6, \quad m_{A_2} = 701.3,$$

$$m_{A_3} = 1683.9, \quad m_{A_4} = 4951.7,$$

in fig.(3), we show the behavior of the lightest pseudoscalar $A_1$ as a function of $v_\chi$, although in fig.(4) we have done the same analyses as function of $v_\chi$ and $v_\chi'$. The current 95% CL mass bound on the lightest pseudoscalar at MSSM is 90.3GeV [19].

On the single charged sector considering the first matrix, Eq.(C.2), the physical states have the following masses in (GeV)

$$m_{H^+_1} = 253, \quad m_{H^+_2} = 1684.9, \quad m_{H^+_3} = 4951.3$$

in fig.(5), we show the behavior of the eigenvalues of $m_{H^+_1}$ (solid line), $m_{H^+_2}$ (dashed line), as function of $v_\chi$ although in fig.(6) we see the dependence of $m_{H^+_1}$ in terms of $v_\chi$ and $v_\chi'$. The current 95% CL mass bound on the lightest charged scalar at MSSM is 79.3GeV [19].

While in the second matrix, Eq.(C.3), we get

$$m_{H^+_4} = 308.4, \quad m_{H^+_5} = 773.4, \quad m_{H^+_6} = 4940.8$$

in fig.(7), we see the dependence of $m_{H^+_4}$ in terms of $v_\chi$ and $v_\chi'$. The current 95% CL mass bound on the lightest charged scalar at MSSM is 79.3GeV [19].

While in the double charged sector, Eq.(D.1), there are

$$m_{H^{++}_1} = 170.7, \quad m_{H^{++}_2} = 770.6, \quad m_{H^{++}_3} = 1650.7.$$
in fig. (8) we show the behaviour of \( H_1^{++} \) as function of \( v_\chi \), although in fig. (9), we see the dependence of \( m_{H_1^{++}} \) in terms of \( v_\chi \) and \( v'_\chi \). The current 95\% CL mass bound on the lightest doubly-charged scalar is 95 GeV and 100 GeV were obtained for left-right symmetric models (the exact limits depend on the leptons flavors) \[19\].

6. Conclusions

On this article we constructed all the spectrum from the scalar sector of the minimal supersymmetric 3-3-1 model, and we also show that all our lightest scalars are in agreement with the experimental limits, as discussed above.

We want also to recall the attention that our results are in agreement with those presented on \[12\]-\[20\].

References

7. Graphics

Appendix A. Scalar in MSUSY331.

Calculating Eq. (14) with the help of Eqs. (16,17) and using as base the following set of scalars $H_\eta, H_\rho, H_\chi, H_\eta', H_\rho', H_\chi'$, we get following mass matrix to the Real
Neutral Fields

\begin{align*}
M_{11} &= \frac{g^2 v_\eta^2}{3} + \frac{1}{6\sqrt{2} v_\eta} (f_1 v_\rho' \mu_\rho v_\chi - 6 k_1 v_\rho v_\chi - f'_1 \mu_\eta v'_\rho v'_\chi + f_1 \mu_\chi v_\rho v'_\chi), \\
M_{12} &= -\frac{g^2 v_\eta v_\rho}{6} + \frac{1}{9\sqrt{2}} (\sqrt{2} f_1^2 v_\eta v_\rho + 9 k_1 v_\chi - \frac{3}{2} \mu_\chi v'_\chi), \\
M_{13} &= -\frac{g^2 v_\eta v_\rho}{6} + \frac{1}{9\sqrt{2}} (9 k_1 v_\rho - 3 f_1 \mu_\rho v'_\rho + \sqrt{2} f_1^2 v_\eta v_\chi), \\
M_{14} &= -\frac{g^2 v_\eta v'_\eta}{3}, \\
M_{15} &= \frac{g^2 v_\eta v'_\rho}{6} - \frac{1}{6\sqrt{2}} (\mu_\rho v_\chi - \mu_\eta v'_\chi), \\
M_{16} &= \frac{g^2 v_\eta v'_\eta}{6} + \frac{1}{6\sqrt{2}} (f'_1 \mu_\eta v'_\rho - f_1 \mu_\chi v_\rho).
\end{align*}

Fig. 3. The eigenvalues of $m_{A_1}$ as function of $v_\chi$.

Fig. 4. $m_{A_1} \times v_\chi \times v'_\chi$. 
This matrix has no Goldstone bosons and six mass eigenstates, which we denote as $H_1^0, H_2^0, H_3^0, H_4^0, H_5^0, H_6^0$. 
Fig. 7. \( m_{H^2} \times v_\chi \times v'_\chi \).

Fig. 8. The eigenvalues of \( m_{H^1}^{++} \) as function of \( v_\chi \).
Fig. 9. $m_{H^+} \times v_{\chi} \times v_{\chi'}$.

Appendix B. Pseudoscalar in MSUSY331.

On this case using the base given by $F_\eta, F_\rho, F_\chi, F_\eta', F_\rho', F_\chi'$, the mass matrix, with the help of Eq. (17), is given by:

\[
\begin{align*}
M_{11} &= \frac{1}{6\sqrt{2}v_\eta}(f_1\mu_\rho v_\rho v'_\chi - 6k_1v_\rho v_\chi), \\
M_{12} &= \frac{1}{6\sqrt{2}}(f_1\mu_\chi - 6f_1v_\chi), \\
M_{13} &= \frac{1}{6\sqrt{2}}(f_1\mu_\rho v'_\rho - 6k_1v_\rho), \\
M_{14} &= 0, \\
M_{15} &= \frac{1}{6\sqrt{2}}(f_1\mu_\eta v'_\chi - f'_1\mu_\rho v_\chi), \\
M_{16} &= \frac{1}{6\sqrt{2}}(f'_1\mu_\eta v_\rho - f_1\mu_\chi v_\rho), \\
M_{22} &= \frac{1}{6\sqrt{2}v_\rho}(6k_1v_\eta v_\chi - f_1\mu_\eta v'_\eta v_\chi + f'_1\mu_\rho v'_\rho v_\chi + f_1\mu_\chi v_\eta v'_\chi), \\
M_{23} &= -\frac{1}{6\sqrt{2}}(6k_1v_\eta + f_1\mu_\eta v'_\eta), \\
M_{24} &= \frac{1}{6\sqrt{2}}(f_1\mu_\eta v_\chi - f'_1\mu_\rho v'_\chi), \\
M_{25} &= 0, \\
M_{26} &= -\frac{1}{6\sqrt{2}}(f'_1\mu_\rho v_\eta + f_1\mu_\chi v_\eta), \\
M_{33} &= \frac{1}{6\sqrt{2}v_\chi}(-6f_1k_1v_\eta v_\rho - f_1\mu_\eta v'_\eta v_\rho + f_1\mu_\rho v_\eta v'_\rho + f'_1\mu_\rho v'_\rho), \\
M_{34} &= \frac{1}{\sqrt{2}}(f_1\mu_\eta v_\rho - f'_1\mu_\chi v'_\rho), \\
M_{35} &= -\frac{1}{6\sqrt{2}}(f_1\mu_\rho v_\eta + f_1\mu_\chi v'_\eta), \\
M_{36} &= 0, \\
M_{44} &= \frac{1}{6\sqrt{2}v_\eta'}(-6f_1k_1v_\eta v_\rho v_\chi - 6k_1'v_\rho v_\chi - f_1\mu_\rho v_\eta v'_\rho + f'_1\mu_\rho v_\rho v'_\rho), \\
M_{45} &= 0, \\
M_{46} &= \frac{1}{6\sqrt{2}}(-6k_1'v_\rho + f_1\mu_\rho v_\rho), \\
M_{55} &= \frac{1}{6\sqrt{2}v_\eta'}(f_1\mu_\rho v_\eta v_\chi + f'_1\mu_\chi v'_\eta v_\chi - 6k_1'v_\eta' v_\chi - f'_1\mu_\eta v_\eta v'_\chi). 
\end{align*}
\]
This mass matrix has two Goldstone bosons, $G_1, G_2$, and four mass eigenstates, $A_1, A_2, A_3, A_4$. 
Appendix C. Single charged fields in MSUSY331.

On this case the basis is given by $\eta_1^+, \rho_1^+, \eta_1^+, \rho_2^+, \chi^+, \eta_2^+, \chi^+$, with the help of Eq. (17), we get

$$M_{15} = M_{16} = M_{17} = M_{18} = 0,$$

$$M_{25} = M_{26} = M_{27} = M_{28} = 0,$$

$$M_{35} = M_{36} = M_{37} = M_{38} = 0,$$

$$M_{45} = M_{46} = M_{47} = M_{48} = 0,$$

$$M_{11} = -\frac{f_1}{18} \sqrt{2} + \frac{g_1}{4} (v_\eta^2 + v_\rho^2 - v_\nu^2) - \frac{\sqrt{2}}{12} (6k_1 v_\eta v_\nu + f_1 v_\rho v_\chi + f_2 v_\rho v_\chi')$$

$$+ \frac{\sqrt{2} f_1}{12v_\eta} (v_\rho v_\mu v_\chi + v_\mu v_\chi' v_\eta),$$

$$M_{12} = -\frac{f_1^2}{18} v_\eta v_\rho + \frac{g_1^2}{4} v_\eta v_\rho - \frac{k_1}{\sqrt{2}} v_\chi + \frac{f_1}{6\sqrt{2}} \mu v_\chi',$$

$$M_{13} = -\frac{g_1^2}{4} v_\eta v_\rho,$$

$$M_{14} = -\frac{g_1^2}{4} v_\eta v_\rho + \frac{f_1}{6\sqrt{2}} \mu v_\chi - \frac{f_1'}{6\sqrt{2}} \mu v_\chi',$$

$$M_{22} = \frac{f_1^2}{18} \eta + \frac{\sqrt{2} f_1}{12} \left( \frac{\mu v_\nu v_\eta'}{v_\rho} - \frac{\mu v_\nu v_\eta}{v_\rho} \right) + \frac{g_1^2}{4} (v_\eta^2 - 2v_\rho^2 + v_\nu^2) - \frac{\sqrt{2} k_1}{6} v_\eta v_\chi - \frac{\sqrt{2} f_1}{36} \mu v_\chi v_\eta,$$

$$M_{23} = -\frac{g_1^2}{4} v_\eta v_\rho - \frac{f_1}{6\sqrt{2}} \mu v_\chi + \frac{f_1'}{6\sqrt{2}} \mu v_\chi',$$

$$M_{24} = -\frac{g_1^2}{4} v_\eta v_\rho,$$

$$M_{33} = -\frac{f_1^2}{18} v_\eta' + \frac{g_1^2}{4} (v_\eta^2 - v_\rho^2 - v_\nu^2) + \frac{\sqrt{2} f_1}{12} \left( \frac{\mu v_\rho v_\eta'}{v_\eta} + \frac{\mu v_\rho v_\eta}{v_\eta} \right) - \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\chi,$$

$$M_{34} = -\frac{f_1^2}{18} v_\eta v_\rho + \frac{g_1^2}{4} v_\eta v_\rho + \frac{f_1}{6\sqrt{2}} \mu v_\chi - \frac{k_1'}{\sqrt{2}} v_\chi',$$

$$M_{44} = -\frac{f_1^2}{18} v_\eta + \frac{g_1^2}{4} (v_\eta^2 - v_\eta^2 + v_\eta^2) + \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\eta v_\chi - \frac{k_1'}{\sqrt{2}} v_\chi' + \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\eta,$$

$$M_{55} = -\frac{k_1}{\sqrt{2}} v_\rho v_\mu + \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\eta v_\chi - \frac{f_1^2}{12} v_\rho^2 + \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\eta v_\chi' + \frac{\sqrt{2} f_1}{12} \mu v_\rho v_\eta,$$

$$M_{56} = -\frac{k_1}{\sqrt{2}} v_\rho + \frac{f_1}{6\sqrt{2}} \mu v_\rho v_\chi - \frac{f_1^2}{18} v_\eta v_\chi + \frac{g_1^2}{4} v_\eta v_\chi,$$

$$M_{57} = -\frac{g_1^2}{4} v_\eta v_\rho,$$
Analysing the matrix elements above, we conclude that the mixing occurs in the set $\eta_1^+, \rho^+, \eta_1'^+, \rho'^+$ and in the set of $\eta_2^+, \chi^+, \eta_2'^+, \chi'^+$ and it is in agreement with the results presented in [14].

This results mean that in the most general case, the singly charged scalars are obtained by diagonalizing one $8 \times 8$ matrix. Ignoring the null elements mixing, the matrix presented above decompose into two series of $4 \times 4$ matrices.

The mixing between the particles $\eta_1^+, \rho^+, \eta_1'^+, \rho'^+$ is giving by the following matrix

$$
\begin{pmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{pmatrix},
$$

(C.2)

using the values given at Eq.(C.1) we get one Goldstone boson, $G_1^+$, and three mass eigenstates $H_1^+, H_2^+, H_3^+$. While the second mixing is given by

$$
\begin{pmatrix}
M_{55} & M_{56} & M_{57} & M_{58} \\
M_{65} & M_{66} & M_{67} & M_{68} \\
M_{75} & M_{76} & M_{77} & M_{78} \\
M_{85} & M_{86} & M_{87} & M_{88}
\end{pmatrix},
$$

(C.3)

This matrix, with the help of Eq.(C.1), has one Goldstone, $G_2^+$, three mass eigenstates $H_4^+, H_5^+, H_6^+$. 
Appendix D. Double charged fields in MSUSY331.

On this case, with the help of Eq. (17), our results are

\[
\begin{align*}
\mathcal{M}_{11} &= -\frac{k_1}{\sqrt{2}} v_\eta v_\chi - \frac{\sqrt{2} f_1}{12} \left( \frac{\mu_\eta v_\eta v_\chi}{v_\rho} - \frac{\mu_\rho v_\rho v_\chi}{v_\rho} - \frac{\mu_\chi v_\eta v_\chi}{v_\rho} \right) - \frac{f_1^2}{12} v_\chi^2 + \frac{g^2}{4} (v_\rho^2 + v_\chi^2) , \\
\mathcal{M}_{12} &= -\frac{k_1}{\sqrt{2}} v_\eta - \frac{f_1}{6 \sqrt{2}} \mu_\eta v_\eta - \frac{g^2}{4} v_\rho v_\chi , \\
\mathcal{M}_{13} &= -\frac{g^2}{4} v_\rho v_\rho , \\
\mathcal{M}_{14} &= \frac{f_1}{6 \sqrt{2}} \mu_\rho v_\rho + \frac{f_1}{6 \sqrt{2}} \mu_\chi v_\eta - \frac{g^2}{4} v_\rho v_\chi , \\
\mathcal{M}_{22} &= -\frac{k_1}{\sqrt{2}} v_\eta v_\chi + \frac{\sqrt{2} f_1}{12} (\mu_\rho v_\rho v_\eta - \mu_\eta v_\rho v_\chi) + \frac{\sqrt{2} f_1}{12} \mu_\chi v_\eta v_\chi - \frac{f_1 f_2}{12} v_\rho^2 + \frac{g^2}{6} (v_\rho^2 + v_\chi^2 , \\
\mathcal{M}_{23} &= \frac{f_1}{6 \sqrt{2}} \mu_\chi v_\eta + \frac{f_1}{6 \sqrt{2}} \mu_\rho v_\rho - \frac{g^2}{4} v_\rho v_\chi , \\
\mathcal{M}_{24} &= -\frac{g^2}{4} v_\chi v_\chi , \\
\mathcal{M}_{33} &= -\frac{k_1}{\sqrt{2}} v_\eta v_\chi + \frac{\sqrt{2} f_1}{12} \mu_\rho v_\rho v_\eta + \frac{\sqrt{2} f_1}{12} \mu_\chi v_\eta v_\chi + \frac{f_1 f_2}{12} v_\rho v_\chi + \frac{g^2}{6} (v_\rho^2 + v_\chi^2) , \\
\mathcal{M}_{34} &= -\frac{k_1}{\sqrt{2}} v_\eta - \frac{f_1}{6 \sqrt{2}} \mu_\rho v_\rho - \frac{f_1}{12} \mu_\eta v_\eta - \frac{g^2}{4} v_\rho v_\chi , \\
\mathcal{M}_{44} &= -\frac{k_1}{\sqrt{2}} v_\eta v_\chi + \frac{\sqrt{2} f_1}{12} \mu_\rho v_\rho v_\eta - \frac{\sqrt{2} f_1}{12} \mu_\chi v_\eta v_\chi + \frac{f_1 f_2}{12} v_\rho v_\chi - \frac{f_1^2}{12} v_\rho^2 \\
&+ \frac{g^2}{6} (v_\rho^2 + v_\chi^2 + v_\eta^2). \quad (D.1)
\end{align*}
\]

This mass matrix has one Goldstone boson, \(G_1^{++}\), and three mass eigenstates \(H_1^{++}, H_2^{++}, H_3^{++}\).